

A Portfolio Optimization Approach to Senatorial Campaign Strategy

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During the 2008 election cycle, the National Republican Senatorial Committee allocated a total of \$39,730,148 to Republican Senatorial Candidates. Not to be outdone, the Democratic Senatorial Campaign Committee provided a total of \$77,457,960 to Democratic candidates.

This paper will examine the manner in which such sums were disbursed. Firstly, it will examine the distribution of funds to different candidates. Since candidates received vastly different allocations, it will test different theories of why some candidates received more funds than others, and the extent to which these findings reaffirm or contrast with the prevailing wisdom. Secondly, it will explore whether the funds could have been allocated in a more efficient manner. Such analysis depends largely upon the context in which “efficiency” is defined, and the paper will account for such uncertainty. Under several different assumptions, this paper will employ a theoretical optimization and compare the optimization’s recommended allocations with those that were actually made by the two committees.

This study will focus on the spending by each of these committees after October 1, 2008. By this time, the two major party candidates in each race had all been determined, and vast sums of money had already been spent both by and on behalf of the candidates. Furthermore, the dynamics of each campaign and the election season in general had been established.

There were a total of 35 Senate elections in 2008. 33 of them were scheduled to be held that year, in addition to 2 special elections to fill the unexpired terms of Trent Lott (R-MS) and Craig Thomas (R-WY). 23 of the seats being contested were Republican-held seats, whereas 12 of them were previously held by Democrats.

Furthermore, of the ten seats NRSC chairman John Ensign (R-NV) identified on June 11 as being the most competitive, nine of them were Republican held seats. The one exception was the Louisiana Senate seat of Mary Landrieu (D). The other nine Republican held seats to which Ensign was referring were the races occurring in Maine, New Hampshire, Minnesota, Alaska, Oregon, Colorado, New Mexico, Louisiana, North Carolina and Virginia. As such, there is little wonder that Ensign proclaimed on June 11 that losing only three seats “would be a terrific night” for the Republicans.¹

As pessimistic as Ensign was in June, the situation would only deteriorate from a Republican perspective by October 1. New Mexico, Louisiana, and Virginia were by that point considered to be likely Democratic wins, whereas of the ten races to which Ensign alluded, only Maine was considered to be safe for the Republicans. Furthermore, while the Republicans remained the favorites to defend their seats in Georgia and Texas, this was far from a certainty. As defined by the election forecasting website [fivethirtyeight.com](http://www.fivethirtyeight.com), these were the closest races on October 4, 2008. The projected column margin column represents the estimated margin on that day.² The rightmost column represents the implied probability of the Democratic candidate winning as expressed by intrade.

¹ Carnavale, Mary Lu. “Sen. Ensign Says GOP Majority Would be ‘Fairly Miraculous’”. <http://blogs.wsj.com/washwire/2008/06/12/sen-ensign-says-gop-majority-would-be-fairly-miraculous/>

² <http://www.fivethirtyeight.com/2008/10/senate-polling-update-104.html>

Table 1: Summary of Senate races as of 10/4/2008

State	Republican Candidate	Democratic Candidate	Projected Margin	Intrade Probability (D)
Alaska	Ted Stevens	Mark Begich	D + 2.6	65%
Colorado	Bob Schaffer	Mark Udall	D + 6	80%
Georgia	Saxby Chambliss	Jim Martin	R + 9.9	15%
Kentucky	Mitch McConnell	Bruce Lunsford	R + 5.9	20%
Minnesota	Norm Coleman	Al Franken	R + 1.4	38%
Mississippi	Roger Wicker	Ronnie Musgrove	R + 4.9	36%
North Carolina	Elizabeth Dole	Kay Hagan	D + 1.4	59%
New Hampshire	John Sununu	Jeanne Shaheen	D + 5.4	30%
Oregon	Gordon Smith	Jeff Merkley	D + .7	52%
Texas	John Cornyn	Richard Noriega	R + 11	15%

Each of the two committees spent different amounts of money in each of these races. Firstly, this study will analyze the manner in which the money was allocated. Secondly, it will attempt to measure what, if any, effect that this money had in terms of influencing the electorate in each of the races. Thirdly, it will analyze whether if money had been disbursed differently, whether either of the committees could have attained a better outcome.

A better outcome, for the purposes of this study, will be defined as winning the maximum number of seats. The mission statements of each of the two committees seem to corroborate this as being a fair measurement of success:

The NRSC provides invaluable support and assistance to current and prospective Republican U.S. Senate candidates in the areas of budget planning, election law compliance, fundraising, communications tools and messaging, research and strategy.³

Our mission is to elect Democrats to the United States Senate. We are the largest organization solely dedicated to electing a Democratic Senate in the country. From grass-roots organizing to candidate recruitment to providing campaign funds for tight races, the DSCC is working hard all year, every year to increase the number of Democratic Senators.⁴

The DSCC's position seems more clearly defined than the NRSC's. It, in no uncertain terms states its objective, and only objective, of increasing the number of Democratic Senators. While the Republicans don't explicitly endorse such a mission, their statement certainly does not contradict it a seat maximizing strategy on both sides will be the working assumption of this study.

Literature review

Jacobson (1985)⁵ examined the effect of spending on election outcomes, and concluded that “[t]aken at face value, the evidence is overwhelming that the challenger's level of spending has a strong impact on the vote, whereas that of the incumbent has virtually no impact at all. He also sets up regressions that quantify the effects (or in the case of incumbents, non-effects) of such fundraising.

³ <http://www.nrsc.org/about/>

⁴ <http://www.dsc.org/about>

⁵ Jacobson, Gary C. “Money and votes reconsidered: congressional elections, 1972-1982.” *Public Choice*.

Such a conclusion must be reviewed carefully, because of the problem of selection bias. It may be the case that the only challengers who are able to raise money are those with a viable chance of winning, and that their fundraising ability is an effect of their viability, not a cause of it. And it makes sense that in terms of pure correlation, there exists none between incumbent spending and vote share. On the one hand, it is possible that an incumbent could spend his or her way to a large vote share, but on the other hand, it is also possible that a candidate could be so strong that he does not even attract a challenger, in which case his large vote share would be associated with very little money spent at all.

Levitt (1994)⁶ examines this problem while attempting to circumvent this bias by examining repeat challengers. Under the assumption that candidates have a constant intrinsic appeal over time, he compares multiple elections between identical candidates in order to control for all independent variables other than the amount of money spent. His findings indicate that the effect of money raised in such cases is minimal.

Goldstein and Freedman (2000)⁷ did a case study of the 1996 Senatorial Elections in order to address “the effect of political advertising and the role of campaign spending in congressional elections.” Such a unique data set led them to attribute an effectiveness to advertising expenditures that had not previously been inferred.

Such a relationship makes sense. After all, it would be quite a leap to assume that incumbents spend considerable time and energy raising money for no particular reason.

⁶ Levitt, Steven. “Using Repeat Challengers to Estimate the Effect of Campaign Spending on Election Outcomes in the U.S. House.” *The Journal of Political Economy*.

⁷ Goldstein, Ken and Freedman, Paul. “New Evidence for New Arguments: Money and Advertising in the 1996 Senate Elections.” *The Journal of Politics*.

And this conclusion serves as a necessary assumption for this study, because if spending had no effect, then the optimization of resources is an altogether fruitless pursuit.

Gerber⁸ (1998) uses an instrumental variable approach to estimate the effect of money in Senatorial elections. Taking this approach, he finds that, in fact, spending yields relatively equivalent returns to incumbents and challengers. This is directly contrasts the findings of most others (including the aforementioned Jacobson), who find that challengers receive better returns than incumbents do. Gerber finds that in the average Senate election, the incumbent's spending advantage yields a 6% increase in the incumbent's vote share.

Erikson (2000)⁹ explores a game-theoretical model for explaining the spending patterns of both parties in a given congressional race. Such a model allows for explanation of the simultaneity problem that has confounded political scientists doing analyses such as those that have been described above. Essentially, what this does is controls for differences in the effect of money spent in races that are expected to be close as opposed to races that are not expected to be close.

Such considerations should be taken seriously in the analysis of this paper. If the optimizations performed in the final portion of it are to make recommendations for how the committees could better optimize their expenditures, it is important that the model does not make an incorrect causal inference that dollars spent would have an effect that could not be realistically expected. In this study, the subset of data used encompasses

⁸ Gerber, Alan. "Estimating the Effect of Campaign Spending on Senate Election Outcomes Using Instrumental Variables." *The American Political Science Review*.

⁹ Erikson, Robert S and Palfrey, TR. "Equilibrium Effects in Campaign Spending Games: Theory and Data." *American Political Science Review*.

racers are expected to be at least somewhat close, so this problem somewhat takes care of itself.

Additionally, the game theoretical framework Erikson establishes can be used as a way of evaluating the spending decisions of the DSCC and NRSC in this study.

Specifically, such considerations are necessary in evaluating the decisions because it is not sufficient to simply state that had one party spent more money in a given race, it could have gained a corresponding share of the vote based on what this study measures the effect of a dollar spent on a race to be. Such an analysis would assume that the opposing party wouldn't match that increased spending with increased spending of its own. Essentially, the consideration must be made that spending decisions aren't made in a vacuum, and that the other party can be expected to respond to any action with an action of its own.

Such literature can serve as a fine basis for a model that would attribute a basic expectation in terms of votes (and thus, probability of winning a given election) to dollars spent by the NRSC or DSCC on a given election. However, to my knowledge, nobody has ever taken the approach of applying such models to the allocation of a committee's funds. In fact, most of the aforementioned analyses are either purely theoretical in nature, or use such models as a means of exploring the ramifications of Campaign Finance Reform. Silver (2008)¹⁰ introduced the concept that “[s]ome [Senate races] really do count more than others,” but never set up a formal allocation model to account for such differences. Furthermore, while Silver clearly recognizes the fact that money spent in a close race has more leverage over the outcome than money spent in a less close

¹⁰ <http://www.fivethirtyeight.com/2008/06/moving-beyond-seat-count.html>

race does, he never formally proposed a framework for quantifying the relationship between money spent and probability of winning.

Furthermore, studying the effect of spending money on winning elections is a more interesting problem by using a portfolio approach because it adds a budgetary constraint to the formulation of the problem. While different authors have devised different ways of measuring the effectiveness of campaign spending on winning elections, they would all agree that spending more money can only add to the probability of winning (or, at the very least, cannot detract from it). In this analysis, however, spending money in a race can, in fact, have adverse consequences, if spending such money entails bearing the opportunity cost of not spending the money in a race in which dollars spent would have higher leverage.

Therefore, such an approach adds a significant contribution to the existing literature, even without weighting races differently, as Silver suggests doing. This part of the solving the problem will need to optimize each committee's budget across the various candidates. Such an optimization can be written mathematically, as follows:

For each candidate i

Where $X1_i$ is the matrix of variables for the probability of winning (other than dollars spent).

Where $\beta1_i$ is the matrix of coefficients for the predictors for the probability of winning (other than dollars spent).

Where T_i is the vector of variables for the probability of winning for dollars spent (i.e., the actual number of dollars being spent).

Where $\beta2_i$ is the vector of coefficients for the predictors for the probability of winning for dollars spent

And total number of expected seats is given by

$$\sum_{\text{Candidates } i} \text{Seats}_i$$

And

- $\sum_{\text{Candidates } i} T_i$ must not exceed the budget

Dantzig and Infanger describe a useful optimization approach for using linear programming to solve such problems.¹¹ Such techniques would be easy to implement when considering the optimization of resources that the two committees would face. Furthermore, the risk constraints for which financial planners attempt to account could be similar to constraints that a party could use if it wanted to hit a “magic number.” That is, for example, if the Democrats didn’t want to maximize the mean number of seats they wished to hold, but wished to maximize their probability of winning 60 seats. Conversely, perhaps the Democrats would hope to allocate their resources in such a way that would simply maximize the probability of holding onto 51 votes. Of course, the latter scenario was unlikely to be the case in 2008.¹² While these could be real situations, by the time October 1 rolled around, the Democrats were firmly poised to hold the majority.

The data

The data that this paper will use come from the Federal Elections Commission website.¹³ For both committees, the data list every expenditure made to a candidate’s campaign for every Senatorial election from the 1998 cycle to the 2008 cycle.

Additionally, it has data for all contributions made to all campaigns during this period.

¹¹ Dantzig, GB & Infanger, G. “Multi-stage stochastic linear programs for portfolio optimization.” *Annals of Operations Research*.

¹² Such a dimension is likely beyond the scope of this paper, but could be a topic ripe for future research.

¹³ www.fec.gov

Such data is important because it is important to know how well each candidate was able to raise funds independently of the DSCC and NRSC. For example, the DSCC's lack of support for a certain candidate may simply be attributable to the fact that the candidate was financially viable in his or her own right, not because the committee viewed the election as being uncompetitive.

For each race, data has been collected for the final result of the election. Adjusted polling averages were collected from fivethirtyeight.com. Additionally, intrade.com provided trading data for all competitive senate seats, which serves as a probabilistic indicator of the winner of each senate race during the duration of the campaign.

How did the committees spend their money?

Table 2: DSCC and NRSC spending between 10/1 and Election Day

Race	DSCC Spending	NRSC Spending
ALASKA	\$1,339,067.00	\$259.00
COLORADO	\$1,628,885.00	\$2,160,721.00
GEORGIA	\$5,759,584.00	\$1,439,509.00
KENTUCKY	\$4,485,081.00	\$18.00
LOUISIANA	\$1,437,833.00	\$2,392,867.00
MAINE	\$575,481.00	\$23.00
MINNESOTA	\$6,934,855.00	\$4,453,211.00
MISSISSIPPI	\$5,134,383.00	\$3,363,662.00
NEW HAMPSHIRE	\$4,543,076.00	\$4,237,905.00
NORTH CAROLINA	\$8,205,475.00	\$3,360,006.00
OREGON	\$7,349,072.00	\$3,649,659.00
OTHER	\$777,861.00	\$771.00
TEXAS	\$39,900.00	\$36.00
Total	\$48,210,553.00	\$25,058,647.00
Correlation: .71		

Looking at table 1, several observations can be made. Firstly, the DSCC outspent the NRSC during this time period by nearly a two to one margin. Secondly, the correlation between the two committees' spending in the different races is fairly strong (.71). This makes sense, because if one party believes it can influence the Electorate in a given race by spending money, it naturally follows that the other party would reach a similar conclusion.

Table 3: Campaign Cash on hand as of 10/1

State	Republican Cash On Hand: 10/1	Democrat Cash On Hand: 10/1
Alaska	\$1,242,352.00	\$778,961.00
Colorado	\$2,733,942.00	\$545,963.07
Georgia	\$1,187,567.20	\$92,339.00
Kentucky	\$5,765,608.69	\$1,244,059.27
Minnesota	\$3,995,948.63	\$2,782,815.66
Mississippi	\$1,662,635.57	\$460,258.82
North Carolina	\$1,715,797.00	\$880,282.74
New Hampshire	\$3,629,029.28	\$1,763,014.00
Oregon	\$1,503,232.18	\$767,213.00
Texas	\$7,322,195.00	\$951,404.00

Table 4: Spending by each campaign after 10/1

State	Republican Candidate Expenditures Post 10/1	Democratic Candidate Expenditures Post 10/1
Alaska	\$932,564.00	\$1,335,323.00
Colorado	\$3,177,281.00	\$1,662,731.25
Georgia	\$1,939,144.02	\$2,501,487.00
Kentucky	\$8,484,505.54	\$4,317,737.40
Minnesota	\$5,425,990.00	\$5,222,772.00
Mississippi	\$1,930,593.02	\$963,836.52
North Carolina	\$6,310,348.00	\$3,257,376.30
New Hampshire	\$4,066,306.74	\$3,233,796.00
Oregon	\$2,239,412.13	\$2,196,114.00
Texas	\$7,355,705.00	\$1,480,134.00
Total	\$41,861,849.45	\$26,171,307.47

This leads into the broader question of what types of races are going to attract large amounts of party committee funding. The most obvious and widely cited criterion is that the race is competitive. A dollar spent in a close race is more likely to influence the outcome than a dollar spent in a less close race. While this particular subset of candidates represents only the closest races, even within this subset there is a great degree of variance in terms of how close the race was expected to be on October 1, 2008. This theory largely explains the strong correlation between party expenditures, because each party is more inclined to spend money in the closer races.

Secondly, a dollar spent in a smaller state “buys” more votes than a dollar spent in a larger state. On the surface, it would seem that this would lead to more money being spent in larger states. However, it is not always this simple. For example, the DSCC didn’t make a serious investment in Texas, even though it was projected to be a closer race as of October 1, 2008 than Maine and Louisiana. However, overcoming an 11 point deficit in such a large state would be such an expensive process that it would have drained precious resources from the coffers that could have been disbursed to other races. Therefore, in Texas, it appears that the size of the state contributed to the DSCC wanting to spend less, rather than more, money.

Thirdly, a factor to consider is the particular financial situation of the prospective recipient’s campaign. For example, the NRSC spent very little money in Kentucky during October and November. This didn’t mean that the race wasn’t competitive, it simply was a byproduct of the fact that Mitch McConnell’s (R-KY) campaign committee had raised enough of its own money that it didn’t need and party funds that could have been spent on other races with less well financed Republican candidates. McConnell’s

committee spent \$8,484,506 during October and November, more than any other committee.

Fourthly, a factor to consider is the financial situation of a prospective recipient's opponent's campaign. Once again, an examination of the situation in Texas provides a look at this phenomenon. At the end of September, John Cornyn (R-TX) had \$7,322,195 of cash on hand, and had raised a whopping \$16,858,905 over the course of the campaign. Therefore, if the DSCC had decided to deploy resources to this race and the margin had in fact tightened, Cornyn had ample resources to counteract this investment, and the fundraising apparatus to raise even more money. Such a situation likely contributed to the DSCC's decision not to become heavily involved in Texas.

Finally, other, race specific reasons could motivate whether the party committees wanted to spend money in a given state. While such a statement violates the assumption that all races are in fact equal, that is not entirely likely to be the case. One race that stands out in terms of this phenomenon is the race in Alaska, where the NRSC decided not to get involved. Senator Ted Stevens (R-AK) had recently been convicted on federal charges of making false statements. At this time, the Republican Party was making every attempt to distance itself from Stevens, and likely led to the decision not to support Stevens, even though he was involved in a close race.

Looking at the NRSC's expenditures, it appears that they were based almost entirely on the basis of how well funded its candidate was rather than the Democratic challenger. The one obvious exception is Alaska, but once Alaska is removed from the data set, knowing only the relative cash position of the candidates serves as an excellent predictor of whether or not the NRSC will spend money, in the cases of Kentucky and

Texas, they clearly stayed away because of their preferred candidate already had massive cash advantages.

So, NRSC expenditures can be explained by a 3 step process.

1. Don't give money to convicted felons.
2. Don't give money to candidates who are already extremely well funded.
3. Once candidates in categories 1&2 are eliminated, allocate money based on how close the races are.

As for the DSCC, it invested significant money into nine of the ten races being discussed. However, upon closer examination, there appears to be no real relationship between the competitiveness of the race and the amount of money the DSCC invested.

Alaska, once again, proves to be problematic in terms of this analysis, but this time for a different reason. Because it is such a small state, it is likely that it takes far less money to compete there than it would in a bigger state. Thus, it is likely that the DSCC would have spent more money there had they deemed it necessary, but that they simply felt that it wasn't necessary.

However, perhaps there is something else at work here. Alaska and Colorado are the states in which the Democrats had been at a relatively small financial disadvantage (as measured by money disbursed prior to 10/1). As such, these are the types of states where the democrats simply had to hold their ground and match Republican expenditures, rather than overcome a massive cash disadvantage.

What is the effect of all money spent during the last month of the campaign?

In Erikson's analysis on House races, he points out that spending of incumbents and challengers has roughly the same effect once the election is projected to be decided by ten points or less. All of the races here are projected to be within eleven points as of October 1, 2008. However, the circumstance under which each race is close varies from race to race. Specifically, some races are close largely because each side had spent relatively comparable amounts prior to October 1, whereas in some cases, the race was close in spite of the fact that one side had outspent the other by a large margin.

Intuitively, one would expect that each additional dollar spent in a race would have diminishing marginal returns. Indeed, all of the literature referenced here uses the natural logarithm of spending as the explanatory variable for vote share, rather than just dollars spent outright.

What this means in terms of the analysis at hand is that one would expect that the marginal effect of a dollar spent in a race in which a given candidate was being badly outspent would have a greater effect than a dollar spent in a race in which the spending was previously relatively evenly distributed between the two sides, or in a race where the candidate was outspending his or her opponent.

An example of this phenomenon can be found by comparing the dynamics of the races in Georgia and Mississippi. The projected margin in Mississippi on October 1, 2008, was that the Republican candidate would win by 4.9 points, whereas in Georgia, the margin was projected to be a Republican win by 9.9 points. However, when one considers that prior to 10/1 that Jim Martin (D) had been outspent 4 to 1 in Georgia, whereas Ronnie Musgrove (D) had actually outspent Roger Wicker in Mississippi, one

could say that the Democrat in Georgia was actually performing better, given this handicap. Furthermore, it is possible that such a candidate could potentially benefit from closing the spending gap with his opponent.

Thus, a good way to measure the effect of money spent after October 1 is to explore how it altered the dynamics of spending relative to what they were before October 1. These tables summarize this breakdown, along with the percentage of money in the race being spent by or on behalf of the Democratic candidate. Note that Louisiana is also included here. Although the race was projected to be a 14 point Democratic lead on October 1, it is included because each committee spent significant funds there during October.

Tables 5&6: Spending by and on behalf of each campaign, prior to and after 10/1

State	Total Democrat Pre 10/1 Spending	Total Republican Pre 10/1 Spending	Democratic Share of Pre 10/1 Spending
AK	\$3,342,697.00	\$4,206,834.00	0.44
CO	\$13,737,794.75	\$5,563,900.00	0.71
GA	\$2,411,356.00	\$9,363,489.98	0.20
KY	\$5,906,311.60	\$10,691,686.46	0.36
LA	\$8,225,875.00	\$5,905,121.00	0.58
MN	\$15,453,397.00	\$15,955,736.00	0.49
MS	\$5,120,298.48	\$4,716,573.98	0.52
NC	\$10,019,078.70	\$15,224,688.00	0.40
NH	\$9,242,998.00	\$7,191,690.26	0.56
OR	\$9,702,040.00	\$11,306,016.87	0.46
TX	\$2,473,039.00	\$10,776,646.00	0.19

State	Total Democrat Post 10/1 Spending	Total Republican Post 10/1 Spending	Democratic Share of Post 10/1 Spending
AK	\$2,674,390.00	\$932,823.00	0.74
CO	\$3,291,616.25	\$5,338,002.00	0.38
GA	\$10,757,669.00	\$5,099,704.02	0.68
KY	\$8,802,818.40	\$8,484,523.54	0.51
LA	\$5,871,963.00	\$3,325,467.00	0.64
MN	\$12,157,627.00	\$9,879,201.00	0.55
MS	\$6,098,219.52	\$5,294,255.00	0.54
NC	\$7,800,452.30	\$10,548,253.00	0.43
NH	\$11,439,271.00	\$7,426,312.00	0.61
OR	\$9,545,186.00	\$5,889,071.13	0.62
TX	\$1,520,034.00	\$7,355,741.00	0.17

In order to define how these dynamics changed in each race after October 1, Change shall be defined as follows:

$$\text{Change} = (\text{Dem Share Post 10/1} - \text{Dem Share Pre 10/1}) / (\text{Dem Share Pre 10/1})$$

The most drastic shift occurred in Georgia, where Jim Martin went from being outspent by a 4 to 1 margin to outspending Saxby Chambliss by a 2 to 1 margin.

Testing this metric in a regression framework, it is in fact a statistically significant predictor that the Democratic candidate will gain in the race if he or she changes the ratio of spending to his or her opponent relative to what it was prior to October 1.

$$\text{Gain} = B_0 + B_1 * \text{Change} + B_2 * \text{PVI} + B_3 * \text{Blunder}$$

Where Gain is defined as the gain that the Democratic candidate made in the election relative to October 4.

Where Change is the change in campaign spending dynamics as defined above.

Where PVI is the Partisan Voting Index in each state.¹⁴ A neutral state has a value of zero, Democratic leaning states have positive values, Republican leaning states have negative values.

Where Blunder is a dummy variable, 1 for Alaska and North Carolina, 0 for all other races.

¹⁴ Cook Political Report

	coef.est	coef.se
(Intercept)	0.88	1.21
Change	3.59	1.31
PVI	0.52	0.17
Blunder	5.09	2.44
residual sd = 2.96, R-Squared = 0.69		

As can be seen from the regression results, all three variables are statistically significant at the 95% level. PVI and whether the Republican campaign committed a blunder were important to control for, and improved the model fit. Obviously a Republican blunder would be expected to help the Democrats. Additionally, previous research by Silver indicates that the electorate has tended to move toward its partisan leanings in recent elections, decreasing the likelihood that the correlation examined here is spurious.¹⁵

Change being positive and statistically significant means that by altering the spending dynamics in the race, Democratic candidates could, in fact, expect to see that translate into an increased vote share at the polls on Election Day. In the case of Georgia, the regression model estimates that such a shift in the dynamics of financial support netted Jim Martin, the Democratic candidate, an additional 8.9 points of support (3.59×2.47).

PVI being positive and statistically significant meant that the electorate did in fact tend to break toward the Democratic candidate in Democratic leaning states and the

¹⁵ <http://www.fivethirtyeight.com/2008/09/was-obamas-50-22-state-strategy-mistake.html>

Republican candidate in Republican leaning states, relative to where each candidate stood in the polls in early October.

Ted Stevens was convicted on felony charges in October 2008, and Elizabeth Dole (R-NC) ran the now infamous “Godless” Ad against her opponent, Kay Hagan.¹⁶ These two events clearly altered the dynamics of each race, causing their opponents to outperform what would be expected against a candidate who did not commit such blunders. As measured by the regression, such blunders boosted the Democrats’ gains in those races by about 5 points.

The R^2 value of .69 indicates that the model fits the data reasonably well, and indicates that knowing these factors alone can allow one to make a reasonable prediction of how the final results would turn out. For example, following this equation, it would be no surprise that the Democrat would become more competitive in Georgia than in Texas simply as a byproduct of far outspending the Republican counterpart in Georgia.

Given the results of this regression equation, and the residual standard deviation of 2.96, not only can the final outcomes be predicted, but the probability of each side winning the election can be inferred via a normal distribution centered on the point estimates with the residual standard deviation representing the uncertainty.

¹⁶ <http://blogs.abcnews.com/politicalradar/2008/10/dole-sued-for-g.html>

Table 7: Expected results as measured by regression on 10/4

State	Prob of D Win	Expected D Margin
AK	0.93	4.23
CO	0.96	5.21
GA	0.13	-3.29
KY	0.00	-9.19
LA	1.00	9.81
MN	0.69	1.48
MS	0.00	-8.60
NC	0.98	6.60
NH	0.99	6.30
OR	0.95	4.88
TX	0.00	-14.58
NC (No Blunder)	0.70	1.52
AK (No Blunder)	0.39	-0.90
GA (Outright Win)	0.01	-3.29

Also included are the hypothetical results if “blunders” had not occurred in Alaska and North Carolina. While Stevens’ legal troubles were a known fact going into October, the “godless” ad was the type of exogenous shock that could have not been predicted on October 1. Therefore, when assessing the wisdom of each side spending the sums of money they did on North Carolina, perhaps it would be fair to evaluate such decisions in the context of there being no such blunder.

Also included is the probability to Jim Martin winning the Georgia Senate race outright. The dynamics of the special election did not favor Martin, therefore, it could be said that for the Democrats to really have a chance to win the seat, they really needed to actually make up more additional ground on Chambliss than just winning a simple plurality. Perhaps such a consideration should have been made before the DSCC decided to plow so many resources into Georgia.

Table 8: Comparison between regression predictions and Intrade Implied Probability for Democratic Candidate to win.

State	Regression Model	Intrade
AK	0.93	0.65
CO	0.96	0.8
GA	0.13	0.15
KY	0	0.2
LA	1	0.82
MN	0.69	0.38
MS	0	0.36
NC (No Blunder)	0.7	0.59
NH	0.99	0.7
OR	0.95	0.52
TX	0	0.15
Correlation	.90	

In order to test the extent to which such a regression estimate was reliable, the correlation with intrade prices was tested. The high correlation (.90) indicates that the model does a reasonable job of estimating the probability of the outcome of a race as of October 4. Furthermore, the fact that the regression model tends to be more polar, thus implying that intrade overstates the underdog’s probability of winning a given race, is consistent with previous research.¹⁷

Optimization Summaries

Several different optimizations were run from the perspective of each committee. Here only two for each side are presented. The first for each side works under the assumption of what each side would have done in retrospect, with no constraints, with perfect information (namely, that Elizabeth Dole would commit the blunder that she did). The second set for each side assumes that such a blunder would be unknown, and that the

¹⁷ <http://online.wsj.com/article/SB122531904458481925.html#articleTabs%3Darticle>

NRSC would not invest any resources in Alaska regardless of the optimal strategy for maximizing seats won.

DSCC Optimization 1, Perfect Information

State	Optimized Spending	Original Spending	Optimized Prob of Win	Original Prob of Win	Gain
AK	2,625,202	1,339,067	0.95	0.93	0.03
CO	1,438,553	1,628,885	0.96	0.96	0.00
GA	23,040,401	5,759,584	0.46	0.13	0.34
KY	0	4,485,081	0.00	0.00	0.00
LA	0	1,437,833	1.00	1.00	0.00
MN	12,203,115	6,934,885	0.77	0.69	0.08
MS	0	5,134,383	0.00	0.00	0.00
NC	2,824,033	8,205,475	0.96	0.99	-0.03
NH	515,018	4,543,076	0.96	0.99	-0.02
OR	5,564,642	7,349,072	0.94	0.95	-0.01
TX	0	39,900	0.00	0.00	0.00
Total	48,210,964	46,857,241	7.02	6.64	0.38

NRSC Optimization 1, Perfect Information

State	Optimized Spending	Original Spending	Optimized Prob of Win	Original Prob of Win	Gain
AK	8,520,269	259	0.50	0.07	0.43
CO	0	2,160,721	0.02	0.04	-0.01
GA	4,526,258	1,439,509	0.98	0.87	0.11
KY	0	18	1.00	1.00	0.00
LA	0	2,392,867	0.00	0.00	0.00
MN	5,975,617	4,453,211	0.34	0.31	0.03
MS	0	3,363,662	0.99	1.00	-0.01
NC	0	3,360,006	0.00	0.01	-0.01
NH	0	4,237,905	0.01	0.01	-0.01
OR	6,035,146	3,649,659	0.07	0.05	0.03
TX	0	36	1.00	1.00	0.00
Total	25,057,291	25,057,853	4.91	4.36	0.56

DSCC Optimization 2, No Knowledge of “Godless” Ad

State	Optimized Spending	Original Spending	Optimized Prob of Win	Original Prob of Win	Gain
AK	2,335,107	1,339,067	0.95	0.93	0.02
CO	1,083,919	1,628,885	0.96	0.96	-0.01
GA	18,586,015	5,759,584	0.42	0.13	0.29
KY	0	4,485,081	0.00	0.00	0.00
LA	0	1,437,833	1.00	1.00	0.00
MN	9,825,100	6,934,885	0.74	0.69	0.04
MS	0	5,134,383	0.00	0.00	0.00
NC	12,778,799	8,205,475	0.78	0.70	0.08
NH	0	4,543,076	0.96	0.99	-0.03
OR	4,744,918	7,349,072	0.93	0.95	-0.02
TX	0	39,900	0.00	0.00	0.00
Total	49,353,857	46,857,241	6.73	6.36	0.37

NRSC Optimization 2, No Knowledge of Godless Ad, Cannot invest in Alaska

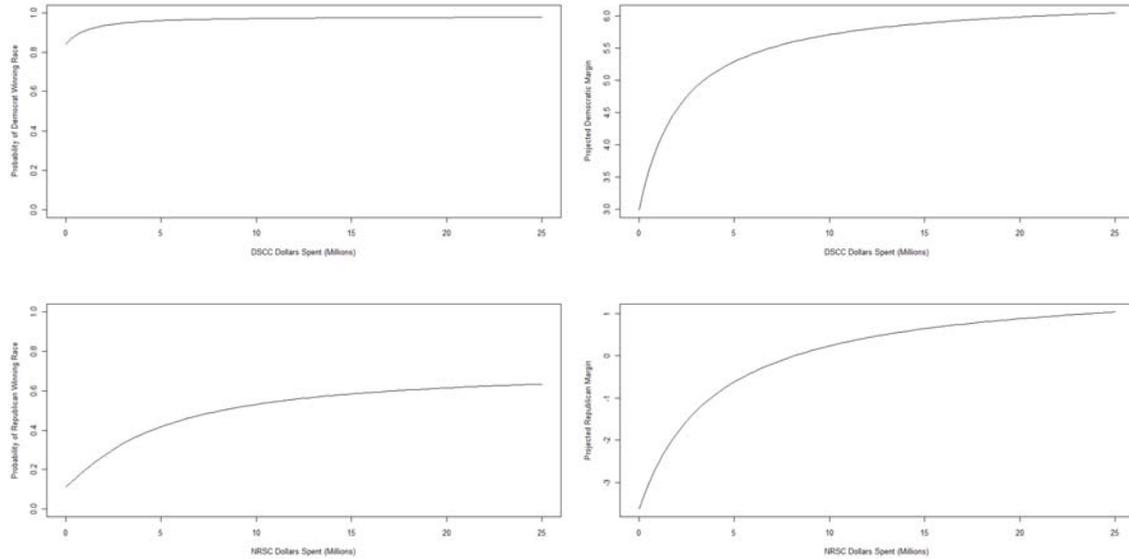
State	Optimized Spending	Original Spending	Optimized Prob of Win	Original Prob of Win	Gain
AK	0	259	0.07	0.07	0.00
CO	0	2,160,721	0.02	0.04	-0.01
GA	3,873,580	1,439,509	0.97	0.87	0.10
KY	0	18	1.00	1.00	0.00
LA	0	2,392,867	0.00	0.00	0.00
MN	8,614,900	4,453,211	0.39	0.31	0.08
MS	0	3,363,662	0.99	1.00	-0.01
NC	12,556,700	3,360,006	0.50	0.30	0.20
NH	29	4,237,905	0.01	0.01	-0.01
OR	12,670	3,649,659	0.01	0.05	-0.03
TX	0	36	1.00	1.00	0.00
Total	25,057,878.05	25,057,853.00	4.96	4.64	0.31

Could each side have deployed its resources more effectively?

The statistician George Box once said that “all models are false, but some models are useful.” Such a line of reasoning is appropriate when using these optimizations as a way of evaluating the spending of the DSCC and NRSC in each state during this time period. This study is not meant to be a mathematically infallible road map for how the DSCC and NRSC should spend their money. Nevertheless, some conclusions from the optimizations can yield useful strategic information for how each committee could best deploy its resources. As such, a state by state breakdown of what insight could be gained from the optimizations follows.

Alaska

Figure 1: Effect of Committee Dollars spent on outcome in Alaska, In terms of margin of victory and projected winning percentage. Each is based on the assumption that the other side would not adjust its spending



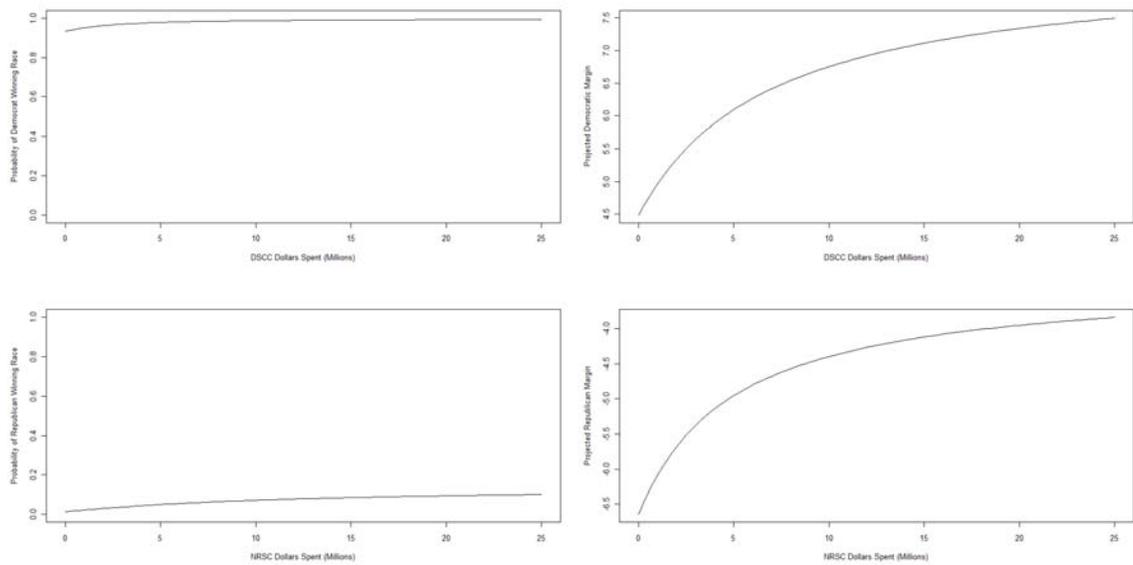
The first iteration of the optimization from a NRSC perspective recommended that the NRSC invest very heavily in Alaska. Clearly, the NRSC made a very concerted effort not to do this. Such a decision may well have cost them this seat. This is not to criticize the NRSC for making an irrational decision; openly supporting a convicted felon may have had intangible costs.

An optimization from a DSCC perspective shows that increasing their spending in the race from \$1.3 million to \$2.8 million would have been the proper decision in the context of maximizing the number of seats the Democrats would win. According to the model, such an investment would have increased Mark Begich's chances of winning from 92.7% to 95.2%.

Owing to Alaska’s tiny population, it is possible that the model overstates the marginal effect of a dollar spent there. Nevertheless, it seems likely that the Democrats probably should have spent more money here to increase the likelihood of a victory, and that money would have been better spent here than in states like Mississippi or Kentucky.

Colorado

Figure 2: Effect of Committee Dollars spent on outcome in Colorado, In terms of margin of victory and projected winning percentage. Each is based on the assumption that the other side would not adjust its spending



The NRSC seemed to waste money here in what was likely a lost cause. All iterations of the optimization confirm this as well, as they all suggest that the Republicans should have spent no money there. Even though this was the race in which the Republican candidate had been outspent by the widest margin prior to October 1, there was simply too big a lead for Bob Schafer to overcome in a state where the partisan dynamics did not work for the Republicans. Interestingly, given the NRSC spending, the optimizations indicate the DSCC spending was a wise counterpunch, as optimized

spending remains above \$1,000,000, even in a world in which North Carolina is much closer.

Georgia

Figure 3: Effect of Committee Dollars spent on outcome in Georgia, In terms of margin of victory and projected winning percentage. Each is based on the assumption that the other side would not adjust its spending

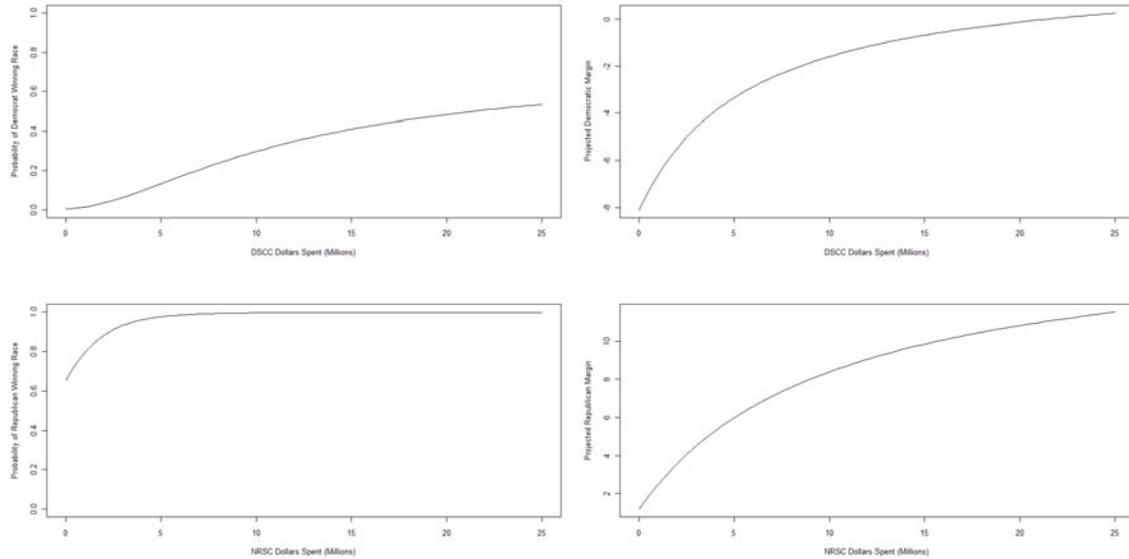
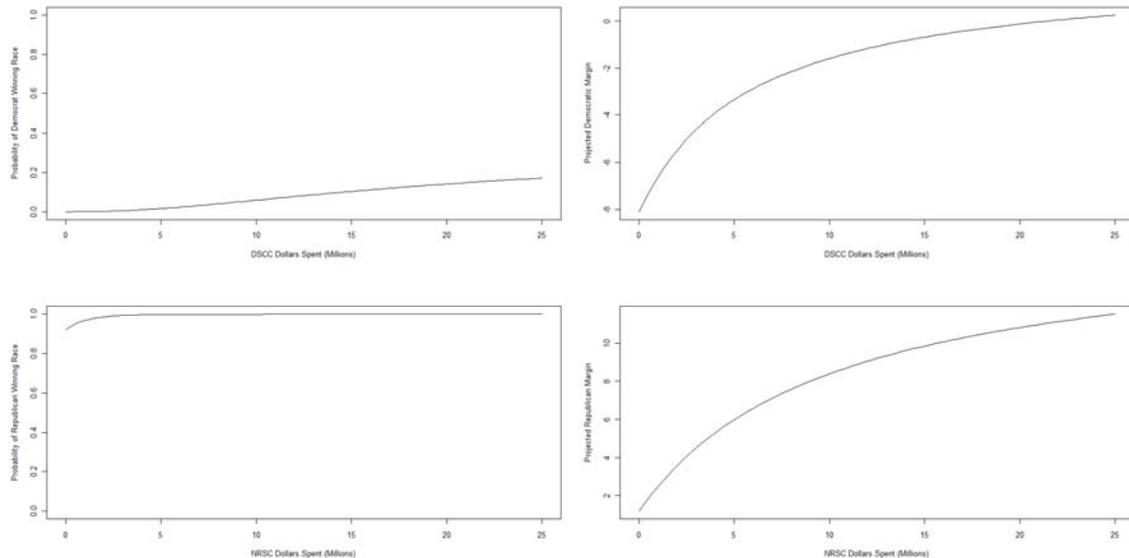


Figure 4: Effect of Committee Dollars spent on outcome in Georgia, but under the assumption that the Democrats would need an outright win



Whether or not each side should have spent more or less in Georgia depends upon whether or not the goal of each committee was to get more votes than its opposition, or to win an outright majority.

In the context of the regression and optimization models, Georgia is an interesting state to analyze. On the one hand, Jim Martin (D) trailed Saxby Chambliss (R.) by ten points at the beginning of October, leaving a massive deficit to be overcome. On the other hand, Chambliss had badly outspent Martin going into October. As such, the regression model predicted that Martin could have gained significant ground by changing the spending dynamics in the campaign. Furthermore, Chambliss did not have large cash reserves going into October, meaning that if the DSCC chose to invest heavily in Georgia, it could succeed in changing those spending dynamics.

If the Democrats were aiming simply for a plurality of the vote, an optimization from their perspective indicates that they should have invested a whopping \$23 million in the race. This figure is likely overstated, as it would have been a massive amount to spend in such a short period of time. However, there is validity to the fact that the DSCC could have likely effectively spent much more than they actually did. Firstly, not much money had previously been spent in Georgia, which logically dictates that a large infusion of cash could have been deployed effectively. Secondly, Georgia is a relatively large state (smaller than only Texas and North Carolina of the states in this study), which again largely dictates that cash could have been spent effectively. So, while \$23 million may have been a stretch, the DSCC could have effectively spent much more than the \$5.8 million that they actually did. Had the \$23 million been spent, according to the

regression model, Martin's chances of gaining more votes than Chambliss would have increased from the 13% originally predicted to 46%.

Unfortunately for Martin, simply getting more votes than Chambliss would not be sufficient, as Georgia requires the winning candidate to obtain 50% of the vote, or else a runoff is held. Furthermore, Martin's chances of winning a runoff certainly seem dubious in retrospect (in reality, he lost the runoff by 14 points). Such a result was not totally unexpected, as much of Martin's success in the general election was riding the coattails of Democratic Presidential candidate Barack Obama, who was, of course, not on the ballot for the runoff. Intrade pegged to odds of Martin winning the runoff at 95.5% the day after the general election.

To account for this reality, an optimization was run from a DSCC perspective where only an outright win is counted as a Democratic pickup. In such a scenario, the optimal allocation for the DSCC in Georgia drops to \$0. Such a feat would have required Martin to gain 13 points rather than 10, and the marginal cost of gaining those extra points would have simply been too high to justify at the expense of investing in other states.

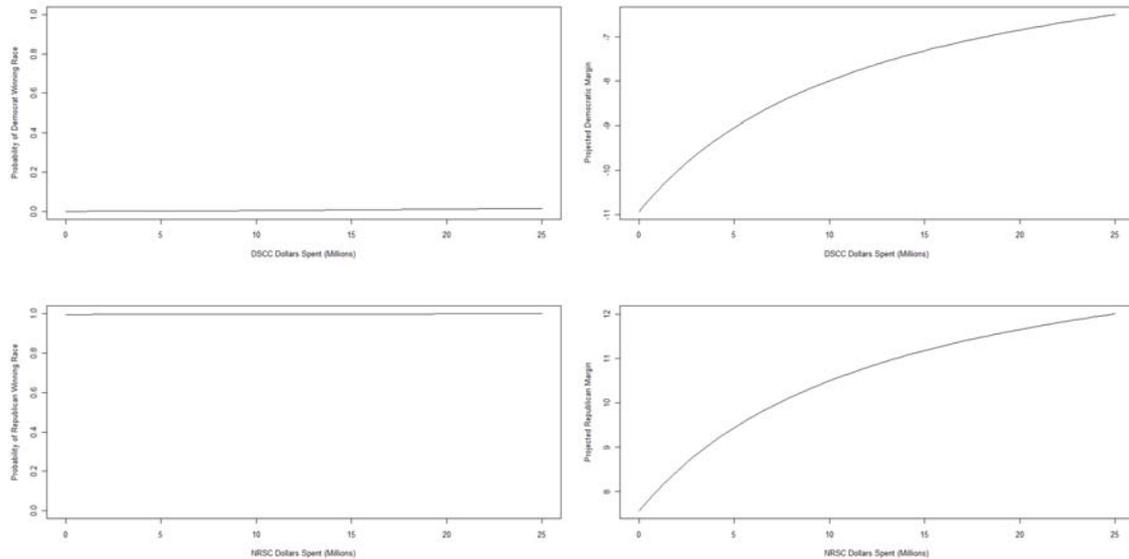
So, as it stands, the DSCC investment of \$5.7 million in Georgia seems questionable. Either it should have stayed away altogether if they realistically felt that winning an outright majority was necessary, or they should have made a larger investment (taking funds from places like Louisiana and Kentucky) had they deemed winning a plurality to be sufficient.

As for the NRSC, which spent \$1.4 million, it should spent more money if they wanted to win a plurality, or simply avoid a runoff. On the other hand, if they were

confident that they could win a runoff (which, in retrospect, they had every right to be), there was no need to invest additional funds, and they probably could have gotten away with spending even less money.

Kentucky

Figure 5: Effect of Committee Dollars spent on outcome in Kentucky, In terms of margin of victory and projected winning percentage. Each is based on the assumption that the other side would not adjust its spending



Kentucky seemed to be a state where the DSCC should have either stayed away entirely, or else spent a lot more money than they did. As it stood, they managed to tread water in the race, but never were a real threat (they stood to win less than .01% of the time according to the model with the original spending in place).

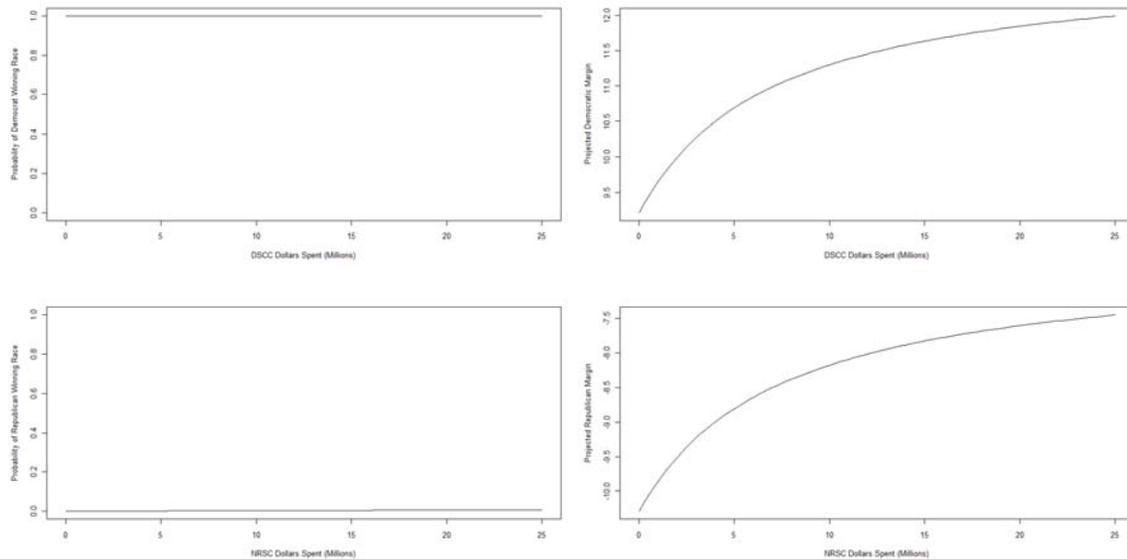
The results of the optimizations clearly indicate that staying away altogether would have been the best option, whatever increased odds of winning that could have been attained by increased investment wouldn't have compared to the returns that would have been realized in other states. Furthermore, this model actually understates the mistake that the DSCC made by investing so much money in Kentucky because it

doesn't take into account that Mitch McConnell could have spent even more money to counteract the DSCC investment had the race actually tightened. So this spending appears to have been foolish.

The NRSC didn't have to make much of a decision in Kentucky considering McConnell's considerable cash holdings. That said, not getting involved was clearly the correct decision, as confirmed by the optimizations.

Louisiana

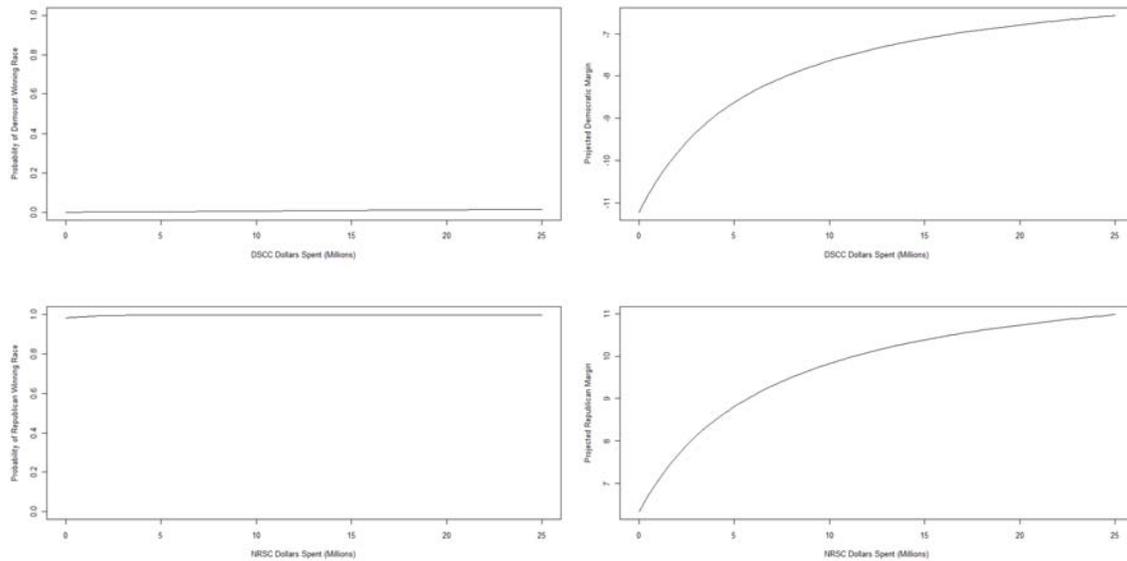
Figure 6: Effect of Committee Dollars spent on outcome in Louisiana, In terms of margin of victory and projected winning percentage. Each is based on the assumption that the other side would not adjust its spending



No deep analysis is needed here; it was a firm Democratic seat on October 1, and it remained so throughout the process. Both sides wasted money here. The fact that Republican John N. Kennedy ended up closing the gap to lose “only” by six points doesn't justify the expenditure.

Mississippi

Figure 7: Effect of Committee Dollars spent on outcome in Mississippi, In terms of margin of victory and projected winning percentage. Each is based on the assumption that the other side would not adjust its spending



This was a race in which both parties invested considerable resources, neither of which seemed to be justified.

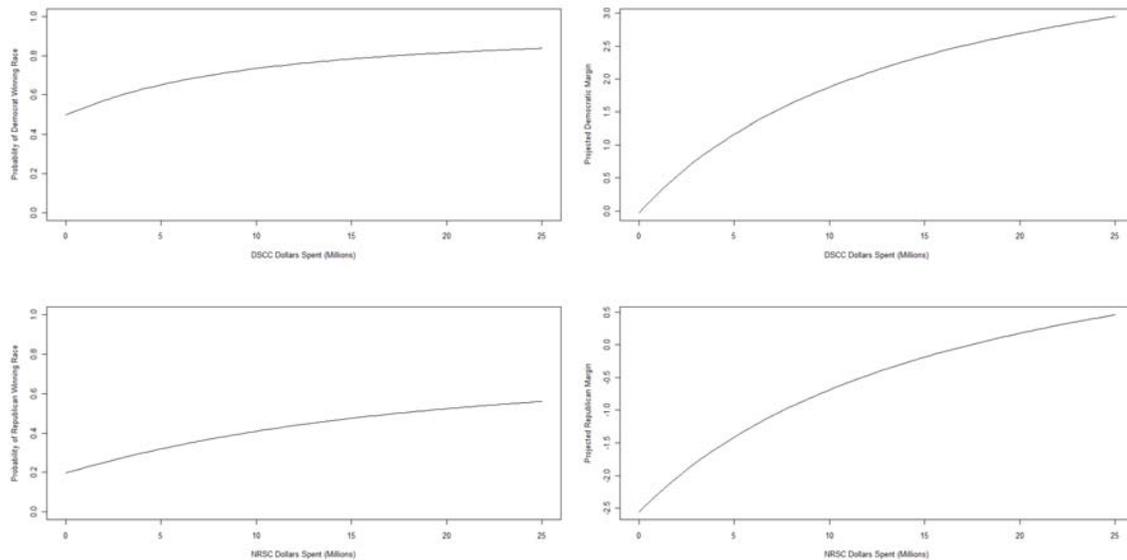
Starting with the Democrats, the \$5 million they spent here would have been much more wisely invested in states such as Alaska, Minnesota, Georgia, or North Carolina (the latter two depending on what assumptions they worked under). Democratic expenditures already exceeded Republican expenditures prior to October 1 in Mississippi, and the partisan dynamics certainly worked against the Democrats.

The NRSC appears to have been equally foolish. According to the optimization model, the Roger Wicker would have won the election over 99% of the time regardless of what the NRSC spent, where in reality the NRSC spent over \$3 million.

However, game theoretical considerations must be taken into account here as well. It is possible that had the NRSC telegraphed that they weren't going to spend any money in Mississippi, that the DSCC would have then invested even more money in an attempt to make it competitive. To test this, an optimization was performed from a DSCC perspective in which the NRSC didn't invest any money in Mississippi. Even under this scenario, it would have been optimal for the DSCC to not invest any money in Mississippi. Thus, the NRSC clearly shouldn't have wasted any money defending such a safe seat, because even if the DSCC had invested there, it would have been suboptimal and thus benefited the Republicans.

Minnesota

Figure 8: Effect of Committee Dollars spent on outcome in Minnesota, In terms of margin of victory and projected winning percentage. Each is based on the assumption that the other side would not adjust its spending



This was an interesting race to analyze in the context of this study's regression and optimization models. On the one hand, the marginal effect on vote share of an additional dollar spent would be expected to be quite small, given how much money was already being spent by the two sides. On the other hand, this race was expected to be (and ended up being) very competitive, so every additional vote that could be garnered had a high degree of leverage over who would ultimately get more votes.

All versions of the optimization model indicate that the two campaigns should have spent more money here. While there obviously is some frontier at which the value of a marginal dollar spent would eventually be meaningless, it is unlikely that such a frontier had been met, even in this very expensive race. And with a final margin of victory that ended up being 215 votes, each side would have been better off picking up a few hundred votes here than a few thousand in states like Louisiana or Colorado.

North Carolina

Figure 9: Effect of Committee Dollars spent on outcome in North Carolina, In terms of margin of victory and projected winning percentage. Each is based on the assumption that the other side would not adjust its spending

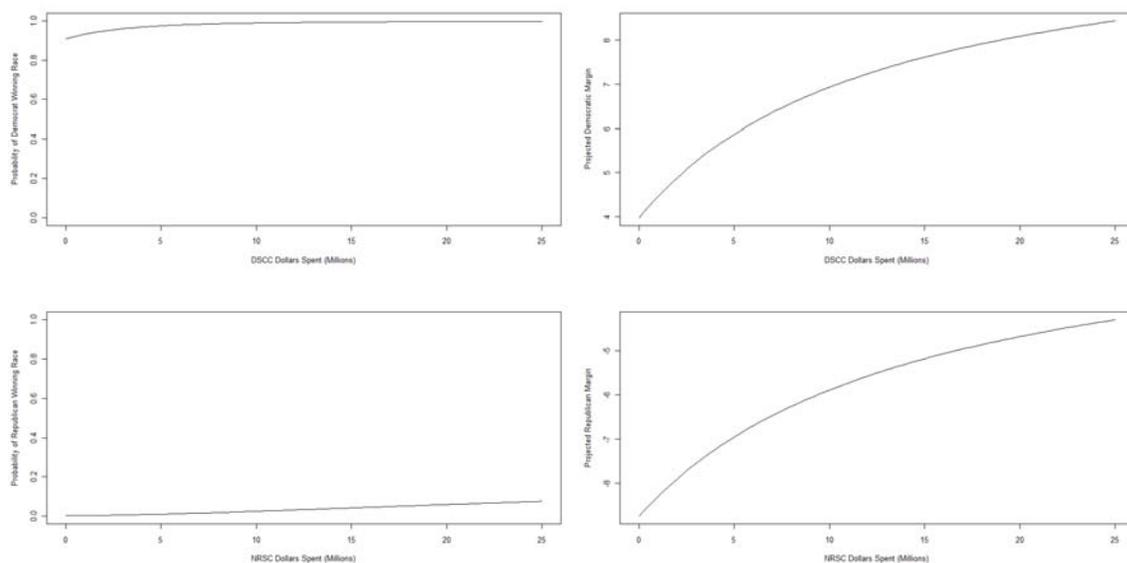
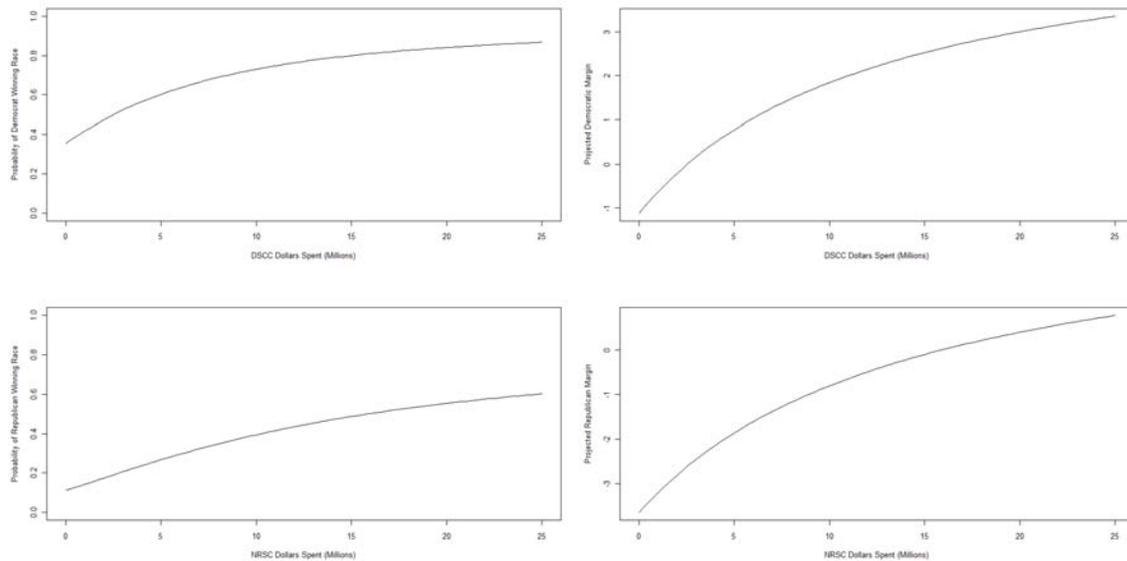


Figure 10: Effect of Committee Dollars spent on outcome in North Carolina, assuming no knowledge that the dynamics of the campaign would change owing to the “Godless Ad”.



In retrospect, knowing that this race would be decided by such a large margin largely due to the “Godless” ad, neither party should have invested any money here. However, nobody knew that this would transpire on October 1, and that being the case, the optimization indicates that each side should have invested significantly more money in the race. The margin in this state was expected to be tight on October 1, and the partisan dynamics did not favor the Republicans by a wide enough margin to make a substantive difference in the expected results.

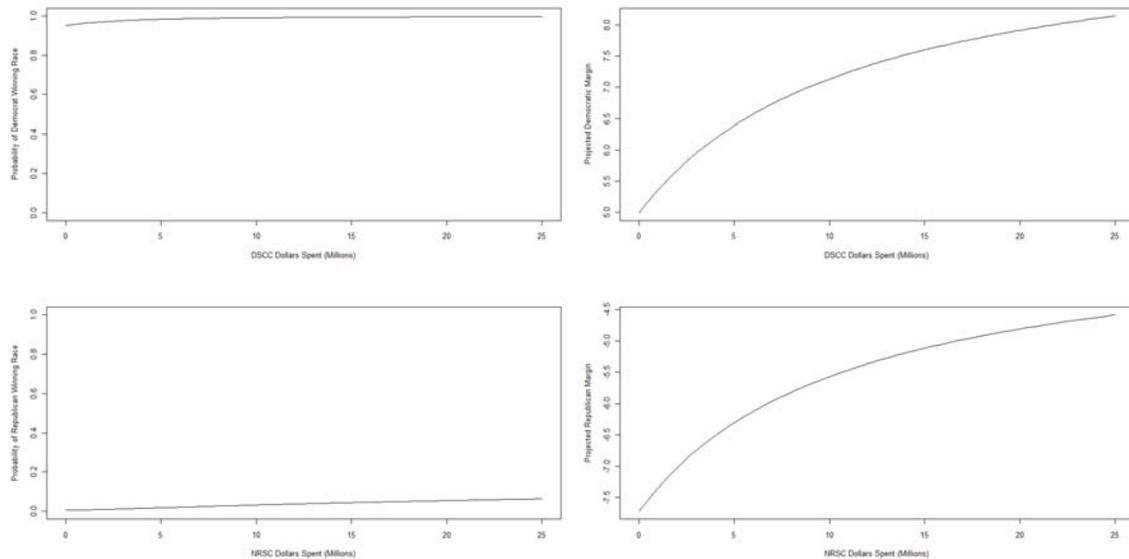
Interestingly under the assumption that Dole *would* commit such a blunder, optimization from a DSCC perspective still indicates that it should spend nearly \$3 million. It is a sharp decrease from the \$8 million it actually spent, but it isn’t optimal under such a scenario to drop down to spending nothing, as it is in Kentucky, Louisiana and Mississippi. According to the model, such an investment gave Kay Hagan a 96.2%

chance of winning the election, as opposed to the 91.4% chance that she would have had if the DSCC had not invested any money. By investing the \$8 million that the DSCC actually did, Hagan ended up having a 98.9% chance of emerging victorious.

Without the godless ad occurring, the model estimates that Hagan would have had about a 70% chance of winning. Under such an assumption, the model targets North Carolina as a state that should have been more heavily invested in by both sides.

New Hampshire

Figure 11: Effect of Committee Dollars spent on outcome in Minnesota, In terms of margin of victory and projected winning percentage. Each is based on the assumption that the other side would not adjust its spending



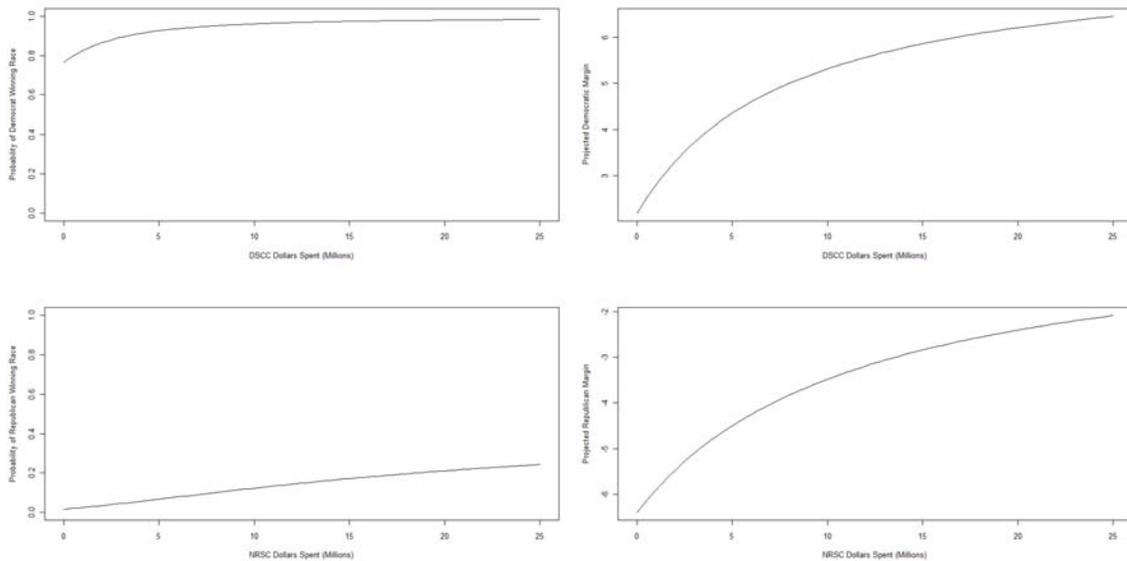
Like Colorado, Kentucky and Mississippi, this is a state where both sides appear to have wasted resources, as Jeanne Shaheen was in a commanding position as of October 1. Under all assumptions, the Republicans should have given up on winning this seat and focused on more winnable races.

The same logic largely applies to the DSCC. From a DSCC perspective optimization that assumes that the NRSC would spend what they actually did (\$4.2 million), the optimal allocation for the DSCC is \$618,000, down from the \$4.5 million that they actually invested. If the NRSC had (properly) invested nothing in the race, the DSCC would have been wise not even to have spent the six figure sum.

Like in Mississippi, game theoretical considerations were tested, to see if the NRSC would have been better off investing in the race had the DSCC not spent any money. However, like in Mississippi, even under such a scenario, the NRSC still should have saved their money. Therefore, the optimizations indicate that both sides erred in investing any money here.

Oregon

Figure 12: Effect of Committee Dollars spent on outcome in Oregon, In terms of margin of victory and projected winning percentage. Each is based on the assumption that the other side would not adjust its spending



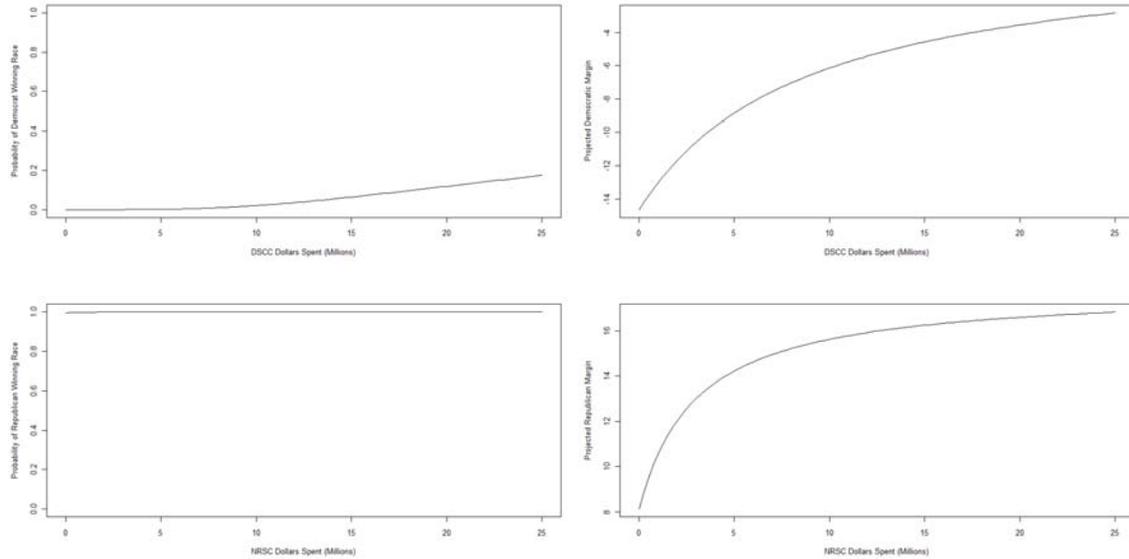
Jeff Merkley (D) was a strong favorite over incumbent Gordon Smith (R.) by October 1, but the outcome of the race was much more in doubt than the races in Texas, Mississippi, Kentucky, Louisiana and Colorado were.

The optimizations yielded different results for the allocations that the NRSC should have made to the race in Oregon depending upon the other assumptions. Under the assumption where it was known that North Carolina would become uncompetitive owing to the “Godless” ad, the optimal allocation for the NRSC increases, but under the more realistic scenario in which such a blunder on Dole’s part was not anticipated, the optimal allocation in Oregon drops.

From a DSCC perspective, its recommended allocation of funds to Merkley decreases under any optimization scenario (as opposed to what they actually allocated), and drops to a greater extent if North Carolina is considered competitive, because the optimization thought that funds disbursed in North Carolina would have higher leverage than the money that was in actuality spent in Oregon.

Texas

Figure 13: Effect of Committee Dollars spent on outcome in Texas, In terms of margin of victory and projected winning percentage. Each is based on the assumption that the other side would not adjust its spending



Texas shared the characteristic with Georgia that the Republicans had heavily outspent the Democrats prior to October 1. Therefore, it is possible that had heavy DSCC been able to shift the dynamics of spending in the race, that Rick Noriega could have made significant gains on John Cornyn.

However, the big difference between the two states is that Cornyn had significant cash on hand when compared to Saxby Chambliss in Georgia. Therefore, the spending dynamic simply could not have been shifted that radically, because Cornyn's campaign spent \$7.4 million on the race after October 1, whereas Chambliss' campaign only spent \$1.9 million. Furthermore, such an optimization does not even account for the fact that Cornyn could have spent even more money had he actually needed to. Therefore, it was a wise decision for the DSCC not to spend money in Texas, and this decision is

confirmed by the optimizations, all of which indicate that the DSCC shouldn't have spent any money there. Given this decision by the DSCC, the decision by the NRSC not to spend any money was an even easier one.

Conclusions

Research by Daniel Kahneman and Amos Tversky indicates that people are more likely to feel the pain of something lost than the pain of potentially not gaining something they never had in the first place.¹⁸ Such a theory would indicate that the NRSC and DSCC would be more likely to overinvest in races where they were leading in order to protect such a lead, rather than to make a calculated risk into a race where they trailed.

The DSCC fell victim to such a behavioral bias in New Hampshire and Louisiana, where it perhaps could have made a tactical gamble in Georgia that could have paid large dividends. Republicans, on the other hand, seem guiltier of overinvesting in races where they never had a chance rather than spending too much money protecting leads (with the notable exception of Mississippi).

Another concept that comes to mind in evaluating the allocation decisions of the two committees is the gambling statistical concept of non-self-weighting strategies.¹⁹ Such a concept states that entities should weight their allocations toward a small number of opportunities in which the expectation and variance are favorable, rather than spread them out more evenly. In most realms, entities are more likely to over-diversify. Clearly, both committees appeared to over-diversify during the 2008 Senatorial election cycle.

¹⁸ Kahneman, Daniel, and Amos Tversky (1979) "Prospect Theory: An Analysis of Decision under Risk", *Econometrica*, XLVII (1979), 263-291.

¹⁹ Malmuth, Mason. *Gambling Theory and Other Topics*. Las Vegas, Two Plus Two Publishing, 2004.

One interesting conclusion of these optimizations is that under any set of assumptions, the highest impact that proper portfolio optimization could have had on the expected number of seats won would be about .3 seats. So, while there are definitely gains to be made, it is not as if hiring a team of portfolio engineers would yield five additional seats every cycle.

Keeping in mind Box's insight about the value of models, both the NRSC and the DSCC would be wise to carefully examine how they allocate their funds, taking an approach similar in structure (even if different in the details) to this study. If such analysis could lead to an additional seat won every three election cycles (.3 wins per cycle), such analysis would be highly worthwhile. Rod Blagojevich correctly surmised that a Senate seat is a "[bleeping] valuable thing", and correctly allocating precious funds can have as great an effect as crafting advertising messages, coordinating get out the vote efforts, or trying to raise additional funds.

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