Reliability analysis of basal-heave in a braced excavation in a 2-D random field

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ARTICLE INFO

Article history:
Received 7 May 2011
Received in revised form 11 July 2011
Accepted 11 August 2011
Available online 16 September 2011

Keywords:
Basal-heave
Excavation
Probability
Reliability
Shear strength
Slice surface
Spatial variability
Two-dimensional random field

ABSTRACT

A simplified approach to consider the effect of spatial variability in a two-dimensional (2-D) random field for reliability analysis of basal-heave in a braced excavation in clay is presented. In contrast to the random field modeling (RFM) approach, which necessitates use of Monte Carlo simulation (MCS), the proposed approach employs the equivalent variance technique to consider the effect of spatial variability so that the analysis for probability of basal-heave failure can be performed using well-established first-order reliability method (FORM). Case study shows that this simplified approach yields results that are nearly identical to those obtained from the MCS-based RFM approach. The proposed approach is shown to be effective and efficient for probabilistic analysis of basal-heave in a braced excavation in a 2-D random field.

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1. Introduction

Basal-heave failure in a braced excavation in clay may be induced by insufficient shear strength, which supports the weight of soil within the critical zone around the excavation. During an excavation, soil outside the excavation zone moves downward and inward because of its own weight and surcharge; this tends to cause soil inside the excavation zone to heave up. Collapse of the bracing system may occur if the amount of basal-heave movement is excessive. Traditionally, basal-heave stability is evaluated using the deterministic approach that treats all input parameters as non-random variables, the result of which is generally expressed in terms of factor of safety ($FS$). To compute $FS$ against basal-heave, simplified methods based on limiting equilibrium [1–4] are widely used. In principle, “failure” (i.e., the occurrence of excessive basal-heave) is said to occur if $FS < 1$. In practice, a minimum $FS$ against basal-heave is often required and is generally specified in design codes (e.g., [5–7]).

Because of uncertainties in soil parameters, failure may still occur, even though the computed $FS$ is greater than 1 or the minimum $FS$ specified in the design code. To this end, it may be useful to perform a probabilistic analysis that considers explicitly the parameter uncertainty in a basal-heave design. Probabilistic approaches evaluate the basal-heave stability problem, and express the results in terms of the probability of failure. Several studies of basal-heave problems using probabilistic approaches, especially reliability methods, have been reported (e.g., [8,9]). In a recent reliability analysis of basal-heave failure, it was found that the failure probability tends to be overestimated if the effect of spatial variability is neglected [10]. However, none of these studies consider the effect of the 2-D random field.

Uncertainty of soil parameters stems not only from inherent variability but also from spatial variability. Inherent variability of soil parameters is interpreted by their probability distributions or sample statistics (e.g., mean value and standard deviation). Spatial variability is generally described by the scale of fluctuation, which is the maximum distance within which the spatially random parameters are correlated [11]. Spatial variability may be modeled with the random field theory [12]. Recent studies of random field modeling (RFM) based on Monte Carlo simulation (MCS) by Griffiths and his colleagues [13,14] demonstrate that spatial variability plays an important role in reliability-based design in geotechnical engineering. Neglecting spatial soil variability in reliability analysis of geotechnical problems can lead to either
overestimation or underestimation of the failure probability in a given design, depending on the specified limiting FS value [15,16]. For example, for the basal-heave stability analysis in a braced excavation in clay, the computed failure probability can increase significantly if the adopted limiting FS is reduced from 1.2 to 1.0 [8]. Another recent study on basal-heave stability using the slip circle method [9] reported that the traditional reliability analysis that does not account for the effect of spatial variability tends to overestimate the failure probability. Although the conventional RFM using MCS generally yields solution that is “closest to the truth,” it is computationally demanding, and thus, less desirable in practice. As an alternative, simplified approaches based on the equivalent variance technique have been employed (e.g., [17,18]). With the equivalent variance technique, the spatial variability of the soil parameters can be averaged by multiplying a variance reduction factor to the variance of a soil parameter. Recent studies to compare RFM and simplified approach [10,19,20] show that the latter implemented can capture the overall trend of the former. Although the RFM is a more rigorous approach to account for spatial variability, its use in geotechnical reliability-based design is quite limited because of much greater computation effort that is required [17]. In contrast, simplified approach that uses reduced variance to deal with the effect of spatial variability, in lieu of MCS, can significantly reduce the computational effort.

In this paper, a simplified approach to consider the effect of spatial variability in a two-dimensional (2-D) random field for reliability analysis of basal-heave in a braced excavation in clay is formulated. This simplified approach is demonstrated through a case study. As the first step, the 2-D RFM analysis is performed in a study of basal-heave stability to provide a benchmark. Then, variance reduction factors for both vertical and horizontal directions, at which the simplified approach yields results that match well with those obtained with RFM, are back-calculated. Next, assumptions of the characteristic lengths of both vertical and horizontal directions in the stability analysis are verified based on the back-calculated variance reduction factors. Finally, a simplified approach which combines the first-order reliability method (FORM) and the equivalent variance technique to account for spatial variability is proposed for reliability analysis of basal-heave stability. The proposed approach is easy to use, requires less computational effort, and yields results (in terms of probability of basal-heave failure) that are nearly identical to those obtained with the MCS-based RFM method.

### 2. Slip circle method for basal-heave analysis – A deterministic approach

In this study, the slip circle method [5–7] for determining FS against basal-heave in soft clay is adopted for its simplicity and suitability for modeling the random field of undrained shear strength. Similar to other semi-empirical methods [1–4], FS in the slip circle method is defined as the ratio of resistance over load:

$$FS = \frac{M_R}{M_D}$$  \hspace{1cm} (1)

where $M_R$ and $M_D$ are the resistance moment and the driving moment respectively. The resistance moment $M_R$, may be computed as (in reference to Fig. 1):

$$M_R = r \cdot \int_{0}^{\pi/2} s_u \cdot r \cdot d\beta$$  \hspace{1cm} (2)

where $r$ is the radius of the slip circle, and $r = H_w - H_s$ in which $H_w$ is the length of the diaphragm wall, $H_s$ is the depth of the final strut; $\beta$ is the angle from $\delta \theta$ to the current slice as shown in Fig. 1 and arc length $ds = d\beta \cdot r$. The driving moment $M_D$ is caused by the weight of the soil and possible surcharge:

$$M_D = W \cdot \frac{r^2}{2} + q_s \cdot \frac{r^2}{2}$$  \hspace{1cm} (3)

where $W$ is total weight of the soil in front of the vertical failure plane and above the excavation surface, $q_s$ is the surcharge.

As reflected in the above formulation [Eqs. (1)–(3)] of the slip circle method, the undrained shear strength ($s_u$) of clay plays a critical role in the design of a braced excavation against basal-heave. In other words, FS is a function of $s_u$ and other parameters.

![Fig. 1. Geometry of slip circle method and 2-dimensional random field modeling region for basal-heave stability analysis.](image-url)
3. Two-dimensional random field modeling of $s_u/\sigma_p$

3.1. Conventional random field modeling of $s_u/\sigma_p$

As noted previously, the first step toward developing a simplified reliability-based procedure for evaluating the probability of basal-heave failure in an excavation in clay with significant spatial variability is to establish a benchmark using the MCS-based RFM. A two-dimensional RFM approach is deemed especially suitable for the basal-heave problem analyzed with the slip circle method (Fig. 1).

The undrained shear strength generally increases with depth for most normally consolidated clay but the ratio of undrained shear strength over the effective overburden stress $(s_u/\sigma')$ remains roughly constant [21]. Thus, in this paper the parameter $s_u/\sigma_p$ is modeled using lognormal random field, and all other input parameters are modeled as spatially-constant lognormal variables or constants. The assumption of lognormal distribution for inherent soil variability assures positive soil parameters and has been widely advocated by past studies (e.g., [22]). In RFM, the uncertainty of $s_u/\sigma_p$ is represented by its spatially-constant mean $\mu_{ln}$ and coefficient of variation $COV_{ln}$ and its scale of fluctuation $h$. Thus, the basal heave problem here involves a stationary random field modeling of $s_u/\sigma_p$. The standard deviation and mean of the equivalent normal distribution of $s_u/\sigma_p$, denoted as $\ln(s_u/\sigma_p)$, are expressed as:

$$\sigma_n = \sqrt{\ln \left( 1 + COV^2_{ln} \right)}$$  \hspace{1cm} (4)

$$\mu_n = \ln \mu_{ln} - \frac{1}{2} \sigma_n^2$$  \hspace{1cm} (5)

where “$n$” denotes normal distribution and “$ln$” denotes lognormal distribution. The lognormally distributed random field of $s_u/\sigma_p$ can be generated through the following transformation [23]:

$$s_u/\sigma_p(x_i) = \exp \left\{ \mu_n + \sigma_n \cdot G_n(x_i) \right\}$$ \hspace{1cm} (6)

where $x_i$ is the spatial position at which $s_u/\sigma_p$ is modeled; $G_n(x_i)$ is a normally distributed random field with zero mean, unit variance and correlation function $\rho(\tau)$. In this paper, the exponential correlation function, which is commonly used in random field modeling, is selected [24]:

$$\rho(\tau) = \exp \left( -\frac{2\tau}{\theta} \right)$$ \hspace{1cm} (7)

where $\tau$ is the absolute distance between any two points in the random field and $\theta$ is the scale of fluctuation. As shown in a previous study, the vertical and the horizontal scales of fluctuation in the field for clay are generally different [25]. In the 2-D RFM, Eq. (7) may be modified to consider the unequal scales of fluctuations [26]:

$$\rho(\tau) = \exp \left[ -2\sqrt{\left( \frac{\tau_v}{\theta_v} \right)^2 + \left( \frac{\tau_h}{\theta_h} \right)^2} \right]$$ \hspace{1cm} (8)

where $\tau_v$ and $\tau_h$ are the absolute vertical and horizontal distances between any two points in the random field, respectively; and $\theta_v$ and $\theta_h$ are the vertical and the horizontal scales of fluctuation, respectively. In this study, the correlation matrix built with the correlation function is decomposed by Cholesky decomposition which has been proved simple and effective [20,27]:

$$L \cdot L^T = \rho$$ \hspace{1cm} (9)

With the matrix $L$, the correlated standard normal random field can be obtained by linearly combining the independent variables as follows [27]:

$$C_n(x_i) = \sum_{j=1}^{M} L_{ij}Z_j \quad i = 1, 2, \ldots, M$$ \hspace{1cm} (10)

where $M$ is the number of points in the random field; $Z_j$ is the sequence of independent standard normally distributed random variables.

The normalized undrained shear strength $s_u/\sigma_p$ at each spatial position in the random field can be obtained with Eq. (6) for a specified mean, standard deviation, and scale of fluctuation using Monte Carlo simulation. In each simulation, the same mean, standard deviation, and scales of fluctuation of $s_u/\sigma_p$ are used. The statistics of output such as $FS$ [Eq. (1)] can be obtained after a sufficient number of simulations are carried out. The failure probability $p_f$ is computed as the ratio of the number of simulations that yield failure ($M_f < M_0$ or $FS < 1$) over the total number of simulations $N$. The number of MCS samples should be at least 10 times of the reciprocal of the target failure probability [28,29]. In this study, the level of failure probability of interest is greater than $10^{-4}$, therefore $N$ is set at $10^5$.

Fig. 2 shows the results of random field modeling at four combinations of $\theta_v$ and $\theta_h$ given as an example the mean of $s_u/\sigma_p = 0.3$ and coefficient of variation (COV) = 0.3: (a) $\theta_v = 0.25 m$; (b) $\theta_v = 0.10 m$; (c) $\theta_h = 2.5 m$; (d) $\theta_h = 10 m$. The RFM region shown in Fig. 2 includes 36 by 18 square elements with element size of 1 m. Considering that the aforementioned RFM procedure is defined at the point level, the local averaging over the square element size is performed to obtain the locally averaged statistics. The local averaging is realized through multiplying a variance reduction factor to the variance of a normal variable. Then the statistics of the equivalent lognormal variable are computed [15,26]. As shown in Fig. 2, the darker color represents higher $s_u/\sigma_p$ and lighter color represents smaller $s_u/\sigma_p$. The effect of scales of fluctuation is apparent in the 2-D RFM: either in the vertical or horizontal direction, smaller $\theta$ corresponds to more drastic variation of $s_u/\sigma_p$ in that direction of the random field; conversely, a larger $\theta$ corresponds to more uniform $s_u/\sigma_p$ in that direction of the random field. In either direction, the spatial variation in the case of smaller $\theta$ is much more significant than that for larger $\theta$.

3.2. Simplified approach based on equivalent variance technique

The second step toward developing a simplified reliability-based procedure for evaluating the probability of basal-heave failure is to establish a simplified solution that matches closely with the MCS-based RFM solution.

The simplified approach is based on the concept of spatial averaging in which the spatial variability of the soil property is averaged in order to approximate a random variable that represents a soil parameter [12]. The averaged variability of the soil property over a larger domain can be quantified with an equivalent variance technique in which the variances of soil parameters may be reduced by multiplying a factor known as variance reduction factor ($I^2$). With two inputs: scale of fluctuation and characteristic length, the variance reduction factor that adopts an exponential form is given as follows [30]:

$$I^2 = 1 + 2L \theta - 1 + \exp \left( \frac{2L}{\theta} \right)$$ \hspace{1cm} (11)

where $\theta$ is the scale of fluctuation and $L$ is the characteristic length.

In the 2-D random field, the variance reduction factor is expressed as the product of the variance reduction factors in the vertical and horizontal directions, computed with respective scales of fluctuation and characteristic length [12]:

$$I^2 = I_{vertical} \cdot I_{horizontal}$$
where \( C_2^v \) and \( C_2^h \) are the vertical and horizontal variance reduction factors, respectively. The reduced variance \( r_2^2 \) can be obtained with the following equation:

\[
r_2^2 = C_2^v / C_1^2 \quad (13)
\]

where \( r_2 \) is the variance of the soil parameter of concern (\( s_u / \sigma_u^* \) in this study). In this paper, the positive square root of the variance reduction factor is referred to herein as the reduction factor \( (\Gamma) \) to differentiate it from the variance reduction factor \( (R^2) \).

It is noted that if the equivalent variance technique is used to simplify the effect of the spatial variability of a lognormal distributed parameter, as is the case in this paper, only the variance of its equivalent normal distribution, defined previously, should be reduced through the use of variance reduction factor.

For the simplified approach using the equivalent variance technique, the analysis can be conducted either with MCS or with a reliability method. The latter is preferred, as it requires far less computational effort and is more practical than the MCS-based RFM. Implementation of the reliability methods in a spreadsheet has been demonstrated to be a practical approach to geotechnical problems (e.g., [31–33]). Past investigators [10,19,20] have shown that reliability analysis with the equivalent variance technique can capture the overall trend of the MCS-based RFM. In this paper, the two approaches are compared within the context of 2-D random field modeling.

### 4. Simplified reliability method for assessing probability of basal-heave failure in braced excavation in 2-D random field

#### 4.1. RFM approach for probability of basal-heave failure

The probability of basal-heave failure in a braced excavation in clay is first analyzed herein using the conventional random field modeling with Cholesky decomposition method. The geometry and input data for the excavation case employed in this study is illustrated in Fig. 1 and listed in Table 1, respectively. The undrained shear strength \( s_u / \sigma_u^* \) is modeled as a spatially random variable, and the unit weight of soil and surcharge are modeled as spatially-constant random variables. All other geotechnical and structural parameters are treated as constants for simplicity, since the uncertainties in these parameters are relatively negligible.

#### Table 1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Notations</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight of soil</td>
<td>( \gamma )</td>
<td>19 kN/m³</td>
</tr>
<tr>
<td>Surcharge</td>
<td>( q_s )</td>
<td>10 kPa/m</td>
</tr>
<tr>
<td>Depth of GWT</td>
<td>( D )</td>
<td>2 m</td>
</tr>
<tr>
<td>Final excavation depth</td>
<td>( H_f )</td>
<td>18 m</td>
</tr>
<tr>
<td>Final strut depth</td>
<td>( H_s )</td>
<td>15 m</td>
</tr>
<tr>
<td>Penetration depth</td>
<td>( H_p )</td>
<td>15 m</td>
</tr>
</tbody>
</table>

\* In this paper, many basal-heave problems defined with this set of input parameters and geometry are analyzed. The difference in these problems is in the choice of the mean value of the normalized undrained shear strength \( (s_u / \sigma_u^*) \), which results in different factors of safety (FS).
Typical COV of the undrained shear strength $s_u$ is about 0.3, although it could be as high as 0.8 [25]. In this paper, the COV of $s_u/\sigma_v^r$ is first set at 0.3 and the effect of assuming higher COVs is examined later. Based on a statistical study by [25], the average vertical and horizontal scales of fluctuation for clay are 2.5 m and 50.7 m, respectively. In a 2-D RFM, both vertical and horizontal spatial variability are considered. It should be noted that for basal-heave analysis, only the random field shown in Fig. 1 (the box region) needs to be modeled since the resistance moment [Eq. (2)] comes only from this region. Here, the RFM region in Fig. 1 is subdivided into 36 by 18 square elements (horizontal direction by vertical direction). The size of the square elements is 1 m. For this 2-D RFM, the total number of elements (648 elements in this case) is comparable to the suggested maximum number by [27] for the Cholesky decomposition operation. Although a larger modeling region can be adopted, the region shown in Fig. 1 is the minimum region that covers the slip circle where the resistance moment $M_R$ is derived.

To provide a reference, the effect of vertical and horizontal scales of fluctuation is first examined separately (i.e., treating it like 1-D RFM). Thus, when vertical or horizontal spatial variability is considered, the other direction is assumed to be spatially constant. To study the effect of spatial variability, a series of scales of fluctuation ($h_v = 1$ m, 2.5 m, 10 m, and 100 m) for each direction is investigated.

Thus, given a set of input data for a braced excavation (Fig. 1 and Table 1), the probability of basal-heave failure is computed for a design with a given FS [say, $FS = 2.0$ as per Eq. (1)] and a given scale of fluctuation that reflects the 1-D random field of $s_u/\sigma_v^r$. This probability can be calculated using the MCS. For each given scale of fluctuation and design FS, $10^5$ simulations are conducted. The probability of failure is determined by the ratio of the number of failure cases (defined here as $FS < 1.0$) over the total number (i.e., $10^5$). This process is repeated for each series of scale of fluctuation and each series of “designs” (signaled by a series of FS values, which was realized by assuming different mean $s_u/\sigma_v^r$ values while keeping the mean values of all other parameters the same). For a single run of MCS, the execution time for $10^5$ simulations is approximately 4 min on a laptop PC equipped with an Intel Pentium Dual CPU T2390 running at 1.86 GHz using MATLAB [34]. The results are shown in Fig. 3a for vertical spatial variability and Fig. 3b for horizontal spatial variability. The results presented in this figure provide the engineer a basis for selecting a factor of safety for design against basal-heave using the slip circle method [Eqs. (1)–(3)]. This basis is the target probability of failure that considers the spatial variability of soil parameters. The design, based on the target probability of failure is referred to herein as the probability-based (or reliability-based) design against basal-heave failure.

The effect of scales of fluctuation in a 1-D random field is quite obvious: a smaller scale of fluctuation results in a smaller $p_f$ at the same $FS$. As shown in Fig. 3a, if the target $p_f$ is set at $10^{-3}$, the required $FS$ is about 1.85 at $h_v = 2.5$ m (note: this $h_v$ value is the mean of the vertical scale of fluctuation for clay as per [25]), and is about 2.65 at $h_v = 100$ m (note: this $h_v$ value is close to a spatially-constant condition). Similar conclusions may be drawn from Fig. 3b for the effect of horizontal spatial variability. The implication is that the required $FS$ will be overestimated for a target $p_f$ if the effect of spatial variability is ignored. Thus, traditional reliability analysis that considers variation of input soil parameters (for example, through
but not spatial variability exhibited in a random field can overestimate the probability of failure for a given deterministic-based design (i.e., a given $FS$).

The effect of 2-D spatial variability is examined next. To begin with, three different horizontal scales of fluctuation ($\theta_h = 2.5 \text{ m}, 50 \text{ m}, \text{ and } \infty$) are considered simultaneously with the average vertical scale of fluctuation ($\theta_v = 2.5 \text{ m}$ as per [25]) to study the effect of $\theta_h$ at fixed $\theta_v$. The results are shown in Fig. 4a. Afterwards, three different vertical scales of fluctuation ($\theta_v = 2.5 \text{ m}, 50 \text{ m}, \text{ and } \infty$) are considered simultaneously with the average vertical scale of fluctuation ($\theta_v \approx 50 \text{ m}$ as per [25]) to study the effect of $\theta_v$ at fixed $\theta_h$. The results are shown in Fig. 4b. The effect of the scales of fluctuation in a 2-D random field is also obvious: at a fixed scale of fluctuation in one direction, a smaller scale of fluctuation in the other direction results in a smaller $pf$ at the same $FS$. Furthermore, the required $FS$ will still be overestimated for a target $pf$ if the soil parameter is modeled with only a 1-D random field, as opposed to a 2-D random field. Therefore, it is essential to consider 2-D spatial variability in the probability-based (or reliability-based) design against basal-heave failure in a braced excavation.

One concern with traditional reliability-based design in geotechnical practice in the past is that the computed failure probability is often high in a design that satisfies the minimum $FS$ specified in the codes, but failure seldom occurs in such cases. For example, this concern has been reported by [8–10] in their study of basal-heave stability in an excavation. Overestimation of variation in soil parameters is often pointed out as a possible cause for having a higher computed probability of failure. Based on the results presented (Figs. 3 and 4), it is evident that negligence in the effect of spatial variability of soil parameters can lead to an overestimation, perhaps to a high degree, of the failure probability. Thus, to apply the traditional simplified reliability-based method for evaluating the failure probability, an adjustment is needed to model spatial variability.

4.2. Equivalent simplified approach with equivalent variance technique

While the RFM analysis generally yields the most accurate results, it requires use of the MCS. On the other hand, the simplified approach can be implemented with reliability-based methods. Past studies have shown that simplified approaches with a proper variance reduction can match well with the RFM solution (e.g., [10]). In other words, for a RFM solution, an equivalent solution using simplified approach is possible. Therefore, the analysis for the probability of basal-heave failure in a braced excavation in a random field can be performed using traditional reliability-based methods, provided that an equivalent simplified approach can be established first.

The desired equivalency between simplified approach and RFM solutions in this case is “equal” probability of failure, or $pf = P[M_R < M_B]$. For the basal-heave problem analyzed herein using the slip circle method (Eqs. (1)–(3)), the undrained shear strength is the only soil parameter that is treated as spatially random, while all other parameters are set as constants so that the spatially random effect of the undrained shear strength is examined explicitly. Here, in search for the equivalency between the RFM approach and the simplified approach, $M_R$ is treated as a constant and $M_B$ is

![Flow chart for searching for the reduction factor for a given pair of standard deviation $\sigma$ and scale of fluctuation $\theta$.](image-url)

Given a pair of $\sigma$ and $\theta$

Random field modeling

- Generate correlation matrix for $\theta$
- Cholesky decomposition
- Generate standard normally distributed random numbers
- Generate lognormally distributed $s_u$
- Run stability analysis with slip circle method using Monte Carlo simulations

Simplified approach

- Assume the interval of reduction factor $\Gamma$ for $\theta$: $[\Gamma_L, \Gamma_U]$
- Compute the midpoint: $\Gamma_p = (\Gamma_L + \Gamma_U)/2$
- Obtain reduced $\sigma_{\Gamma} = \Gamma_p \cdot \sigma$
- Generate lognormally distributed $s_u$
- Run stability analysis with slip circle method using Monte Carlo simulations

Obtain standard deviation for $M_B$: $\sigma_{NS}$

Obtain standard deviation for $M_B$: $\sigma_S$

$\sigma_{NS} \leq \sigma_S$?

Yes

- If $\sigma_{NS} - \sigma_S > 0$, $\Gamma_L = \Gamma_p$
- else $\Gamma_U = \Gamma_p$

No

Obtain reduction factor $\Gamma$ for $\theta$
treated as a random variable. Therefore, the equivacency in the computed failure probability can be achieved if $M_R$ determined from the two approaches (simplified approach and RFM) agrees with each other at the same level of soil variability (i.e., the same level of COV and scale of fluctuation). Because the mean of $M_R$ is approximately the same regardless of which of the two approaches is employed [This is apparent since in this study $M_R$ is linearly correlated with $s_h$ as per Eq. (2)], it is considered appropriate and adequate to use the variation (or more precisely, the standard deviation) of the computed $M_R$ as a basis for establishing the equivacency.

It should be noted that the mean of $M_R$ (and thus $FS$) may not be independent of the scale of fluctuation $\theta$ as observed from the results of the slip circle method, since the shear zone may develop through softer areas [20]. If other approaches such as finite element method are employed for basal heave analysis, the computed mean of $M_R$ is likely to change with $\theta$ [20]. This effect is not accounted with the slip circle method, which is a limitation of the proposed approach. However, this issue is beyond the scope of this paper.

Equivacency between simplified approach and RFM solution may be achieved by applying variance reduction to the former, which requires determination of the reduction factor ($\Gamma$). For the basal-heave problem, $\Gamma$ values at various levels of variability (in terms of standard deviation $\sigma$ and scale of fluctuation $\theta$ of $s_h/\sigma$) can be back-calculated using the procedure illustrated in Fig. 5. As shown in Fig. 5, the flow sequence on the left summarizes the procedure of RFM with the Cholesky decomposition method [Eqs. (4)–(10)]. The standard deviation of $M_R$, denoted herein as $\sigma_{M_R}$, is obtained from $10^5$ MCS of the basal-heave analysis for a braced excavation in clay with a given pair of standard deviation $\sigma$ and scale of fluctuation $\theta$ of $s_h/\sigma$.

In Fig. 5, the flow sequence on the right summarizes the procedure of simplified approach using the equivalent variance technique. First, an interval of the reduction factor, $[\Gamma_L, \Gamma_U]$, is assumed for this case (with the same $\sigma$ and $\theta$ of $s_h/\sigma$). $\Gamma_L$ and $\Gamma_U$ are the assumed lower and upper bounds, which may be set at 0 and 1, respectively. Then the bisection method is used to search for the equivalent variance: the interval is divided into two segments by the midpoint $\Gamma_p = (\Gamma_L + \Gamma_U)/2$, and the variance is reduced with $\Gamma_p$. With the reduced variance $\sigma_{M_C}$, MCS may be performed without the Cholesky decomposition. The standard deviation of $M_R$, denoted herein as $\sigma_{M_S}$, is then obtained from $10^5$ MCS of the basal-heave analysis of the same case, as in the RFM analysis (left side of the flowchart shown in Fig. 5). If the reduction factor $\Gamma_p$ is correct, the two standard deviations, $\sigma_{M_S}$ and $\sigma_{M_C}$, will be equal to each other for the given pair of standard deviation $\sigma$ and scale of fluctuation $\theta$ of $s_h/\sigma$. In this paper, the $\Gamma$ value at which $\sigma_{M_S} = \sigma_{M_C}$ is $\Gamma = \sigma_{M_S}/\sigma_{M_C}$. As shown in Fig. 5, if the above stopping criterion ($|\sigma_{M_S} - \sigma_{M_C}|/\sigma_{M_S} < 10^{-5}$) is not satisfied, the interval of $\Gamma$ is shortened by setting $\Gamma_L = \Gamma_{p,L}$ (for $\sigma_{M_S} > \sigma_{M_C}$) or $\Gamma_U = \Gamma_{p,U}$ (for $\sigma_{M_S} < \sigma_{M_C}$). The new midpoint $\Gamma_p$ is then computed and the aforementioned procedure is repeated until the final reduction factor for a given pair of $\sigma$ and $\theta$ of $s_h/\sigma$ is obtained. It should be noted that for this “equivacency” analysis, the simplified approach is implemented with the MCS. As will be shown later, the simplified approach can also be implemented with FORM to further reduce the computational effort.

Through the above back-calculation procedure (Fig. 5), the reduction factor $\Gamma$ for an equivalent simplified approach can be obtained. Fig. 6a and b shows the back-calculated $\Gamma$ values for various pairs of $\sigma$ (or COV) and $\theta$ of $s_h/\sigma$, for vertical and horizontal spatial variability, respectively. Two observations are made: (1) the inherent variability rarely influences the variance reduction at the same $\theta$ level; (2) the reduction factor $\Gamma$ depends only on $\theta$ at the same COV level. These observations are consistent with the variance reduction models presented in the literature (e.g., Eq. (11) from [30]).

Finally, a comparison is made between reduction factors ($\Gamma$), computed using the variance reduction function [Eq. (11)] and those computed using Eq. (11) with assumed characteristic lengths ($L_v = 18$ m or $L_h = 36$ m).

As shown in Fig. 6, the assumptions of $L_v = 18$ m and $L_h = 36$ m yield reduction factors consistent with those that are back-calculated from the equivalency analysis. The implication is that for basal-heave analysis in a 2-D random field using the slip circle method (Fig. 1), vertical characteristic length $L_v$ can be taken as the vertical distance between the depth of the final strut and the bottom of the diaphragm wall (the length $\overline{d}$ as in Fig. 1); horizontal characteristic length $L_h$ is assumed to be the horizontal scale of the slip circle ($L_h \approx \overline{\sigma} \approx 36$ m as in Fig. 1). The rationale for these assumptions is that these lengths are the vertical and horizontal scales of the region that contributes to the resistance moment in the random field. With the assumed characteristic lengths, reduction factors are computed with Eq. (11), and the results are shown in Fig. 6 for comparisons with those that are back-calculated based on the equivalency analysis presented previously.
4.3. Practical reliability analysis of basal-heave considering 2-D spatial variability

Based on results presented in the previous sections, a step-by-step procedure is established for the simplified reliability analysis of basal-heave in a braced excavation in clay considering 2-D spatial variability:

1. Select an analytical model for basal-heave analysis (for example, slip circle method as per [5]).
2. Obtain spatially-constant input parameters and random variables, such as unit weight of soils and applied surcharge.
3. Characterize a spatially random variable (in this case, the normalized undrained shear strength) with its mean, COV and horizontal and vertical scales of fluctuation ($h_h$ and $h_v$) for this 2-D random field.
4. Determine the vertical and horizontal characteristic lengths ($L_v$ and $L_h$) based on the selected random field region (such as the one shown in Fig. 1). Apply the equivalent variance technique [Eqs. (11)–(13)] to determine the reduced variance ($r^2C$) of normalized undrained shear strength.
5. When the equivalent variance technique is used to simplify the effect of the spatial variability of a lognormal distributed parameter, the variance of its equivalent normal distribution should be reduced in this process.
6. Conduct FORM analysis (for example, using a spreadsheet implementation as shown in Fig. 7) using the reduced variance of the normalized undrained shear strength. The reliability index and probability of failure can be determined using FORM.

As an example, basal-heave in a braced excavation shown in Fig. 1 with input parameters listed in Table 1 is analyzed. All parameters except $s_u/\sigma_v$ are treated as spatially-constant random variables or simply constants. The parameter $s_u/\sigma_v$ is modeled with a 2-D random field, and characterized by a mean value of 0.30, a COV of 0.3, and scales of fluctuation $h_h = 2.5$ m and $h_v = 50$ m. As shown in Fig. 7, the equivalent simplified approach that considers the spatial variability of $s_u/\sigma_v$ is realized using the variance reduction function [Eq. (11)] with characteristic lengths $L_v = 18$ m and $L_h = 36$ m. The reduced variance of $s_u/\sigma_v$ in the equivalent simplified approach is thus obtained. FORM analysis is then performed using the spreadsheet as shown in Fig. 7, which yields reliability index $\beta = 1.8856$ and probability of failure $P_f = 0.0297$. It should be noted that the model bias (BF) of the deterministic slip circle method is ignored at this point (as shown in Fig. 7, mean value of model bias is set as 1.0 and the COV of the model bias is set to 0.0); the effect of model bias is examined later.

To further examine the capability of the spreadsheet that implements the FORM procedure with the equivalent variance technique, the basal-heave problems that were analyzed with the MCS-based RFM approach (Fig. 4), are re-analyzed. Comparisons with the previous results from Fig. 4 are shown in Fig. 8 for four different scenarios of constant vertical and horizontal scales of fluctuation: (a) $h_h = h_v = 2.5$ m; (b) $h_h = 2.5$ m, $h_v = 50$ m; (c) $h_h = h_v = 50$ m; (d) $h_h = 50$ m, $h_v = 2.5$ m. Note that Case (b) represents the mean values for $h_v$ and $h_h$ suggested by [25]. As in Fig. 4, a series of basal-heave problems (signaled by different FS values, which were realized by assuming different mean $s_u/\sigma_v$)
values while keeping the mean values of all other parameters the same) are analyzed. In Fig. 8, the results show that regardless of the chosen scales of fluctuation and design safety level (FS value), the probabilities of basal-heave failure obtained from the spreadsheet agree very well with those determined with the MCS-based RFM approach. At any given target probability of failure (Fig. 8), the maximum difference in the required FS between the two approaches (RFM vs. simplified approach) is less than 5%. Thus, the simplified approach is deemed effective.

4.4. Effect of model bias on the computed probability of failure

The analyses of basal-heave failure presented so far are performed assuming no model bias in the adopted slip circle method. In the reliability analysis shown in Fig. 7, the model bias is implemented with a bias factor (BF), which is treated as a random variable. To implement the assumption of no model bias, the mean of the model bias factor is taken as unity ($\mu_{BF} = 1.0$) with no variance ($\text{COV}_{BF} = 0.0$). Most geotechnical analysis models are biased one way or the other because they often represent a conservative approximation of the actual conditions. If the model bias exists but is not accounted for, the computed probability of failure may be either underestimated or overestimated. This is an additional source of uncertainty that could influence the selection of a required factor of safety for a target failure probability in a reliability-based design.

A recent calibration study on the slip circle method [35] reports that the mean value ($\mu_{BF}$) and the COV of model bias ($\text{COV}_{BF}$) of the method are 1.39 and 0.21, respectively. To examine the effect of this model bias on the failure probability determined using the FORM-based spreadsheet solution, Case (b) in Fig. 8b, in which $\theta_v = 2.5\ m$, $\theta_h = 50\ m$, is reanalyzed with this model bias factor. The results with and without this model bias factor are compared in Fig. 9. It is observed that at the same FS level, the failure probability is smaller if the model bias of the slip circle method is considered. Thus, negligence of the model bias in the reliability analysis of basal-heave failure can lead to an overestimation of the failure probability. As shown in Fig. 9, however, the effect of the model bias is less significant at smaller target probability level (e.g., $p_f = 10^{-3}$ to $10^{-4}$). For example, at the target probability of failure $p_f = 10^{-4}$, the difference in the required FS for the basal-heave design between the two conditions (with and without model bias consideration) is less than 1.5%. As a reference, at this target probability of failure, the difference in the required FS for the basal-heave design between the two conditions of spatial variability (with and without considerations of spatial variability of the
undrained shear strength of clay) is approximately at 50%. Thus, at the generally accepted level of failure probability ($p_f = 10^{-2}$–$10^{-4}$), the need to consider spatial variability in the analysis is clearly demonstrated while the effect of model bias of the slip circle method is relatively insignificant.

5. Concluding remarks

Traditional reliability analysis that considers variation of input soil parameters, but not spatial variability in a random field, can significantly overestimate the probability of basal-heave failure for a given deterministic design with a certain factor of safety (e.g., Figs. 3 and 4). Negligence of the model bias of the slip circle method also leads to an overestimation of failure probability (Fig. 9), albeit at a much lesser degree. The results help explain the unreasonably high probability of failure that is often reported in a traditional reliability analysis of a basal-heave design that achieves a satisfactory safety factor (for example, $FS > 1.2$). Thus, spatial variability must be properly accounted for in the reliability analysis.

When proper characteristic lengths are selected, the reduction factor ($F$) computed using Eq. (11) agrees extremely well with that which was back-calculated from the “equivariance” analysis using the MCS-based RFM solution (Fig. 6). This study found that for basal-heave analysis in a 2-D random field using the slip circle method (Fig. 1), the vertical characteristic length $L_v$ can be taken as the vertical distance between the depth of the final strut and the bottom of the diaphragm wall (length $d$ in Fig. 1) and the horizontal characteristic length $L_h$ can be taken as the horizontal scale of the slip circle (length $r$ in Fig. 1). With these characteristic lengths, the reduction factor ($F$) can be accurately evaluated using the equivalent variance technique.

It should be noted that within the framework of the slip circle method, the mean of the computed $M_0$ is found independent of scale of fluctuation. In reality, this mean $FS$ may be influenced by the scale of fluctuation, as reported by [20]. This is a limitation of the slip circle method and the proposed simplified approach in dealing with the effect of spatial variability. Further investigation of this issue is warranted.

In this paper, a procedure for conducting reliability analysis of basal-heave in a braced excavation in a 2-D random field is presented. This reliability-based analysis procedure that considers spatial variability in the 2-D random field is based on an equivalent simplified approach. The equivalency in the computed probability of basal-heave failure is established through equivalent variance technique. The results presented in this paper show the probability of basal-heave in a given braced excavation in clay with spatial variability can be determined through traditional reliability analysis of basal-heave using the equivalent simplified approach, in lieu of the Monte Carlo simulation (MCS) analysis. The developed procedure is implemented in a spreadsheet, which is shown to be an effective and efficient tool for performing reliability analysis that takes into account 2-D spatial variability of soils.

The spreadsheet that implements the developed simplified approach facilitates reliability-based design of braced excavation against basal-heave, as the probability of basal-heave failure can be easily calculated even when spatial variability must be considered. Reliability-based design can be realized by satisfying a target reliability index or the acceptable probability of failure against basal-heave. In this regard, it is important to recognize that traditional reliability analysis that does not account for the effect of spatial variability tends to over-estimate the probability of basal-heave failure for a given deterministic design (i.e., a given $FS$). The developed reliability-based procedure (and spreadsheet), on the other hand, can accurately evaluate the probability of basal-heave failure because soil spatial variability is properly counted for through variance reduction. The developed reliability-based procedure is easy to use, especially with a spreadsheet tool which requires far less computational effort than the MCS-based RFM approach. Thus, this simplified approach can be a practical tool for reliability-based design against basal-heave failure.

Acknowledgments

The anonymous reviewers are thanked for their constructive comments that have helped sharpen the work presented in this paper. The authors wish to thank Duncan Wu and Chang-Yu Ou of National Taiwan University of Science and Technology, and Jianye Ching of National Taiwan University for helpful discussions of the subjects presented.

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