

Assessing the Costs of Fractional Reserve Banking:  
A Theoretical Exposition and Examination of Post-Meiji Japan

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## Section I: Introduction

Fractional reserve banking – a system in which banks retain only a small percentage of the deposits their clients have entrusted to them while lending out the remainder at interest – has come under fire in recent years from some elements that once remained fringe, but have since entered into the mainstream under the pretense of serious economic inquiry. The “Austrian School of Economics” has seen a resurgence in interest, particularly among those who identify as part of the “Tea Party” in America, many of whom have an interest in finding appealingly simple explanations to the recent crisis. Congressman Ron Paul’s renewed calls to “End the Fed” were a charming distraction years ago; now they are at the head of an organized effort to audit the Federal Reserve, a move which many economists regard as dangerous because of the potential for political interests to begin to dictate monetary policy (see, *e.g.*, Krugman 2011). This outcome is particularly undesirable, apart from the potential for special interests to corrupt their motives, when members of Congress do not possess the analytical framework to accurately assess proper monetary policy.

The so-called “Austrian School of Economics” embodied in the works of Ludwig von Mises, Friedrich Hayek, Murray Rothbard and others, created a brand of economics that appeals to many dilettante economists (Krugman 1998). It is appealing first because it is openly hostile to the methodology of mainstream economics, namely that of highly rigorous mathematical modeling and statistical inference to test, refine and calibrate those models. This appeals to many who feel excluded from a field of social science that until the middle of the 20<sup>th</sup> Century was quite open to lay study. This hostility to quantitative

analytical rigor gives them *carte blanche* to engage in economic analysis without having to endure the quite difficult mastery of econometric and advanced mathematic techniques that serve as the toolkit for the modern economist. Second, and more importantly, the Austrian School offers appealingly simple explanations for economic disturbances. Austrian Economists will hardly mention anything about the shadow banking system's role in the recent financial crisis, and will instead point toward the Federal Reserve's influence and government intervention in the free market as the primary culprits.

This paper seeks to answer a central question posed by Austrian Economists, namely, whether the practice of fractional reserve banking – a central target of critique by the group – in fact has an overall negative impact on aggregate welfare, or whether the practice in fact produce a net increase in welfare. The paper intends first to provide a brief overview of the arguments advanced by Austrian economists, including a history of the origins of fractional reserve banking and some critiques of it throughout the 20<sup>th</sup> Century (Section II). This paper then will expand on the model provided by Diamond and Dybvig (1980, 1983) to illuminate the net gain to all parties once fractional reserve banking, coupled with deposit insurance, is established, particularly with an eye toward the seigniorage losses and “inflationary tax” so often criticized by the members of the Austrian School of Economics (Section III). The paper will seek to examine a straightforward theoretical analysis of the costs and benefits of this practice, particularly with an eye to compare the abovementioned losses to the productivity gains provided by increased access to credit. The paper finally makes a brief examination of the time in post-Meiji Japan, in which data are available for a sharp expansion of the banking sector

(Section IV). In this, the theoretical findings of section III are tested to see a correspondence with observed historical events. Section V provides concluding remarks.

## **Section II: The History of Fractional Reserve Banking**

Fractional reserve banking has endured as the dominant form of banking in the West for centuries, and Austrian Economists have been tireless critics of the practice. This section follows the work of prominent Austrian monetary historian Murray Rothbard in his book *A History of Money and Banking in the United States : The Colonial Era to World War II* (2005) and other Rothbard literature, in order to provide the strongest possible sympathy for Austrian economists' case.

The practice of fractional reserve banking originated with goldsmiths in 17<sup>th</sup>-Century England. Gold and silver served as money at the time, and since goldsmiths usually had high quality safes, many individuals chose to deposit their gold with them rather than purchasing their own safes for storage (Rothbard 1994). The individuals would pay a small fee for the storage service and would receive a receipt detailing how much gold they had on deposit. Over the years, individuals realized that when they wanted to make purchases using gold on deposit, rather than taking the time to travel to the goldsmiths to withdraw their gold, they could simply sign over these gold warehouse receipts to the individual from whom they were purchasing. This act became widespread and eventually formed the basis for checks and paper money.

Once the practice of signing over gold warehouse receipts became common, goldsmiths realized only a small percentage of clients would typically come in at any given time to withdraw their gold. The rest would simply trust that the goldsmiths were safeguarding it – as the goldsmiths had contractually agreed to do in the first place. Tempted by the access to money, many decided to embezzle some of the funds, with the idea that they would lend them out at interest and, once the long-term loans were finally repaid, return the funds and keep the interest. They managed to get away with the practice for such a long time that, by the time the practice caught the attention of the legal system, it had essentially become institutionalized and, of course, these individuals running banks had a great deal of money at their disposal with which to buy the influence of politicians unsure about its legality.

Historically, fractional reserve banking became firmly established in the United States in the 18th Century precisely because of a dire need for seigniorage during wartime (Rothbard, 2005, who despite his politics manages to give a fair rendering of the history of the colonial era banking system). Since the United States' monetary system was based on commodities (gold and silver) which could not be produced at will by the government, the federal government resorted to a series of liaisons with banks that printed excess bank receipts and helped generate covert seigniorage to fund the government's war expenditures. In return, the government repeatedly intervened in defense of the regime, allowing banks to break contract and declaring bank holidays regularly, propping up a system (as Austrians argue) due to the symbiotic relationship that developed.

An important conceptual distinction to be made with the practice, and one that is ceaselessly pointed out by those in the Austrian School of Economics, rests in the fact that through this process, banks *create* money, “out of thin air” as the saying goes. Individuals at the time, trusting that their gold was deposited safely, were spending the receipts as if the gold were actually there. Meanwhile much of that gold was being put back into circulation, lent out at interest.

A simple illustration would be as follows: depositors had \$1,000 in funds, all of which they deposited with goldsmiths (and later bankers) in exchange for receipts. As a result, \$1,000 worth of gold receipts was in circulation. However, goldsmiths (bankers) decided to lend out \$900 of actual gold out at interest. As a result, both the \$1,000 in gold receipts (which people trusted to be 100% backed by gold) and \$900 in hard gold (which of course people accepted as real money) were in circulation, resulting in a total of \$1,900 or a 90% increase in the money supply. Since the system created more money chasing the same amount of goods, one of the results of this was that individuals’ purchasing power decreased, much in the same way it would if corrupt individuals were counterfeiting large sums of money and passing them as legitimate. Herein rests the primary critique of those in the Austrian camp.

The history of government acceptance (and the opinions of government officials on the practice) is complex, but one salient feature of government tolerance for the practice specifically in the United States, pointed out repeatedly by Rothbard, rested in the fact that this fractional reserve banking system allowed for the government to raise revenue through effective seigniorage in a way that would otherwise have been impossible. Seigniorage, or a government’s act of raising revenue simply through

printing money, is not possible in a monetary system based on commodity money (such as silver and gold), as was the case in the United States at the time (Rothbard 2005).

However, governments soon realized that they could grant banks certain rights to expand the money supply (through fraudulent gold receipts) in exchange for banks' commitments to buy debt from the government to fund war efforts. Since the situation, at least in the short-term, was a win-win for both parties, the practice persisted and continued to gain in force.

A limited number of descriptive criticisms of fractional reserve banking exist in more mainstream literature, though all of it predates the emergence of vastly increased analytical rigor in the field since the 1970s (*e.g.*, Lucas 2000). Notably in the United States, proponents of the so-called "Chicago Plan" submitted to Franklin Roosevelt in the wake of the waves of bank failures in the early 1930s strongly advocated a transition to a fully 100% reserve system – *i.e.*, a system in which every dollar deposited at a bank would be backed by a dollar in reserves – on grounds of improving monetary stability (Allen 1991, Phillips 1995). This plan, however, was eventually dropped in favor of deposit insurance and expansion of the Federal Reserve's powers of market intervention. This later solution had its critics at the time, but as will be shown in Section III below, this provides a net welfare gain for all.

Irving Fisher, influenced by the ideas proposed in the Chicago Plan, went further to postulate that among the benefits of a 100% reserve system was the possibility of a vast increase in the potential for seigniorage than exists under a fractional reserve regime (Fisher 1935). The shift to a 100% reserve regime would drastically reduce the public debt of the United States, Fisher argued, effectively moving in the reverse of the path

shown on Chart 1 to be discussed below. Fisher in fact spent the remainder of his life passionately advocating for the establishment of a full reserve system, but as the moment of crisis of the Great Depression had passed, so had the possibility of galvanizing public action.

Milton Friedman followed suit with Fisher's proposal, describing in his *Program for Monetary Stability* (1959) that the additional stabilizing benefits of a 100% reserve system would include a more direct control over the optimal quantity of money by effecting an end to the dependence of the money supply on the lending behaviors of banks. In the current system, while monetary contraction by the Federal Reserve is easily achieved, it can only expand the money supply in the short-term by expanding base currency and *hoping* the banks decide to lend it out (and thus increase the money supply) – a limitation with which we have recently been confronted. By requiring 100% reserves, Friedman argued, the Federal Reserve would be allowed to increase the money supply at will, a feature which would prove extremely useful especially in times of crisis, which have historically occurred precisely when the banking industry is most reluctant to lend (*cf.* Friedman and Schwartz 1963). Friedman also described the net benefits to retiring public debt that would occur, and described an orderly transition over the course of two or three years that would create high-powered money to supplement the difference between M0 and M1. He pointed out at the time of writing that if the 100% reserve plan had been achieved by printing money (*i.e.*, seigniorage) to purchase government bonds, approximately \$150 billion, practically the entire sum of marketable government bonds at the time, would have been retired and thus the public debt greatly lessened.

An intimate relationship exists between the fractional reserve regime and purely monetary inflation – namely, that a significant portion of the seigniorage is funneled through banks who “issue” the money first through loans rather than the government through paper. As Friedman (1971) has pointed out, general consensus among economists holds that inflation essentially operates as a tax on money holdings. Expressed in a way which is more germane to the line of reasoning this paper seeks to explore, long-term growth of the money supply (which comprises both inflation and monetary expansion effected in response to real output growth) functions as a tax whenever any monopolistic or oligopolistic structure holds complete control over the issuance of new money, and hence over the distributive structure of inflation.

There are three possible mechanisms for redistribution of this “tax” in a modern economy. First, there is what Friedman (1969) describes as simply dropping money out of a helicopter – *i.e.*, a process through which all individuals might receive the newly created funds directly, through a mechanism akin to the effect recently achieved by funds from the Economic Stimulus Act of 2008, deposited directly into taxpayers’ accounts. Obviously, for the general population’s welfare, this would likely be optimal.

Second, there is seigniorage, whereby the government finances part of its expenditures through printing money. Numerous studies, most prominently among them Cagan’s classic essay “The Monetary Dynamics of Hyperinflation” (1956) have shown that excessive attempts at seigniorage finance past the maximum amount given by the inflation-tax Laffer curve can lead to catastrophic results. However, others have argued persuasively that a moderate amount of steady seigniorage created with the full knowledge of the public can actually be beneficial to the public, when compared against

other forms of taxation, due to the structure's avoidance of collection and enforcement costs (see, *e.g.*, Romer 2006, p. 538-552 for an overview of the various economic perspectives on seigniorage).

The third possible mechanism – and the one that is observed in the contemporary economic structure of all developed countries – is one in which the benefits of this tax, *i.e.*, the expansion of the money supply, are shared between government and the banking structure. In an economy with a fractional reserve banking system, every dollar issued by the government becomes several dollars – depending on an array of determining factors. Interestingly enough, though the banking sector is an indirect beneficiary of increases in the money supply, classic studies of seigniorage and the dynamics of inflationary finance such as Bailey (1956) focus solely on growth of high-powered money, and completely ignore the dynamic role banks play in such historical experiences, as well as under more stable situations in which seigniorage is sustainable.

Commercial banks enjoy a unique legal privilege in their ability use their corporate liabilities as a form of currency. This privilege positions them as one of the key determinants of the money supply, as their decisions to lend or not lend on the basis of purely private interest have an aggregate effect on the stock of money and thus on public welfare. From the perspective of profit maximization, banks have a natural motivation to minimize their level of deposit reserve holdings (whose opportunity cost is the nominal interest rate), while loaning out as much of their funds as possible in the market at interest. While in the short-run this behavior can yield greater profits for banks at large, this has led to repeated incidences of market instability and fluctuations in the money supply documented so thoroughly in the United States experience by Friedman

and Schwartz (1963), where runs on the bank regularly had the unintended side effect of effecting significant deflation. Arguably, the Austrian argument against fractional reserve banking could be seen merely as a hold-over from pre-Depression times.

Diamond and Dybvig (1983) and Diamond (2007) describe how government insurance on deposits addresses this issue. Without deposit insurance, they explain, if investors believe more depositors will seek to liquidate their holdings than a given bank has in reserves they will rush to liquidate their holdings even if they have no need of them in the earlier time period. The mere belief that a bank will fail can become a self-fulfilling prophecy, as individuals seek to ensure that they do not lose all their holdings (which, in the case of the bank being forced to call in loans at a steep discount, would necessarily happen for the last depositors as the returns from early liquidation would yield less than the total amount owed all depositors). Thus, while the solution that only those who have utility in period  $T=1$  are those who liquidate at  $T=1$  is clearly a Nash equilibrium, Diamond shows that a bank run might still occur even when not triggered by any real event. Deposit insurance, however, guarantees those who wish to retain their holdings until  $T=2$  will not lose their deposits, so even if a rumor of bank insolvency begins to circulate, individuals with preference for consumption in  $T=2$  will still be comfortable waiting until period 2 to withdraw their funds, and thus bank runs based on rumors rather than real insolvency can largely be avoided.

While bank runs can still become an issue and a source of instability, the general structural issue has largely been solved, as Diamond described. Obviously minimum reserves play a role in insuring that banks do not take on excessive risks, and changes in minimum reserves beyond a certain low level would undoubtedly cause instability in the

banking sector. But as the aim of this study seeks primarily to examine the utility effects of *raising* the minimum reserve requirement, possibly all the way up to 100%, the issue of bank runs will for the time be set aside in this analysis.

### **Section III: Diamond-Dybvig, Seigniorage and Growth**

Despite these descriptive critiques of fractional reserve banking and support of the 100% reserve alternative, virtually no rigorous quantitative critique of the fractional reserve regime exists in today's literature, and this gap in the literature has allowed a certain portion of the population interested in economic matters from an Austrian perspective to persist in their accusation that fractional reserve banking on the whole has a deleterious effect on individual welfare for the sake of banking companies. There are three components in question that will be examined in a stylized rendering of the issue . The first is the seigniorage foregone by the government, given by the difference between the "natural" seigniorage that would occur under a 100% reserve regime with a given inflation target, and the partial loss that is funneled through the banking sector since the banks "create" a fraction of the funds through lending, rather than the government through direct issuance (utility loss). The second issue at hand is the liquidity service provided by banks to individual "investors" (*i.e.*, depositors) who gain from having a riskless and perpetually liquid investment opportunity through bank accounts (utility gain). The third and final component consists of the economic growth that the expansion of the banking sector under a fractional reserve regime provides, which will be estimated

relying on existing literature, particularly the work done by Rousseau (1999) and through an analysis of post-Meiji Japan in Section IV.

In order to attempt a comprehensive cost-benefit analysis, all terms have to be in the same unit of measurement, which poses significant difficulties because the liquidity services described in the Diamond Dybvig model represent are given as an increase in utility, not any monetary sum. Thus, the first step in executing this cost-benefit analysis is to convert the Diamond Dybvig model, in a stylized way, from giving results in terms of utility to a per-capita GDP measure. This conversion is as follows.

As a brief introduction, Diamond and Dybvig's model goes a long way toward modeling in quantitative terms what they describe as the "liquidity services" banks provide through this practice (Diamond and Dybvig 1983, Diamond 2007). Namely, they detail how, since individual depositors do not know whether they will need their funds in time  $T$  or time  $T + 1$  and since only a fraction of depositors will likely withdraw their deposits at any given time, the banks actually *increase* welfare through the credit expansion they create, as well as through being able to offer interest on their deposits (at least nominally) higher than what individual investors could achieve on their own. Further, they argue correctly, depositors would have to pay some kind of fee for the warehouse services banks provided for their money if a 100% reserve system were in place. Overall they conclude that serious welfare gains arise from the banking sector's ability to "create money out of thin air" and lend it at interest, and on this basis they stand in favor of fractional reserve banking. Diamond (1996, 1997, 2005) and Diamond and Rajan (2001) expand this model to its applications on banks' debtor monitoring functions and other aspects that improve general welfare.

Yet, Austrian economists would argue, the Diamond-Dybvig model fails to take into account the other half of the equation in determining overall welfare gains to individuals within the economy – namely, the implicit *costs* of the system. While banks undeniably provide useful access to credit which serves a crucial and necessary function in any modern economy, one must ultimately weigh these benefits against the overall costs to individuals that are exacted, most prominently through bank-created inflation and limits to seigniorage opportunities.

While no study has addressed this cost, a wide range of literature develops technical models which will prove applicable to the study of the monetary dynamics implicit in a change from fractional to 100% reserve system. Bailey (1956) posited a groundbreaking analysis of the welfare cost of seigniorage, describing that despite much previous historical criticism, government raising of revenue by printing money might actually be an optimal method of taxation, as it requires neither collection nor enforcement costs to levy. His general analytical framework has been taken up, replicated and improved upon by a number of more recent studies, most recently in Lucas's (2000) double-log exposition of the model, to confirm that in fact a certain amount of inflationary finance would likely be optimal for government collection of revenue. Moreover, these authors provide mathematical theoretical descriptions of real money demands, further described in Galí (2008) and Woodford (2003), which will prove an indispensable aspect of the study in question.

The following analysis will attempt to give a hybrid comparison of seigniorage loss alone against the utility gain from fractional reserve banking. I will here try to play the “devil’s advocate,” to give the strongest possible argument in favor of the Austrians’

case. The Diamond and Dybvig (1983) model, they would argue, fails to account for the fact that due to the vast amount of inflation which banks are allowed to create over the long term, real returns on investments as theorized in their models are actually below the stated results they offer. More importantly, however, and central to their argument, a fractional reserve banking regime necessarily limits the maximum amount of seigniorage, *i.e.*, inflationary finance, a government can produce.

#### *A Stylized Model of Money Demand with Endogenous Bank-Created Seigniorage*

Because this line of investigation is concerned with long-term impacts of the legal reserve requirement on the seigniorage levels for a given level of inflation, the following exposition will make use of a simple model of a classical monetary economy with perfect competition and fully flexible prices. Galí (2008) provides a simple log-linearized model for real money demand derived from the representative household's optimizing behavior and exhibiting unit elasticity with respect to consumption, given by:

$$(1) \quad m_t - p_t = y_t - \eta i_t$$

where  $m_t$  is log money supply,  $p_t$  is log price level,  $y_t$  is log real output,  $\eta$  is the interest rate semi-elasticity of money demand, and  $i_t$  is the nominal interest rate.

In order to incorporate banks into this model for money demand, one might posit that  $m_t$  is the log aggregate money supply, here called  $M1_t$ , given by a base supply of high-powered money, here called  $M0_t$  and factors determined by the behavior of the banks. A simple expression of the relation between the two variables can be given by the form:

$$M1_t = M0_t * \frac{1 + C_t}{C_t + R_t}$$

where  $C_t$  is the average individual preference for cash on hand versus transaction services provided by banks in the form of demand deposit accounts ( $0 \leq C_t \leq 1$ ), and  $R_t$  is the legally required minimum reserve ratio ( $0 \leq R_t \leq 1$ ). This relation assumes that, since it is in the banks' interest to lend out as much of their deposits as is legally allowed, on average they will do so. Thus, expressing the money demand as a function of base money, (1) becomes

$$(2) \quad \log\left(M0_t * \frac{1 + C_t}{C_t + R_t}\right) - p_t = y_t - \eta i_t$$

For notational simplicity,

$$X_t \equiv \frac{1 + C_t}{C_t + R_t}$$

First differencing of (2) above, with the new notation inserted, yields

$$(3) \quad \log\left(\frac{X_t}{X_{t-1}}\right) + \log\left(\frac{M0_t}{M0_{t-1}}\right) - \pi_t = \Delta y_t - \eta \Delta i_t$$

where  $\pi_t \equiv p_t - p_{t-1}$  is the inflation rate,  $\Delta y_t \equiv y_t - y_{t-1}$  is the rate of growth of real output, and  $\Delta i_t \equiv i_t - i_{t-1}$  is the change in the nominal interest rate. For the sake of simplicity in identifying the phenomena in question, I will assume  $\pi_t$  and  $\Delta y_t$  are constant and relatively small; that is, our model economy exhibits low but constant inflation and low but constant output growth, and both of these variables are expected to remain constant for the indefinite future. Moreover, the model under examination will assume a constant nominal interest rate and thus  $\Delta i_t = 0$  for all  $t$ . This simplifies our expression of the growth of the aggregate money ( $M1_t$ ) supply to

$$(4) \quad \mu_t = \pi_t + \Delta y_t$$

where  $\mu_t \equiv \log \left( \frac{M1_t}{M1_{t-1}} \right)$  is the rate of growth of the money supply.

### *Seigniorage under a Fractional Reserve Regime*

In an economy as described above, wherein currency issued by the government is nested within a banking structure that ultimately determines the aggregate money supply through its lending behavior, variations in the reserve ratio have important implications for the amount of seigniorage yielded by a given rate of growth of the aggregate money supply. The typical textbook equation for seigniorage is given by:

$$(5) \quad \frac{M_t - M_{t-1}}{P_t}$$

But high-powered money is in itself meaningless as an expression of the aggregate money supply. It is only significant in that it – along with the minimum reserve requirement (and bank decisions to lend out all excess reserves, which is assumed here for simplicity) – is one key determinant of the aggregate money supply. For the sake of simplicity of illustration I will set demand for cash on hand ( $C_t$ ) equal to zero, and assume banks lend out the maximum amount of funds (with the resultant money multiplier effect as the money expansion cascaded through the banking sector). Thus simplified,

$$X_t = \frac{1}{R_t}$$

and hence,

$$M0_t = M1_t * R_t$$

When the seigniorage equation is rewritten in terms of the aggregate money supply, the relationship clearly shows that an increase in reserve requirements will result in an increase in seigniorage revenue (*i.e.*, revenue gained by the government), and vice versa:

$$(6) \quad S_t = \frac{M1_t * R_t - M1_{t-1} * R_{t-1}}{P_t}$$

Using the definition of  $\mu_t$ , we can express  $M1_t$  in terms of  $M1_{t-1}$  and  $\mu_t$  and simplify:

$$(7) \quad S_t = \frac{M1_{t-1} * (e^{\mu_t} * R_t - R_{t-1})}{P_t}$$

Further, because of the assumed simplified relation between expansion of the money supply, output and inflation given by (4) above, we can also state that

$$(8) \quad S_t = \frac{M1_{t-1} * (e^{\pi_t + \Delta y_t} * R_t - R_{t-1})}{P_t}$$

From (7) we can see clearly that even if there were no growth in the money supply ( $\mu_t = 0$ , the government could still create seigniorage simply by raising the legal reserve minimum. Further, in an economy with low but positive inflation and low but positive growth (as is posited here), an *increase* in the reserve requirement between time  $t$  and  $t-1$  would yield seigniorage at rate of greater than one-for one – *i.e.*, in addition to the amount naturally associated with a simple government-created monetary phenomenon. This additional yield of seigniorage above strict inflation comes from increase in aggregate productivity, and the relationship states that not only the expansion of the money supply but also the share of wealth ensuing from real output growth, is redistributed to the beneficiaries of  $S_t$  (banks, the government, citizens or a mixture, depending on the distributive structure), though they clearly are the product of labor of the commonwealth as a whole.

## *Money Expansion and Investor Welfare in the Fractional Reserve Regime*

Diamond (2007) condenses the model posited by Diamond and Dybvig (1983) into a simple exposition, and infers from the conclusions of his analysis that banks actually provide greater utility to investors than what investors could receive on their own. While his analysis holds true for the particular type of model under discussion – namely, a savings mutual bank with no deposit accounts and thus no money creation – it did not take into account the question of seigniorage as described above. Rather than examining a steady-state economy with a fractional reserve regime, the difference becomes much clearer in a before-and-after examination, *i.e.*, an economy observed just before a transition out of 100% reserve banking, and in the time period immediately following. Before engaging in the Diamond model directly, for the sake of clarifying whether investor utility is in fact increased or decreased by the establishment of commercial banks, let us first hypothesize a small closed economy in which fractional reserve banking did not exist in an initial state for all dates  $t \leq 0$ . In this model, high-powered currency would be the sole determinant of the money supply and thus  $M1_t = M0_t$ .

In this economy, as before, we assume a monetary policy committed to a target of the money supply, with constant but low inflation and constant but low real output growth. For the sake of simplifying calculations, let us assume  $\pi_t = .025$  and  $\Delta y_t = .025$  for all  $t$ .  $\Delta i_t$  is also 0 for all  $t$  as before, and thus  $\mu_t = .05$  for all  $t$ , as given by (4) above. The model economy in this thought experiment will be composed of 10,000 citizens, which is an arbitrary figure, but one large enough such that each individual cannot affect

the price level or seigniorage yield function through his or her decision whether to deposit funds or not. Each individual has \$100 per term, which can be held as cash on hand or deposited in the newly created commercial banks, to yield interest which the banks set at their discretion. At  $t=0$ ,  $M1_0=M0_0 = \$1,000,000$ . Let us suppose then, that for each date  $t \geq 1$ ,  $\theta$  investors deposit funds in a commercial bank, where  $\theta = .05 * t$  for  $0 \leq t \leq 19$ . Let us also suppose that banks deploy the newly created funds immediately, up to a legal limit described by  $R_t$  above and determined at an arbitrary and fixed level by the government at  $t=0$  for all  $t$ .

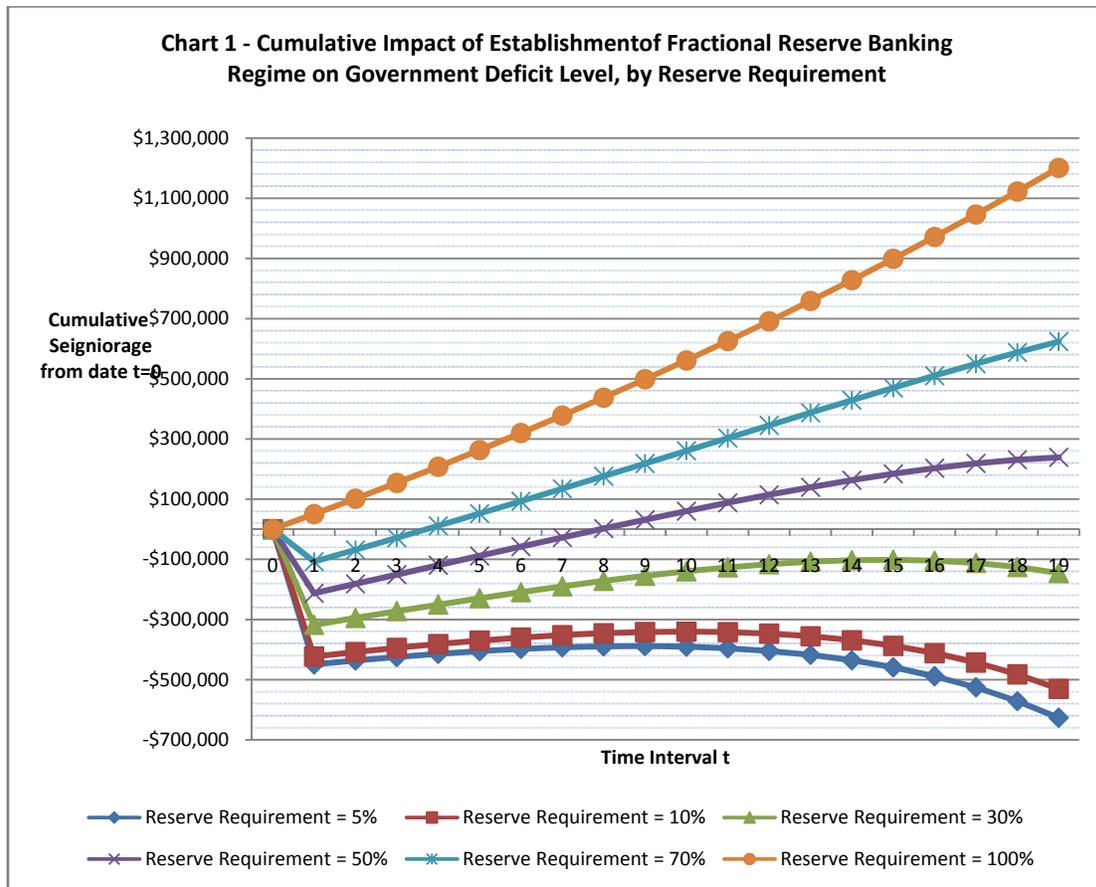
For  $t=1$  and beyond the government, running a fiat system, is faced with a choice. It can accept the increase in the money supply created by the new addition of checking money to the original monetary base, and the rampant inflation that would inevitably ensue. Or it can decrease the monetary base in response to the expansive impact of the new money, in order to keep the overall money supply stable. Let us suppose it chooses the latter, and its mechanism for contracting the money supply is the one observed in modern economies such as the United States today - that is, some kind of monetary authority conducts open market operations to sell debt and retire high-powered money in such a proportion as to offset the expansion of checking money that occurs in each period, up to the point when fractional reserve banking insinuates itself completely into the monetary structure.

Chart 1 below<sup>1</sup> details the theoretical progression of government seigniorage loss – *i.e.*, how much additional debt the federal government must assume in order to maintain

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<sup>1</sup> Table 1 with numerical calculations supporting the graph can be found at the end of this paper.

its inflation target – as a function of varying possible minimum reserve requirements arbitrarily set by the government at  $t=0$ .



From this, we can see that the seigniorage gained at the end of the period is in fact a loss of \$600,000, while the seigniorage gained under a hypothetical 100% reserve regime is \$1,200,000, for a net loss of \$1,800,000 in this fictional economy, or \$180 per capita. Assuming Ricardian equivalence, this amounts to an additional income tax of \$9/year, which can serve as the figure for the “cost” aspect of this cost-benefit analysis.

## *Diamond Dybvig Model*

The Diamond-Dybvig model given in simplified form by Diamond (2007) essentially seeks to explain why investors' general welfare is improved by the so-called liquidity services created by the banks. Diamond posits 100 investors have a risk-averse utility function given by

$$(9) \quad U(c) = 1 - \frac{1}{c}$$

The author describes that in this model  $\tau$  investors will want to consume with utility  $U(c_1)$ , and  $(1-\tau)$  investors will want to consume with utility  $U(c_2)$ , and more importantly, investors do not know which type they will be beforehand. If investors on their own have access only to an illiquid investment with return 1 in period 1 and return  $B^2$  in period 2, banks can create liquidity and increase investors' ex ante utility by maintaining a certain amount of reserves for those type 1 investors, while investing the remainder of deposits in  $B$ . The bank can offer  $r_1 \geq 1$  for the  $\tau$  type 1 investors, and  $r_2 \leq B$  for the  $(1-\tau)$  type 2 investors and thus create liquidity from a return on an illiquid asset. The constraint on the three variables is given as:

$$(10) \quad (1 - \tau * r_1) * B \geq (1-\tau) * r_2$$

Maximization of the ex ante expected utility function

$$(11) \quad \tau * U(c_1) + (1 - \tau) * U(c_2)$$

subject to the restraint given by (10), this yields:

$$(12) \quad \frac{c_2}{c_1} = \sqrt{B}$$

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<sup>2</sup> Note that this was denoted by  $R$  in the Diamond (2007) paper; however, to avoid confusion with the reserve requirement ( $R_t$ ) used herein, I will refer to this yield as  $B$  instead.

Using values  $B=2$  and  $\tau=.25$ , and the fact that  $c_2 = r_2$  and  $c_1 = r_1$ , Diamond solved (10) as an equality for  $r_2$  in terms of  $B$  and  $r_1$  and found optimal values for  $r_1$  and  $r_2$  that yield higher ex ante utility than the values an individual investor could achieve on her own.

While this model does well to explain the liquidity services provided by a mutual savings bank (which Diamond explicitly takes as his model), it does not account for the negative aspects of deposit accounts on the aggregate government income and thus long-term tax outlays. While, as before, I will assume 10,000 individuals in a model economy in which fractional reserve banking is now taken as a given, who cannot on their own affect the money multiplier given by  $X_t$ . However, the government can effect higher reserves through legislation and regulatory changes and thus if it perceives it would increase aggregate welfare it could act on the behalf of its citizens to do so.

We can now integrate Diamond's simple model into the models for seigniorage previously developed, comparing the net decreases in utility given by (11) that would arise from an increase in the required reserve ratio above  $\tau$ . I will here make one peculiar assumption – in order to compare the gain from seigniorage to the loss from the increased reserve requirement I will assume that the additional seigniorage made possible exclusively by the increased reserve requirement be distributed in the manner of a tax break allocated equally across all citizens of the model economy under inspection. Here we will assume in period 0, all members of the economy hold their funds in the bank; in period 1,  $\tau$  will remove their funds from the bank due to cash needs, and in period 2, both type 1 and type 2 investors will reinvest their funds in the commercial bank after type 2 investors have captured their gains in period 2. Let us assume that for all periods  $t \leq 0$ ,  $R_0 = \tau$ , but that in  $t=0$  regulators announce that upon reaching period 1 a new  $R_1 \geq R_0$

will be declared, and that banks must prepare their reserves in advance so that they are in full compliance by time  $t=1$ . I will also assume that in period 1, the  $\tau$  of the Diamond model is equivalent to the  $C_1$  of (2) above solely for period 1, and that for period 2  $C_2 = 0$  as all individuals reallocate their funds back into the bank. This will simplify the utility comparison.

One can describe the level of aggregate real seigniorage yielded exclusively by an increase in the reserve requirement (that is, a change of  $R_t$  above  $R_{t-1}$ ) by subtracting the new seigniorage level from the level that would have resulted from keeping the reserve minimum constant, in the following form:

$$S_t^R = \frac{\frac{M1_{t-1} * e^\mu}{1 + \tau} - \frac{M1_{t-1}}{1 + \tau}}{\frac{\tau + R^*}{P_t}} - \frac{\frac{M1_{t-1} * e^\mu}{1 + \tau} - \frac{M1_{t-1}}{1 + \tau}}{\frac{\tau + R}{P_t}}$$

(Note that this holds true for  $t=1$ ; for  $t=2$  the seigniorage result takes the same form except that now all investors return their funds and  $\tau = 0$ .) Cancelling like terms and simplifying yields

$$S_t^R = \frac{(M1_{t-1} * e^\mu * (R_t^* - R))}{(1 + \tau)P_t}$$

To make this per capita we simply divide by population,  $N$ , and arrive at

$$(13) \quad S_t^{RN} = \frac{(M1_{t-1} * e^\mu * (R^* - R))}{(1 + \tau)P_t * N}$$

Since this is a lump sum reduction in taxation in period  $t$ , it can be considered the same as revenue from another source for the purposes of increasing consumption in time  $t$ .

Next we consider the reduction in yield resulting from an increase in the reserve requirement beyond  $\tau$ , which proportionally decreases the amount of funds the bank can invest in  $B$  and thus decrease the total yield beyond the optimal level described above.

Putting the new reserve criterion into the constraint given by (10) and solving for  $r_1$ , we find:

$$(14) \quad r_1 = \frac{1 - (R^* - \tau)}{\frac{1 - \tau}{\sqrt{B}} + \tau}$$

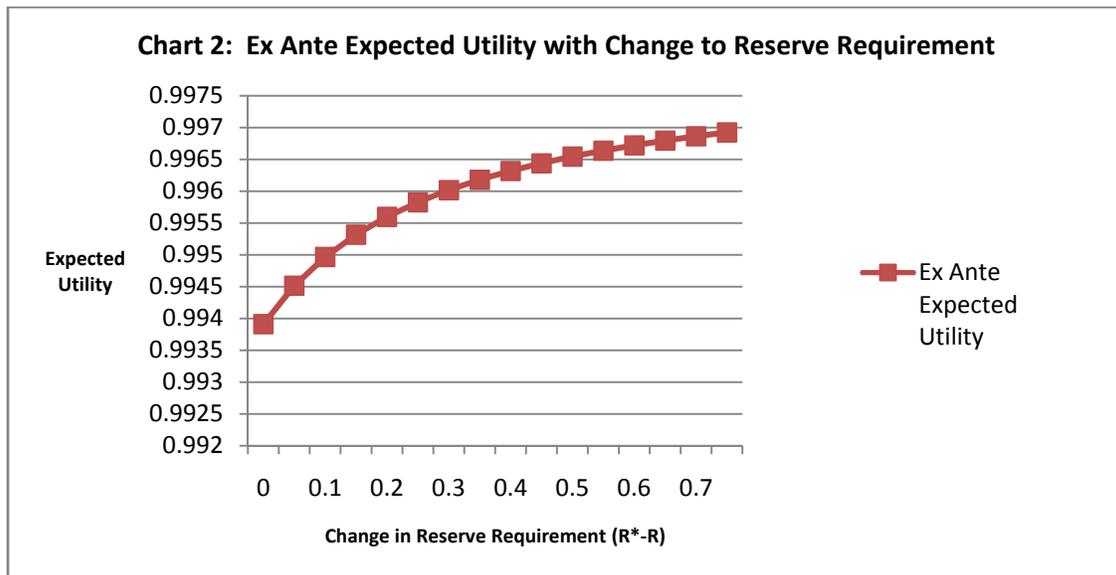
Note here that  $(R^* - \tau)$  is equivalent to  $(R^* - R)$ , an important feature in comparing the change of reserve ratio on overall utility. From (14) we can solve for  $r_2$  for periods 1 and 2 dependent on this change in reserve requirements. For the sake of comparison, while we would naturally observe two cash flows from decreased tax, one for each period, we will state that the utility function only increases in the period in which the investor needed consumption, but that for type 2 investors since they have two periods to accumulate the tax savings they will have two times the seigniorage attributable exclusively to the change in reserve ratios. Inserting these figures into the ex ante expected utility function (11), we can state that

(15)  $E_0U$

$$= \tau * \left( 1 - \frac{1}{\left( \frac{M1_{t-1} * e^{\mu} * (R^* - \tau)}{N * (1 + \tau) * P_t} \right) + 100 * \frac{1 - (R^* - \tau)}{\frac{1 - \tau}{\sqrt{B}} + \tau}} \right) + (1 - \tau)$$

$$* \left( 1 - \frac{1}{\left( \frac{2 * M1_{t-1} * e^{\mu} * (R^* - \tau)}{N * P_t * (1 + \tau)} \right) + 100 * B * \frac{\left( 1 - \tau * \left( \frac{1 - (R^* - \tau)}{\frac{1 - \tau}{\sqrt{B}} + \tau} \right) - (R^* - \tau) \right)}{1 - \tau}} \right)$$

At this point I will take  $\tau$  to equal .25 as in Diamond, and will set B to equal 2 as in Diamond. The utility function with all else held constant, and only  $R^*$  varying, yields the following relation:

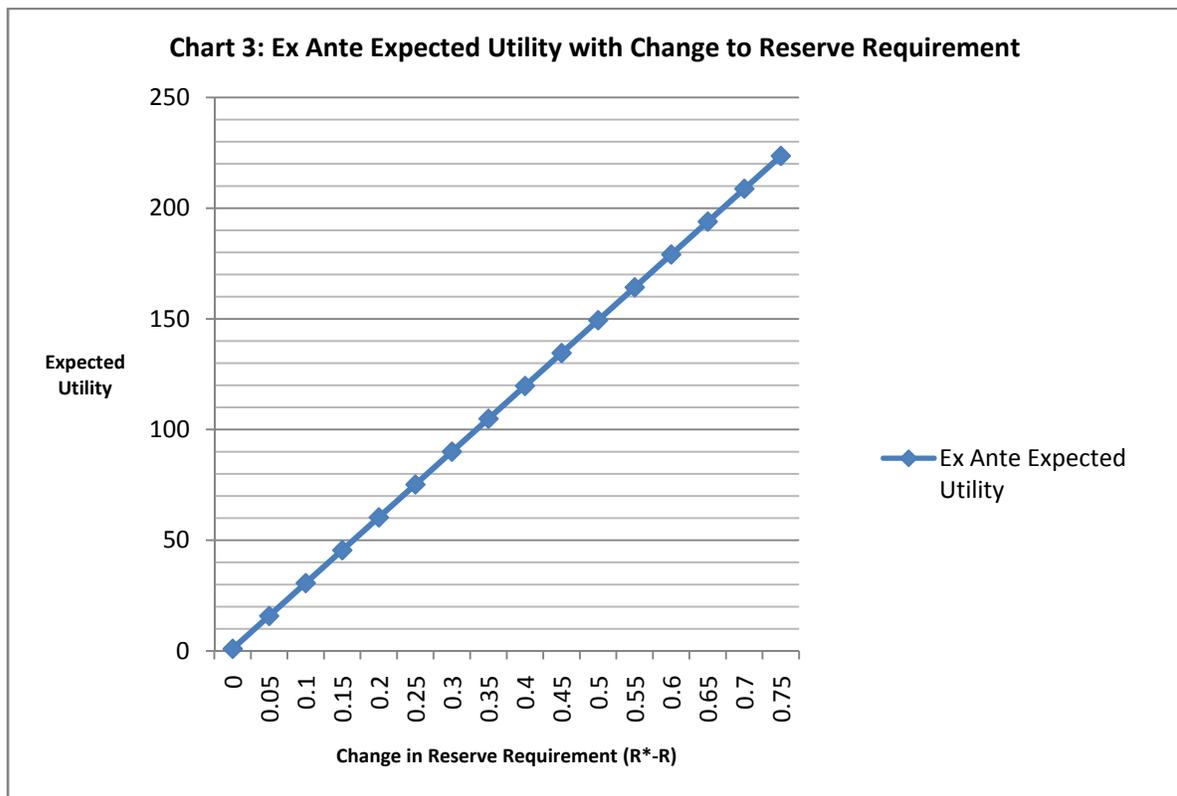


One would note that while expected utility increases as the reserve requirement is increased, according to this model, the change is rather small (within a range of .003).

This would yield much different results with the utility function given by:

$$(16) \quad U(c) = 1 - \frac{1}{c} + S_t^{RN}$$

This function would make more sense given the context as their risk-averse function would not apply to the riskless per capita additional seigniorage that, given our assumptions, would be guaranteed to occur and thus holds no risk. Given these assumption, and inputting our parameters into the new ex ante expected utility function with (16) instead of our modified (9), utility as a function of this reserve ratio change is given by:



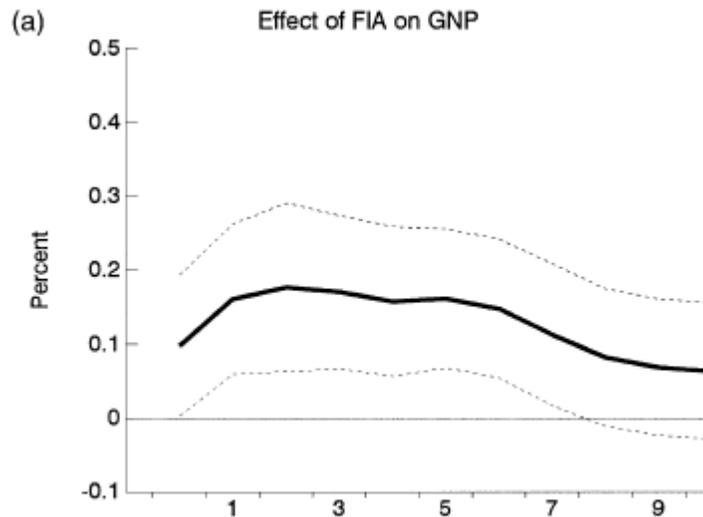
This utility calculation shows more dramatic implications for the utility gains to the average citizen in our model.

To put this in perspective, using the arbitrary money parameters given by our model, the real gains per capita of a shift to a 100% reserve system, using  $t=19$  as the starting time (“ $t=0$ ” in the Diamond model), would yield ( $S_t^{RN}$ ) would equal \$98.92 in time  $t=20$  and \$101.42 in time  $t=21$  for a total yield of \$200.34 per investor. For investors of type 1, who would receive the \$98.92 in period 1 in addition to their withdrawal of \$100 with no interest for a total of \$198.92,  $r_1$  would have to be greater than a highly improbable 1.9892. For investors of type 2, who would receive the total yield of \$200.34 in time 2 in addition to their \$100 with no interest for a total of \$300.34,  $r_2$  would have to be greater than a highly improbable 3.0034. For this to be the case, B (Diamond’s “R”) would have to yield an improbable 4.45 at time  $t=21$  (Diamond’s  $t=2$ ). While theoretically this might render the possible utility of banks under a fractional reserve regime to be greater than that under the 100% reserve system, an asset yielding 345% interest per term seems highly unlikely in the real world. This would seem, at least provisionally, to support the Austrian argument against fractional reserve banking. To complete our analysis, however, we need to examine the final component of our analysis – that of the growth created by the expansion of investment.

### *Endogenous Bank-Led Growth*

The theoretical incorporation of growth into the model above will follow from Rousseau (1999), who argued that the expansion of the financial sector in post-Meiji Japan was the central cause of the vast expansion of the economy that followed. He explains that financial intermediaries have “the dual role of promoting capital accumulation through the mobilization of otherwise unproductive savings” – savings which, it should be noted, would not be invested in a 100% reserve regime – and in “raising total factor productivity

through improvements in project selection and monitoring” (p. 186). His work suggests strongly, through vector autoregression analysis of the effect of the expansion of the banking sector (denoted in his work by “Financial Intermediaries,” or “FIA”) on GNP growth:



(Rousseau 1999, p. 194)

This impulse shock coefficient demonstrates the estimated percent gain in GNP resulting from a one percent increase in bank assets, reaching an almost one-for-one increase by the end of the term under analysis. The \$1.8M “loss” in the stylized model mentioned earlier is now a *gain* in bank assets, and by taking the integral of the impulse shock coefficient given above, we find that, starting in year two (to avoid asymptotic percent growth), the massive shift in investment provided by the banks would result in a growth (not accounted for in the models) of a gain of \$153 per capita per year, vastly covering the seigniorage loss outlined above, and yielding a theoretical result that strongly

suggests that fractional reserve banking with deposit insurance yields on the whole more benefit than cost.

#### **IV. The Case of Japan – Empirics from 1872-1890**

Banking in Japan follows a unique history, and this, in combination with the availability of data at the start of the fractional reserve banking regime, offer up a uniquely compelling case study in the empirical effects of fractional reserve banking. I will examine the monetary effects of this expansion, and find that it conforms quite well to the theoretical model developed in Section II above. I will then make use of literature to point to estimated gains from growth attributable to the expansion of the banking sector at the time.

The first banks in Japan were founded as early as May and June 1869; however, it was the National Bank Act in 1872 and the founding of the Bank of Japan in 1882 that marked the beginning of the modern era of banking in Japan (Takahashi, p. 194-5). In 1872 the effective reserve requirement was .4; this proved to be prohibitively high and prevented the banking industry from being profitable enough to expand to satisfy credit demand. In 1876, however, this reserve ratio dropped to .2 or 20%, and the banking industry witnessed significant growth. From five banks in 1876, the next three years saw a tremendous proliferation of banks, resulting in 151 in 1879 when the Japanese government ceased granting charters for national banks (Rousseau, p. 187). This expansion of the banking sector went in tandem with an expansion of the money supply, and consequently, in combination with the intentional seigniorage to cover government

expenditures to quell the Satsuma Rebellion in 1877, inflation went to new extremes, as Table 1 will show.

A shrewd economist, Japan's Minister of Finance (beginning in 1881) Matsukata Masayoshi began acting against the inflationary forces at hand, by severely curtailing government expenditures, selling off unprofitable government enterprises, and establishing the Bank of Japan and withdrawing money from circulation to restore price stability (McClain 2002; Hirschmeier, 1964, pp. 149-150). The net effect, however, was a reduction in government spending and an increase in government debt, partly from other concerns, but also partly from the cost of the switch to fractional reserve banking.

An examination of the data presented in Table 1 below reveals this cost of displacement, just as the analysis above predicts – though not as drastically, because one feature of real versus stylized results rests in the reality that banks typically do not reach their full minimum reserve ratio potential as models would predict. This fact serves to further undermine the Austrian argument.

As bank notes began to be issued, prices rose dramatically, and in order to restore this, Matsukata was forced to utilize a combination of cost-cutting measures and open market operations; as a result, this change-over essentially cost the Japanese government approximately 26,000,000 yen – the difference offset by the issuance of bank notes – as well as several turbulent years in the correction process.

<i>Table 1: Japan Money Supply Data, 1871-1890</i> <sup>3</sup>							
End of year		Government notes issued	National bank notes issued	Coins in Circulation	Total Money Supply	Price Index (1874-75=1)	Bank notes as percentage of Money Supply
	1871	60,272	0	7,034	67,306		0.00%
	1872	68,400	0	9,484	77,884		0.00%
	1873	78,381	1,362	13,231	92,974		1.46%
	1874	91,902	1,995	14,825	108,722		1.83%
	1875	99,072	1,420	16,544	117,036		1.21%
	1876	105,148	1,744	20,820	127,712		1.37%
	1877	105,797	13,353	24,581	143,731		9.29%
	1878	139,419	26,279	26,647	192,345		13.66%
	1879	130,309	34,046	24,351	188,706	1.1464435	18.04%
	1880	124,940	34,426	20,255	179,621	1.3179916	19.17%
	1881	118,905	34,397	19,377	172,679	1.4518828	19.92%
	1882	109,369	34,385	19,730	163,484	1.334728	21.03%
	1883	97,999	34,276	20,500	152,775	1.1464435	22.44%
	1884	93,380	31,016	21,446	145,842	1.041841	21.27%
	1885	88,345	30,155	24,009	142,509	1.0920502	21.16%
	1886	67,801	29,501	24,085	121,387	1	24.30%
	1887	55,815	28,604	26,153	110,572	1.0125523	25.87%
	1888	46,735	27,680	26,754	101,169	1.0209205	27.36%
	1889	40,913	26,739	25,652	93,304	1.0878661	28.66%
	1890	34,272	25,811	7,034	67,117	1.2133891	38.46%

Following the work done by Rousseau, the 2516% increase in bank assets over this time would amount to a sum that vastly overshadows the seigniorage lost as described in Table 1. This again supports the conclusions of Section III.

<sup>3</sup> Government and Bank Note data from Nihon Ginko, 1962; GNP and price index data from Nakamura, p. 140.

## Section V: Discussion

The theoretical methodology employed in this paper is less than ideal. Specifically, the discourse attempted to fit a secular trend – the long-term accumulation of debt under a fractional reserve banking regime – into the very short-term model provided by Diamond. The long-term costs of government toleration of fractional reserve banking would most likely be better compared to short-term utility gains the bank provides through some kind of discounted-cash-flow model, one which might extend Diamond’s framework to longer time periods in some kind of revolving depositor model, and which would calculate the aggregate utility for the various agents involved as the net present value of investment and taxation configurations based on similar variables as the ones under consideration here. A more empirical approach would also be desirable, as these facile assumptions about the stability of the relationship between the minimum reserve and bank lending activity might undermine the fidelity of these preliminary findings to the phenomena they attempt to describe.

Also, implications of a 100% reserve policy on the instrument of monetary policy deserve serious consideration. While this policy, Austrians contend, would end the perennially frustrating problem of trying to “push on a string” that occurs when financial authorities wish to expand the money supply while banks refuse to lend, informal analysis would lead one to conclude that this would push the lending activities of banks into the “shadow banking sector” (see, *e.g.*, Krugman, 2009), a situation that would exacerbate the issues that led to the most recent financial crisis of 2008, as the banking sector’s practices would no longer be regulated by the government.

Also, in addition to focusing on liquidity services provided to investors, further analysis must also focus on possible alternative credit structures that might serve as a replacement for commercial banks in the event that they no longer had access to deposits as a source for loans. It was argued briefly that a shift away from fractional reserve banking would lead to the shadow banking system taking over traditional banks' lending activities – an undesirable result. Nevertheless, that fact is merely conjectural and further models of credit supply would be needed to provide support for it.

However, while general consensus has long upheld the practice as necessary, and the fractional reserve banking has proven to be a highly successful means of expanding capital in developing economies, those in the Austrian camp speculate that less costly ways of supporting credit might be found. In the United States, for example, the value of foregone seigniorage, calculated as the difference between M1 and M0 as the operationalized form of equation (1), would only amount to under half a trillion dollars (Romer 2006), which may seem a great deal. But in the face of enacting a complete overhaul of the entire financial sector, in combination with the growth potential outlined above resulting from the banking sector, is not significant enough to warrant the change. In pre-modern economies this mode of credit expansion had the benefit of extreme efficiency, and possessed other welfare-enhancing features; it has often facilitated the unification of currency systems (as was the case in Japan), and provides liquidity services whose welfare effects fall outside the scope of this paper.

The issue of foregone seigniorage as a result of fractional reserve banking may seem a trivial question to explore. However, in light of the serious attention the Austrian arguments have been getting in mainstream discourse, as evidenced by the prominence of

the Tea Party in dictating the legislative priorities of the American Right. The lay public of America, once happy to let economists deal with the intricacies of such an abstract subject, have in the wake of the financial crisis become armchair economists, arming themselves with whatever solutions seem most appealing to both their moral sense and their analytical limitations. Consequently, it is of the utmost importance for economists to take a moment from their duties in advanced economic research, and educate the public on what could potentially prove to be dangerously misguided ideas.

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