Fracture & Fatigue

Analysis of Fracture & Fatigue Using Lagrangians using a Simplified Stress Wave Unloading Model

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- This presentation is a summary of:
  - Analysis of Fracture and Fatigue using Lagrangian Mechanics
  - arXiv 1212.6704
  - www.neal-Sturgess.com

- Concentrating on:
  - The Griffith’s crack
  - Long fast cracks
  - Plane Strain fatigue of elastic-plastic materials

- Retrodiction, validation & Prediction
- References in extended abstract

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Recap on Classical Mechanics

• Step 1: Principle of Least Action
• Step 2: Noether’s Theorems
• Step 3: Symmetries (Conservation Laws)
• Step 4: Hamilton’s principle
• Step 4: Euler – Lagrange

Equation of motion
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Notes

• In my view if you do not understand this process you do not understand mechanics

• Classical mechanics is far more than Newton!

• Only taught to engineers in 5 Universities in UK!

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Griffith’s Stationary Crack
Griffith used a Lagrangian but for a propagating crack the physics are different
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Notes:

• Griffith’s modelled a stationary crack, and used a Lagrangian, but did not use Euler-Lagrange – I do not know why; however the physics of moving cracks are entirely different, see Schardin’s picture.

• You can see the von-Mises plastic zones, and the stress waves emanating every time a crack tip bond breaks.

• These stress waves are unloading waves and release the energy from the body to drive the crack.

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Energy released by stress wave
= SE per unit volume x volume unloaded

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Euler-Lagrange for eqn. of motion

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial x} \right) - \frac{\partial L}{\partial x} = 0 \]

\[ L = H - M \]

H = the energy required to create new crack surfaces
M = the strain energy released from the body

The crack acceleration is ignored so the first term is zero.

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\[ H = 2\gamma a \]

\[ M = \frac{\sigma^2}{2E} \text{(vol)} = \frac{\sigma^2}{2E} (\pi r^2) \text{ where } r = a \]

and \( K = \sigma \sqrt{\pi a} \)

\[ L = 2\gamma a - \frac{K^2 a}{E} \]

Stationary crack

\[ \frac{\partial L}{\partial a} = 2\gamma - \frac{K^2}{E} = 0 \quad \therefore 2\gamma E = K^2 \]

Griffith’s-Irwin – in three lines!

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Notes:

• This is major simplification of the problem. Before it was necessary to go through Griffith’s (including his wrong turns), then go through Irwin, then equate their final equations. Here is done in three lines!
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• Long Fast Crack (1)
• A long fast crack requires an inertial correction (see arXiv 1212. 6704)
• Using Mott’s inertial correction gives a Lagrangian of:

\[ L = 2\gamma a - \frac{D\sigma^2\pi a^2}{2E} \left( \frac{\dot{a}}{V_L} \right)^2 - \frac{\sigma^2\pi a^2}{2E} \]

• This analysis ignores crack acceleration

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Notes:

• Mott ignored crack acceleration. Cracks accelerate extremely quickly >10 million G! They can do this because they originally have little mass and so small Kinetic energy.

• The second term in the bracket is the inertial correction

• The unloading wave is modelled as a plane wave.

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• Long Fast Crack (2)

Applying Euler Lagrange gives:

\[ K_{1d} = \frac{K_{1c}}{[1 - 2D\left(\frac{\dot{a}}{V_L}\right)^2]^{\frac{1}{2}}} \]

• This is of the form that the dynamic stress intensity factor has been derived before, but now simply and explicitly in terms of K.
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Notes:

• There are a number of analyses which give a dynamic stress intensity factor as some function of the velocity of the crack, but this is the first time a dynamic stress intensity factor has been derived explicitly as a function of the static stress intensity factor.
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Figure 4.16  Dynamic fracture toughness values for 4340 steel.

Large points are my results

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Crack front modelled as a small part of the area of an arc of a circle.
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• Fatigue results for perfect elasticity are not surprising and are typically:

\[
\frac{da}{dn} = \frac{\Delta K^2}{48 K^2_{1c}}
\]

• Similar to what has been derived before
• Rarely found.

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• Plane Strain - Elastic Plastic
• For the case of plane strain the crack will be modelled as an embedded crack with the volume swept out by the unloading stress wave as a sphere of a radius equivalent to a multiple (n) of the plastic zone radius.
The Lagrangian becomes:

\[ L = 2\gamma\eta a^2 - \frac{\sigma^2}{2E} \left( \frac{4}{3} n \pi r_p^3 \right) \]

\[
\frac{\partial L}{\partial a} = 4\gamma\eta a - \frac{n \zeta^2 \sigma^8 \pi^2 a^2}{153\pi E \sigma_y^6} = 0 
\]

Note stress to power 8

This can be written:

\[
\frac{da}{dn} = C \left( \frac{\sigma}{\sigma_y} \right)^4 \Delta K^4
\]

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To consider the effects of the stress ratio (R ratio effects):

\[
\bar{\sigma} = \frac{\hat{\sigma} + \tilde{\sigma}}{2} = \frac{\hat{\sigma}}{2} \left( 1 + \frac{\tilde{\sigma}}{\hat{\sigma}} \right) = \frac{\hat{\sigma}}{2} (1 + R)
\]

\[
= \hat{\sigma} (0.5 + 0.5R)
\]

\[
\Delta K_{\text{eff}} = \Delta K (0.5 + 0.5R)
\]

This agrees with Elber

\[
\Delta K_{\text{eff}} = \Delta K (0.5 + 0.4R)
\]
Notes:

• Because of the additional term, the formula can be manipulated to give an effective stress intensity factor which broadly agrees with Elber.

• This is the first time a Paris type law has been shown to include the R ratio, which again occurs quite naturally.

• This is an energy analysis and not mechanism dependent. What physical mechanisms give this result is an entirely different matter.
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R ratio effects Al 2024-T3
(Ewalds & Wanhill)

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Can also be expressed in generic form as:

\[
\frac{da}{dn} = C \left( \frac{\hat{\sigma}}{\sigma_y} \right)^n \frac{\Delta K^m}{\sigma_y^2 K_{1c}^2}
\]

Where \( n \) and \( m \) can be fitted to results. Generally, from the theory, \( m = 2 \) for plane stress and \( 4 \) for plane strain.

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• The exponent sizes are purely from the theory based on idealised circumstances, and are generic.

• In reality it is likely that the exponents, and the constant, are highly dependent on the material being fractured, and the precise mechanisms operative.
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\[ \frac{da}{dn} = C_2 \left( \frac{\hat{\sigma}}{\sigma_y} \right)^{3.98} \Delta K^{3.2} \]

Results agree with Brown

Formula normalised on the lowest stress results, and then extrapolated to cover others; good agreement

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Nisitano and Goto plotted results as functions of stress and crack length

$$\frac{da}{dn} \propto \sigma^8 a^2$$

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These results only give $n = 1$?

$$\frac{da}{dn} = 3.25e^{-8} \left( \frac{\hat{\sigma}}{\sigma_y} \right) \Delta K^4$$
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• This is the only set of fatigue results I can find which can be directly compared to the model.

• Tried for 5 years putting the model on ResearchGate and asking for results.

• Had 85 replies, all complimentary, but no one could give me any more results
Three different Crack regimes

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- The fatigue life over wide ranges of $\Delta K$ is NOT an Ogive, there are actually three different types of test superimposed on one set of axes.
- It is pointless to expect one analysis to fit three crack regimes
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- This is a **new result** for fatigue
- Identifies the Yield Stress Ratio (YSR) as an important parameter.
- Pegs fatigue to the Yield Stress.
  - Makes max stress unique!
- Includes R Ratio effects – a first!
- Admits double K form.
- Includes both short & long cracks.
- Includes temperature automatically.
- Important for variable amplitude loading
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Notes:

• This new analysis for fatigue and fracture is a major simplification of fracture mechanics, and will considerably ease teaching the subject, at least to those students who have covered Classical Mechanics.

• It does all what is said in the slide, and future fatigue results ought to be presented in this format.

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• For more information:
• Analysis of Fracture and Fatigue using Lagrangian Mechanics: arXiv 1212.6704
• Follow link on www.neal-sturgess.com
• Follow me on Research Gate

Questions?

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