Paul Hindemith, *The Craft of Musical Composition*, volume 1
(1937; trans. Arthur Mendel, 1942)

pp. 210-15: “Analysis of Wagner’s Tristan Prelude.”
The nature of the atomic structure of music in the individual unrelated tone is already familiar to us, and we have followed the birth of the elements—the tones in the chromatic scale—from the electron-like relations of the overtones. And now we learn the significance of the tones. The order in which the tones of the scale were produced by the progenitor tone is of the greatest importance, in the view which this book represents. It is not only an indication that the tones have a family relationship, expressed in their connections to the principal tone; it is also an index to the ranking of these connections.

To a given tone, the tone an octave higher stands in so close a relationship that one can hardly maintain a distinction between the two. The tone which is only a fifth higher than the given tone is the next most closely related, and there follow in order the fourth, the major sixth, the major third, the minor third, and so on. As the distance from the given tone increases in this series, the relationship diminishes, until, in the tones that stand at the interval of the augmented fourth or diminished fifth, it can hardly be felt at all. This value-order of the relationships is valid under all circumstances. In every combination of tones, some tones must seem subordinate to others. The stronger ones may subject extended series of chords to their domination, or their rule may not last longer than a pulsebeat, but the accompanying tones will always be related to them according to the order laid down in our series. How benumbed our natural sensibilities must be nowadays, when systems of writing are put forward which are based on a complete lack of relationship among the tones. The carpenter would not think of disregarding the natural properties of his wood and putting it together any old way, without regard to its grain. The justification of such attempts to “expand” the musical language will be sought in vain in the tonal material itself. The only excuse for them must be the complaisant ear, which, despite the delicacy of its construction, is robust enough to accept sounds put together without the guidance of taste or instinct, instead of rejecting such sounds with the same unerring discrimination with which the senses of sight and touch would reject a chair thus miserably pieced together. In the domain of tonal relations no expansion or innovation is possible, no question of style is applicable, and there can be no progress, any more than there can be in the multiplication table or the simplest laws of mechanics.

No other system gives us complete proof of the natural basis of tonal relations. All theorists are agreed, it is true, that there are various degrees of relationship, and the order of descending degrees of relationship is the same in all theories. This is remarkable, for in every other respect there is anything but unanimity among musical theorists. It seems as if a true feeling for the relationships had existed even without the only complete explanation of them, here given for the first time. To be sure, theorists have always sought to find those properties of the tonal material which might provide the logical basis for this feeling, and it cannot be denied that at least for the earliest relationships explanations have been found that are just as logical as ours. If, for example, one follows the overtone series up from any given fundamental, the closest relationships will be found between tones 1 and 2, 2 and 3, and 3 and 4, in the familiar order. But if one follows this series further, one will find a series of relationships wholly at odds with the experience of musical practice: tones related to the fundamental tone as 4:5 (major third), 5:6 (minor third), 6:7 (under-size third), 7:8 (over-size whole-tone), and 8:9 (large whole-tone) would be the
next most closely related tones after the fourth. To explain the tone that is really the next most closely related, the major sixth, it would be necessary to interrupt the process of following the overtone series directly upwards (since the major sixth lies between tones 3 and 5); and for the derivation of the remaining tones from a single overtone series, in anything like the order that corresponds to practical musical experience, no rule can be found. This derivation can only be arrived at arbitrarily, and thus would not furnish any theoretical justification for the experience of practice. The establishment of the major sixth as the interval 3:5 tells us the size of this interval, but not the degree of relationship of a tone which is a sixth above a fundamental tone.

We shall henceforth call the significant order in which the twelve tones of the chromatic scale made their appearance, in diminishing degree of relationship to the given tone, Series 1. The values of the relationships established in that series will be the basis for our understanding of the connection of tones and chords, the ordering of harmonic progressions, and accordingly the tonal progress of compositions. Just as in architecture the big supporting and connecting members—piers, columns, girders, and arches—determine the form and size of a building, as well as its interior division into rooms, corridors, and floors, irrespective of the material of which they are built—so tonal relations introduce order into the tonal mass. Rhythm determines only its temporal succession. In our analogy, rhythm would determine the dimensions of the parts of the building and their distance from one another. Of course one cannot separate one function from the other. The supporting and connecting function of the columns cannot be separated from their place in space, and tonal relations must have definite rhythmic dimensions for their effect. But separate forces are nevertheless at work, as we see in places where one is much stronger than the other: where rhythm retreats far into the background in favor of a broad harmonic flow; or, on the other hand, where rhythm is the predominant element, and harmony and melody are hardly more than the coloring of the beats. How the tonal relations operate, we shall learn in Chapter IV.

2

Combination Tones

Up to now we have treated the tones only as members of a family, grouped about a progenitor tone. But a single tone is not music; unrelated and motionless, it cannot be called anything more than an “acoustic phenomenon”. Even tonal relations, which form the basic principle for the organization of tonal combinations, are not in themselves music. The individual tone is not music until it is directly connected with other tones, and tonal relations are not operative until tones and tonal combinations are in motion. The primary building material of music must therefore include a third element, in addition to the tone and the principle of tone-relations. The motion from one tone to another produces melodic tension, and the bridging of a gap in space by the simultaneous juxtaposition of two tones produces harmony. Thus the Interval, formed by the connection of two tones, is the basic unit of musical construction! If we think of the series of tones grouped around the parent tone C (as in Series 1) as a planetary system, then C is the sun, surrounded by its descendant tones as the sun is surrounded by its planets. Series 1 shows us the distance of the planets from the central star. As the distance increases, the warmth, light, and power of the sun diminish, and the tones lose their closeness of relationship. The intervals correspond to the distances of the various planets from the sun. In their melodic function, the two successive tones of an interval are like two planets at different points in their orbits, while the formation of a chord is like a geometric figure formed by connecting various planets at a given instant.

Just as the tone-relations are arranged in descending order of value, so the intervals have a natural order, which we shall call Series 2. Since we possess no material other than the twelve tones of the chromatic scale, they must also present the tonal material for Series 2. But here their significance is entirely different from what it was in Series 1, since we shall use Series 2 to evaluate the distances between the various tones, and not the relationship of each tone to
the progenitor. The basic difference between the two series will be clear to us if we again think of the architectural function of their components in musical construction. Series 1 provided us with the principal members of the structure; Series 2 will furnish the smaller materials: bricks and mortar, rafters, floor-boards, lath and plaster. The stone that can only be made into a wall with thousands of other stones; the shingle that needs hundreds of other shingles and cross-pieces before the roof is complete—these things are not governed by the same rules as piers and girders. Even if the larger members are made out of the smaller ones, the properties of each of the latter are of importance only until the next similar unit is reached; the characteristics of these small building materials have little effect on the larger lines of the design. But it is only the knowledge of the properties of the smaller units that enables the builder to make walls, floors, and roofs out of them. We derived Series 1 from the overtone series. For the formation of Series 2 we must proceed to the examination of another natural phenomenon: combination tones.

When a stringed instrument plays a double stop, or two bassoons play together, or simultaneous groups of tones are produced in any other way, additional tones are involuntarily produced which bear the name of combination tones. They are usually so weak that the superficial ear does not perceive them, but this makes them all the more important for the subconscious ear. They are the third point of a triangle whose other two points are in the sounding interval, making possible for the ear a sort of trigonometry by which it is enabled to form a judgement of the purity of an interval. The musician who is not familiar with these phenomena had better impress clearly upon his mind the difference between overtones and combination tones. Overtones are produced in varying number by a single sounding tone; combination tones arise only when two or more tones sound simultaneously. How little the two phenomena have in common is shown by the simultaneous sounding of pure tones, having no overtones, previously mentioned as produced by special electrical means, or by tones poor in overtones, such as those produced by tuning forks. Such tones, poor or lacking in overtones, nevertheless produce combination tones; as a matter of fact, they produce them particularly easily.

Although the nature of combination tones has been known for a long time, they have never been applied in musical theory, in a degree appropriate to their significance, to the explanation of the properties of musical materials and the rules for musical writing. This is owing to the great pains necessary to observe them and the meagre results obtained from so doing. Tuning forks and resonators, the classic tools for the investigation of combination tones, are inaccurate and clumsy weapons, to which they offer stubborn resistance. Today we have better tools, and while the appropriate experiments are not easily available to every musician, description of my own observations will enable him to follow my reasoning.

Many tone-colors are particularly favorable to the production of prominent combination tones. The latter can be clearly heard when two large tuning forks sound simultaneously, and the violinist hears them as soft bass tones when he plays double stops in pure intonation. Once the ear has become aware of them, it hears them easily, and in certain cases it hears the combination tones as strongly or even more strongly than the directly produced tones. This fact is of importance in instrument construction, as we see from a well known device among organ builders: in order to save the great expense and large amount of space required for the deepest labial pipes, builders of small organs who wish to provide for these very low tones take two smaller (higher) pipes which, when sounded together, will produce a combination tone of the desired pitch. From this fact we draw the important conclusion that an interval and its combination tone bear a certain immutable relation to each other, a conclusion confirmed by the following experiment.

Electric instruments for the production of tones permit us to let a given tone (say, c') sound uninterrupted, with constant volume and timbre. In the following drawing, this sound is represented by the horizontal line starting at the point c'.
We then take another tone and move it in a steady upward glide, starting at the unison, $c^1$, and rising to $c^2$ (the oblique line $c^1-c^2$). We can then hear, aided by sufficient amplification, a series of combination tones, also rising, which starts from zero (when the two tones are in unison), comes into being at a pitch so low as to be barely perceptible the moment we diverge ever so slightly from the unison, and forms, against the gradual ascent of the upper voice, a steeply rising curve. Towards the end of this curve the angle of the ascent decreases, and when the two main voices are at the interval of the octave the curve arrives at the pitch of the lower tone of that octave. The graphic representation of this procedure shows that beneath the played fifth we hear the octave below its lower tone, and beneath the fourth the double octave below its upper tone. That is, the simultaneous sounding of the tones $c^1$ (256 vibrations per second) and $g^1$ (384) produces the combination tone $c$ (128), while the simultaneous sounding of $c^1$ (256) and $f^1$ (341.33) produces $F$ (85.33). In other words, the fifth, corresponding to the overtones 4 and 6, carries with it a combination tone equal to overtone 2. Or, in ratios: the fifth 2:3 produces the combination tone 1; the fourth 3:4 (for the sake of comparison, we transpose the fourth $c^1-f^1$ of Figure 31 to g-c', the fourth in our original overtone series) likewise tone 1. From these facts we may deduce the principle governing the relation of the combination tone to the directly produced interval:

*The frequency of the combination tone is always equal to the difference between the frequencies of the directly produced tones of the interval.*

This principle also takes care of the ratios.

Combination tones, being real sounds, obey the same laws as other tones. As component parts of sounding intervals, they produce further combination tones, which are, of course, less intense than the original ones. If a combination tone consists of the difference between the proportion numbers (or frequency numbers) of two tones, then by the same process we have already used we can
easily find the combination tones of the second order. Let us take for example the minor third e\textsuperscript{1}-g\textsuperscript{1} (320–384), which has the ratio 5:6. The first combination tone is the note C, with 64 vibrations. This tone, in connection with one of the directly produced tones, results in a combination tone of the second order. The g\textsuperscript{1} cannot be the tone that produces this second combination tone (in connection with the first), because it would produce nothing more than a change in intensity in the original interval: interval C-g\textsuperscript{1} (1:6), combination tone e\textsuperscript{1} (5). So it must be the interval C-e\textsuperscript{1} (1:5) that produces the new combination tone—c\textsuperscript{1} (4). If we find in this way the differences between the combination tones of the first order and the lower tones of the directly produced intervals, we shall arrive at the curve of combination tones of the second order.

\[ \text{[62]} \]

\[ \text{[63]} \]

This curve goes in the opposite direction to the first: it begins at the unison with the directly produced tone, and sinks slowly, arriving beneath the played interval e\textsuperscript{1}-e\textsuperscript{1} at g, beneath c\textsuperscript{1}-f\textsuperscript{1} at f. Beneath the played fifth it intersects the first curve, and then sinks more quickly to zero, whence its opposite arose.

The series of combination tones of the second order combines with the intervals already present, consisting both of directly produced and of combination tones, to create new orders of combination tones. Theoretically, the system may be extended to infinity.
But we remember that the overtone series, too, is theoretically infinite. In practice it is hardly possible to produce audible combination tones of orders higher than the sixth. For the ear’s sense of intervals the later orders of combination tones are without significance, since they are hardly perceptible even to the fine discriminations of the inner ear (of which we are not conscious), and since furthermore so long as the directly produced intervals are kept close to the simpler ratios of the overtone series, nothing is produced but octave doublings of earlier orders of combination tones. For the purposes of our discussion we shall therefore content ourselves with considering the combination tones of the first and second orders.

3

Inversions

We cannot hear combination tones produced by either the unison or the octave. For the unison, the first order of combination tones is at the zero point, and the second order coincides with the unison of the directly produced tones. For the octave, the second order is at the zero point, and the first coincides with the lower of the original tones. Combination tones represent a clouding or a burdening of the interval. The octave and the unison, as the most perfect intervals, are not subject to any such impurity; the fifth has only one combination tone, since those of the first and second order coincide; all other intervals carry a double burden of varying weight. The clouding of the intervals is not so strong that any effort need be made to suppress the combination tones. Yet of course they must not be so strong as to overshadow the directly produced interval. Provided they remain below the level at which they would become actually disturbing, they give the interval its characteristic stamp. An interval without combination tones would be an abstract concept, as bodiless as the ratio with which we express it numerically. Now, for the musician who, despite the intangibility of his building materials, is a healthy realist in his craft, numbers and intervals are of value only as sounding entities. He will accept calculations employing proportions and curves only if they seem to offer practical advantages in the solution of musical problems. Thus the clouding of the intervals through the combination tones is not something that spoils his enjoyment of the abstract interval proportions; on the contrary, he uses it as a means of more precise perception of the intervals. The differences in the weight of the combination-tone burdens carried by the various intervals enables him to arrange the latter in order, so that starting with the octave, as the clearest, unclouded interval, and passing through the fifth (slightly clouded), each interval in succession carries a greater burden than its predecessors; that is to say, the purity and harmonic clarity of the intervals diminish step by step. In this series—Series 2—we are accordingly setting up a list of the individual building stones according to strength, hardness, and density.

The fifth bears, as we have seen, but a slight burden:

\[ \text{\textbf{5}} \]

Its two combination tones coincide at a point which doubles one of its tones at the lower octave. The fourth, too, shows a doubling of one of its factors:

\[ \text{\textbf{4}} \]

But since these combination tones do not coincide, but are an octave apart, the fourth seems somewhat more heavily laden than the fifth.

\[ \text{\textbf{4}} \]

The major third and the minor sixth, too, show doublings of one of their constituent tones in their combination tones. In the major
third the lower tone is doubled; in the minor sixth, the upper. The minor third and the major sixth, on the other hand, carry two tones not among their original constituents. Thus we see that the intervals seem to pair off according to the arrangement of their combination tones. The two members of each pair have the same combination tones. The only difference is that the second interval of each pair seems to reverse the relations of the combination tones (in this connection we disregard octave transpositions). If in the first interval the combination tone of the first order forms the fundamental of the whole tonal composite, then in the second interval of the same pair this tone takes the place which the second-order tone had in the first interval: in this second interval, the second-order tone becomes the bass. The major third c\(^1\)-e\(^1\) has combination tones in the positions C (first order) and g (second order); and the minor sixth e\(^1\)-c\(^2\) has c (second order) and g (first order), as may be easily seen by transposing upward the minor sixth c\(^1\)-ab\(^1\) in Fig. 37. The intervals thus pair off as follows: fifth and fourth, major third and minor sixth, minor third and major sixth, major second and minor seventh, minor second and major seventh.

Here we have a purely acoustical proof of the invertibility of intervals. That intervals are invertible, and that an interval adds up with its inversion an octave, could hitherto be proved only from the ratios of the overtone series: the inversion of the major third 4:5 is the minor sixth 5:8 (5:2 × 4); the inversion of the minor third 5:6 is the major sixth 6:10 (or, in reduced terms, 3:5). Considering the above-mentioned dislike of most musicians for figures and other abstractions, proof offered by the combination tones ought to appeal to them more than that from the mere ratios, the more so as it provides us at the same time with another important item of knowledge: the two intervals that make up each pair are of unequal value. In order to understand that statement, let us keep two facts clearly in mind:

1. In groups of tones of different pitch sounding simultaneously, the deeper tones, with the slower vibration rates, have greater weight than the higher ones (a fact based on the weight of the vibrating material—the air masses).

2. Combination tones of the first order are significantly louder than those of the second order.

In the combination tones that arise from a major third, the lower, because of its low pitch, has the greater weight:

And it also (as a combination tone of the first order) surpasses its companion in intensity. The combination tones that belong to the inversion of the major third (minor sixth), on the other hand, present a less clear picture. Here too, of course, the lower tone (the doubling of the upper of the two directly produced tones) is the heavier; but it lacks the confirmation of this advantage which its prototype in the major third had. This tone is only the weaker combination tone of the second order, and is surpassed in intensity by the combination tone of the first order, which lies above it. It does not achieve the effect that is really due it, as the doubling of one of the original tones of the interval, because of the competition of its more intense but less significant companion (less significant in its relation to the tonal composite formed by the original interval and its combination tones). The aural impression created by the major third is thus distinctly clearer and more definite than that of the minor sixth. In the interval-pairs, both thirds and seconds show this more favorable disposition of their combination tones as compared with their inversions (sixths and sevenths). Between the fifth and the fourth the difference is not so marked, since the first-order combination tone lies below the second in both, whereas all other inversions exhibit the opposite disposition. But despite this favorable arrangement of the combination tones, the fourth is inferior to the fifth. The coincidence of the two combination tones
of the fifth gives it a purity which makes it superior to all other intervals except the octave.

4

**Interval Roots**

We have learned two lessons from our consideration of the combination tones—the proof of the invertibility of intervals, and the determination of the relative value of the intervals. A third awaits us.

If one of the tones of the directly produced interval is doubled, either in the unison or in a lower octave, by a combination tone, this accretion of strength gives it the upper hand over its partner. In intervals in which such doublings occur, the constituent tones are thus not of equal value. Rather, the tone strengthened by such doublings is to be regarded as the root of the interval, and the other as its subordinate companion. Numerous experiments have convinced me that the feeling that one tone of an interval has more importance than the other is just as innate as the ability to judge intervals exactly—everyone hears the lower tone of a fifth as the principal tone; the ear cannot be persuaded to attribute primary importance to the upper tone. Yet I have never found in any treatise the statement that intervals have roots—a curious circumstance, since this fact is of primary significance for the hearing and evaluation of harmonic intervals, and since its acoustical basis is so easily established:

The lower tone of the fifth is the root. The fact that both combination tones coincide at the lower octave of this tone makes it doubly strong; that is why the impression of the lower tone as root is so unmistakable, and the fifth is so very stable. In the fourth, the upper tone is doubled by the combination tones, and thus it is not the heavier lower tone that has the effect of a root. The interval is thus less steady than the fifth.

In the next pair of intervals, too, the root of the more stable major third is at the bottom, and of the minor sixth at the top.

The new tone introduced by the combination tones converts this interval into a triad, of which the root is doubled. The triad is not present in full strength, to be sure; yet it is so clearly represented that the rules of harmony permit the use of the "triad without its fifth"—a contradiction of the definition of a triad as consisting of three tones, while this interval contains only two. This pair of intervals offers a particularly clear example of the inferiority of the second interval, the minor sixth, in harmonic value. The major third contains in the pleasantest disposition the doubling of its lower tone by the first-order combination tone, while the new tone, which completes the triad, lying a fourth below the lower tone of the original interval, is appropriately weaker, being a combination tone of the second order. In the minor sixth, on the other hand, the root, which, because in the directly produced interval it is higher than its companion tone, is to that extent already weaker, is doubled only by a combination tone of the second order. The latter does, to be sure, lie in the bass, but because of its weaker intensity it is less prominent than the less important first-order tone which completes the triad. Obviously so ambiguous a tonal structure must produce a less satisfying effect than that of the major third.

The next pair of intervals consists of the minor third and its inversion, the major sixth.
Neither of the original tones is represented among the combination tones; instead, a new tone occurs, doubled in the octave, which again converts the interval into a major triad, but this time a major triad of which the root is not one of the original tones. That is, beneath the minor third lies the lower fifth of its upper tone, and beneath the major sixth the lower fifth of its corresponding lower tone. The minor third has the better disposition of the combination tones, since its first-order tone lies in the bass, whereas the lowest tone of the composite formed by the major sixth and its combination tones is the less intense combination tone of the second order. Since the new tone, not represented in the original interval, occurs in two different octaves among the combination tones, and since, lying below the original tones, it possesses greater weight than they, it fulfills all the requirements of a root. This confronts us with the somewhat surprising necessity of placing under every minor third and major sixth a tone which is not represented either in the directly produced interval or, of course, in its notation. Here theory comes into conflict with the practice of composition, which likes to deal with things that are clearly to be heard and seen, and would therefore like to take one of the two tones of the directly produced interval as its root. Practice could perfectly well yield to the theoretical requirement without the slightest hindrance, as long as the minor third and the major sixth appeared alone—that is, in two-part writing only. But as soon as these intervals appear in combination with others, which is by far the most usual practice in our music, the observance of this theoretical requirement would make our analysis far too difficult and too different from our habitual point of view, so that it seems more advantageous to treat both intervals according to the pattern which we have derived from their predecessors in the series. This would make the lower tone of the minor third, and the upper tone of the major sixth, the root.

Another consideration, besides ease in treatment and analogy with the root-determination of the other intervals, speaks for this procedure. This is the fact that whenever minor thirds and major sixths appear as parts of richer tonal combinations, they are almost always subordinate to stronger and more important intervals, so that it is unnecessary to set up a separate rule to cover the few instances in which these intervals are of governing importance, merely to satisfy theoretical requirements. We do not deny the existence of the real roots of these intervals, or their significance, nor does the system here set up as a natural and logical basis for musical composition suffer any lacuna on this account. We are simply making use of a convenient labor-saving device to render these intervals easier to handle. Anyone who rejects this aid, even though it has always been adopted in practice, and prefers to make his study of musical writing more complicated in order to stick absolutely close to theory, may take the real root established by the combination tones as the root of every minor third and major sixth he uses.

Our decision makes it possible for us to use both intervals without hesitation. But if, although we need seek no farther, we insist on looking for an acoustical justification for the step we have taken, we may examine a little more closely the thirds that exist in the lower part of the overtone series. (The basis thus arrived at, however, will not change anything connected with the combination-tone structure of the minor third and the major sixth as explained above, and will not furnish a complete proof.) Within the first 11 overtones alone there are five different sizes of thirds:

\[
\begin{array}{c}
4:5 \\
5:6 \\
6:7 \\
7:9 \\
9:11
\end{array}
\]

the major 4:5, the minor 5:6, the under-sized 6:7, the over-sized 7:9, and the one which is between major and minor, 9:11. To these we may add the Pythagorean third, which we can easily calculate (see page 30), and which is between 4:5 and 7:9 in size. The ear
not only hears all these as thirds; it permits itself to be hoodwinked still further by this beautiful but characterless interval. If we play on the violin or other appropriate instrument a third that is as small as it can be without being a second, and if we then slide the upper tone up to the upper boundary of the third, just below the point where it would become a fourth, we cannot say just where the change from a minor third to a major third takes place.

In the middle space between the outside boundaries there is a field that can belong to either third, and is assigned by the ear to the major or the minor according to the harmonic or melodic context. With an interval that is so indefinite we can well afford to allow ourselves the liberty proposed.

Those intervals whose tones are separated by such great distances that they seem to be octave transpositions of fifths, fourths, etc., present much less happy dispositions of the combination tones than their prototypes, and accordingly justify the widely current practice, as old as musical composition itself, of assigning to them lower harmonic values, diminishing as the distance between their constituent tones increases. Even the octave, which stands above and beyond all calculation of interval values, loses so much of its value when it appears in the form 1:4 that, as its combination-tone structure shows, it is hardly equal to the fifth in clarity. In the form 1:8 the harmonic support of the interval by its combination tones is still less strong, and in the form 1:16 the composite becomes completely dissonant. Although all these forms are exceptional, adapted only to particularly characteristic and striking effects, and consequently seldom concern us as building material for the craft of composition based on a normal harmonic foundation, they are not useless. True, they seldom occur except as parts of chords containing one, two, or even more additional constituents, which take away the empty effect of the extended two-tone interval. But one might assume from the disposition of the combination tones of these intervals that when they did occur unmixed their effect would be much worse than it actually is. The fact that it is not is owing to the overtone series of the lower constituent tone of the interval, which is not disturbed by the comparatively strong combination tones that lie between the two tones of the interval, and which instead takes over their functions and fills in, sparsely but adequately, the large tonal expanse. When the tones of the interval are normally close together, the narrowness of the space and the force of the combination tones prevent any effect of the overtone series on the harmonic content of the interval.

The fifth in the position 1:3 still presents an excellent tonal picture; even 1:6 is good; not until 1:12 does it begin to lose value, although the freely unfolding overtone series of the lower tone of the fifth prevents it from becoming altogether worthless. The fourth is strikingly less fortunate—3:8 is still good, though a bit unstable; but 3:16 is worth nothing (and who would think of introducing the fourth 3:16 into an otherwise smooth two-voiced texture?). That is owing not only to the combination tones, but also to the strong overtone series of the lower tone, in which the upper does not occur. The intervals after the fifth and the fourth lose their harmonic value in such octave transpositions even faster, and the inverted intervals (sixths) are, as one might expect, even worse off than the uninverted thirds.
The question then arises how these widely spread intervals are to be treated, for although in simple writing they never appear, and in the more complicated forms they occur only seldom, still we must know how to use them and what rank to ascribe to them. For this purpose one might determine separately the root of each of them, by experiment and by analogy, and then one would simply have to learn these roots by heart, in order to have them readily at hand when they were needed. But there is no point in going to all this trouble for a few exceptional cases. It is more practical here, too, to take a short cut, and to handle the spread intervals exactly like their close prototypes. This is quite sufficient for the practical purposes of composition.

5

The Minor Triad

In connection with the above description of the nature of the third we must mention a chord that has always given theorists endless trouble—the minor triad. To understand and explain the major triad is a task made easy for us by Nature, who places it in our hands as a handsome gift. But she gives us no hint about the minor triad. It does not occur in the overtone series, at least not in three successive tones. In the upper reaches of the series, minor triads can be constructed by skipping some tones (10:12:15); but this seems too far-fetched an explanation of a chord that appears almost as valuable as the easily explainable major triad.

We have seen that the value of a harmonic interval is determined by the grouping of its combination tones. The euphony of the major triad must accordingly be based not only on its favored position in the overtone series, but also on the disposition of its combination tones. In the major triad c'-e'-g'

the minor third c'-e' produces the combination tone C (first order) and g (second order); the minor third e'-g' produces C and c'; and the fifth c'-g' only c. The directly produced triad is strengthened in the most complete way by the combination tones. How unfavorable, in comparison, is the picture presented by the minor triad. In the triad c'-e'g'

the minor third c'-e' produces the combination tones A, and a; the major third e'-g' produces E, and b; and the fifth c'-g' produces c, as in the major triad. All that the combination tones tell us about the minor triad is that it is of less harmonic value than the major.

Almost all explanations of the minor triad have proceeded from the assumption that it is based on the converse of the natural principle of tonal construction. According to them the minor triad is a mirror image of the major. Within the boundaries of the fifth, the thirds are said to be so arranged that the order “major third, minor third” of the major triad is reversed in the minor. That is not hard to see, but it does not prove anything. In working with an element tied to the principle of gravity as closely as tone is, one cannot simply turn things upside down for the sake of a pretty idea. Every tonal composite is constructed from the bottom up; that is determined by the nature of the tonal world. Tones obey the laws exemplified in the overtone series just as the stones piled one upon another to make a building obey the laws of statics that apply in nature.

The cleverest of all the explanations based on the thirds contained within the fifth in the two triads is the one that says that the origin of the minor triad is in man's desire to transfer to the world of tone the symmetry of his body. Since the major triad, on account of the inequality of its thirds, is an asymmetric structure, its opposite must be erected to restore the balance. This would be
convincing if other instances of such efforts to erect symmetrical counterparts were to be found in music. In the domain of visual forms, symmetry is one of the most important principles of design; tonal and temporal phenomena, on the other hand, seem to avoid it. Except in a few of the simplest basic rhythmic and formal elements (measure-rhythm and the simplest song-forms), it is hardly to be found in the field of aural forms. It is true that every more or less extended musical form consists of parts which offset each other to maintain the tonal balance of the whole. They are usually of unequal weight, since the juxtaposition of equal weights (a truly symmetrical arrangement) does not satisfy the listener. Hence the rule that when a section is repeated, or when a later section corresponds to an earlier one, changes of a formal nature—i.e., abbreviations or expansions—must be made. The extremely few examples of strictly symmetrical structure in musical works of recognized worth (except where other features eclipse the formal design) are the exceptions that prove the rule. Against the theory of symmetrical construction is also the fact that while it is true that we have a minor triad that is the opposite of the major, we do not have any opposite of the complete major tonality, which may be represented by its major triads on the fourth, fifth, and first degrees, in the shape of a really independent minor tonality. In harmonic progressions we make no distinction between the major and minor modes. Only the triad on the first degree tells us which mode is meant; all the other triads exist in both modes with major and minor thirds. A major dominant is the rule in minor, and the minor subdominant is fully at home in major, as are the Neapolitan and other alterations. Indeed, one form of the minor scale, the upward melodic, is the same as the major with the single exception of the minor third, and the fact that the other forms differ more importantly is due to a desire to reconcile the sixth degree with its close relative, the third: it is felt to be undesirable that an augmented fourth should exist between these two degrees. Thus there is no sign of symmetry between the major and minor modes.

A more daring explanation is the one that assumes a different effect of the overtones in the major and minor triads. The three overtones 4:5:6 that make up the major triad (c'-e'-g' in the series based on C) have a common fundamental, of which the first tone of the triad forms the octave, while the second and third tones form the third and fifth.

The minor triad is said to exhibit the opposite relations. When it is in the corresponding close position, its three tones are said to have in the g a common overtone, which bears the relation of fifth, third, and octave, to the root, third, and fifth of the triad respectively. It is not clear why in one case a lower tone (fundamental or combination tone) and in the other a higher tone (overtone) should be adduced. The basic error of this explanation is, however, that it reckons with actual tonal relations which are no sooner cited in the case of one triad than they are disregarded in the case of the other. If the overtones of the minor triad are significant, then so are those of the major triad. But then we should have between the two g's, which occur as the overtones of the c' and the g', a g#, as the fifth overtone of the e':

And here the whole house of cards collapses, for the minor triad should rank higher in the scheme of tonal values than the major, since its overtones are better arranged.

The most far-fetched and at the same time the most interesting explanation of the minor triad is the one based on the undertone series. The latter is the exact inversion of the overtone series:
It seems to me repugnant to good sense to assume a force capable of producing such an inversion. This force would do away with the gravitation that is expressed in the overtone series—and there is no evidence of the operation of such a force. In electro-acoustics there is a familiar phenomenon that it would be easy to mistake for an undertone series. Electric tone-producers can be made to sound, in connection with a given tone, combination tones having a wavelength twice, three times, four times and so on (and accordingly having a frequency one-half, one-third, one-fourth and so on) that of the given tone. This remarkable phenomenon, caused by the intersection points of the electric waves (and consequently of the air-vibrations), can never have for music the same significance as the overtone series. It occurs only under certain conditions, which were not possible before the days of electric tone-production, and which can never be produced by vibrating strings, pipes, or membranes. This “undertone series” has no influence on the color of the tone, and lacks the other natural advantages of the overtone series which arise without any artificial help and are available anywhere and anytime. Thus we have here no proof of any inversion of the overtone series occurring freely in nature. This phenomenon, which actually exists for the ear, does offer, like the purely theoretical undertone series, a tempting picture of the minor triad outlined by its first six tones. And yet even with special apparatus or with the help of the Dualistic theory we learn nothing about the minor triad except that it is the opposite of the major. For that we do not need the undertone series; the simple principle of interval inversion suffices.

What, then, is the minor triad, in reality? I hold, following a theory which again is not entirely new, that it is a clouding of the major triad. Since one cannot even say definitely where the minor third leaves off and the major third begins, I do not believe in any polarity of the two chords. They are the high and low, the strong and weak, the light and dark, the bright and dull forms of the same sound. It is true that the overtone series contains both forms of the third (4:5 and 5:6) in pure form, but that does not alter the fact that the boundary between them is vague. Pure thirds furnish us with pure forms of both major and minor triads. But the ear allows within the triads, too, a certain latitude to the thirds, so that on one and the same root a number of major triads and a number of minor triads can be erected, no two alike in the exact size of their thirds. Triads in which the third lies in the indeterminate middle ground can, like the third itself, be interpreted as major or minor, according to the context. But why the almost negligible distance between the major and minor thirds should have such extraordinary psychological significance remains a mystery.

It seems as if this middle ground between the thirds were a dead point in the scale, to which another similar but less significant dead point corresponds—the middle ground between the two species of sixths. Up to this point the harmonic force of the tonic has been working up from the bottom; here begins the dominance of the fourth and fifth, which extends from the boundary between the thirds to that between the sixths. Thus the minor triad would be associated with rest, and would derive from this fact its heavy, dull character. The major triad, of which the third lies in another field of force, would then receive from the active, life-giving sources other than the tonic its impulse, light, and energy.

6

Seconds and Sevenths; The Tritone

For the practical application of the next two pairs of intervals—major second and minor seventh, minor second and major seventh—it makes no difference which of the tones we take as the root. The combination tones do not point to definite conclusions. Seconds and sevenths are subject to greater variation than any other intervals; in both melody and harmony they occur in the greatest variety of sizes. A glance at our table of combination tones shows us that even
slight changes in the sizes of intervals have important consequences for the disposition of the combination tones.

If we transpose all the seconds that occur in the overtone series between tones 7 and 11 so that they have a common lower tone, then the undersized major second $c'\cdot d'$ (10:11)

has the combination tones $A_b$ and $+b_f$

the major second (minor whole-tone) $c'\cdot d'$ (9:10)

has the combination tones $B_b$ and $b_f$

the major second (major whole-tone) $c'\cdot d'$ (8:9)

has the combination tones $C_1$ and $-b_f$

the oversized major second $c'\cdot d'$ (7:8)

has the combination tones $D_1$ and $a$.

The situation is reversed, of course, for the minor seventh.

The minor second and major seventh exhibit still more complex dispositions of their combination tones. If we do not wish to make our work impossibly complicated, we must renounce hair-splitting distinctions between the various sizes of seconds and sevenths. We shall assume for each interval a normal size representing the average of the possibilities. The choice of a root is made more difficult by the wide choice of combination tones. It would be very tempting to take the lower tone of the second $c'\cdot d'$ as the root, because of the combination tone $C_1$ produced by the major whole-tone. Practical considerations, however, lead me to choose the upper tone as the root. Our familiarity with the dominant seventh chord leads us to hear the lower tone of the seventh belonging to this composite as the root, even when it appears alone:

---

At least this choice seems more natural than the opposite one. As the inversion always has the root in the opposite position, the upper tone becomes the root of the major second. We treat the pair consisting of the minor second and major seventh in exactly the same way: the root of the second is at the top, and of the seventh at the bottom. The objections of the doubter who made himself heard earlier, when we chose the roots of the minor third and major sixth, will no doubt be more vehement this time. In self-defense I can again cite the practice of composers. And to dissipate all doubts I suggest that the attempt be made once to find the true acoustic roots of all the seconds and sevenths one works with, among the many possibilities. Anyone who once carries out this very laborious procedure will soon enough find justification for the easing of his work. For he would have to spend ten times as much time and trouble on determining the exact size of the intervals as on writing them.

To complete Series 2 we still need one interval: the tritone. This is the name given since time immemorial to the augmented fourth, reflecting its construction of three superposed whole-tones. The term does not really fit the enharmonically equivalent diminished fifth. But because of our constant use of chromatic and enharmonic formations, we differentiate between the two intervals nowadays only on paper, so that I do not hesitate to group both intervals under the same tritone. The tritone does not make a pair with any other interval. It stands at the end of the series of pairs, as the counterpart to the octave that stands at the beginning:

The octave is the proudest, the noblest of the intervals, and does not mingle with the others; the tritone is the most distant relative, the eccentric, barred from close association with the interval pairs like Loki among the gods of Valhalla—and similarly indispensable.

The tritone has no root. It is accompanied by combination tones that stand in an unusual relation to it.
When its two tones are in their closest position (5:7), the combination tones form a fifth which combines with the tritone to make a seventh-chord, in which the lower tone of the tritone is the third, and its upper tone (although too low) the seventh. In the opposite, widest position (7:10) the combination tones form a fourth. The latter combines with the tritone to form a four-three chord, in which the lower tone of the tritone is the seventh, and the upper the third of the chord. All the tritone intervals that lie between these two extremes produce seventh chords which are between the two given above. Consequently, the tritone always has a dominant effect. It is characterized by a tendency towards a tonic, a tendency most naturally satisfied by a progression which takes the form of a "resolution" to the progenitor tone of its family (complemented by one or more tones which form with it either an interval or a chord). But already we see the dual nature of the tritone: if the preceding interval-successions have not made the relationship to a progenitor clear, one has the choice between two equally good resolutions. And in the resolution, the ear always hears one of the tones of the tritone as a leading tone to the root of the following tonic chord:

But since the ear cannot at once decide which of the tones of a tritone heard without clear family relations is the leading tone, it is always uncertain in its reaction to this interval. On the one hand the tonal uncertainty of the tritone, which makes it vaguer and more opalescent than any other interval, and on the other its strong urge for resolution, which at the moment of progression monopolizes the attention—this combination of indefiniteness and tension is what distinguishes the tritone, and makes it a foreign body and a ferment among the intervals.

Although this sanctimonious interval, at once obscure and insistent, permits us neither from its aural effect nor from its acoustic construction to declare one of its tones the root, we must at any rate, in order to handle it at all, be able to decide from case to case which of its tones is the more important. The sound of the interval itself artfully conceals from us any answer to this question, and we must draw our conclusions from its environment. From the tone, chord, or interval to which the tritone resolves we see to which family-progenitor it belongs. We shall consider that member of the tritone which proceeds by the smallest step to this progenitor (the root of the resolution interval) as the root representative.

We can easily understand how the tritone has in all periods of music history held its unique position among the intervals. Instrumental music has arrived at a modus vivendi with it, aided more or less by the mechanization of its method of determining the pitches of the tones. But to the singer, especially the choral singer, it is still loathsome. Musical theory has always been at odds with the "diabolus in musica", and has always treated it with a peculiar mixture of love and hatred. Theorists at first tried to get around it. The Greeks avoided it by the interpolation of a complementary tetrachord (symemmenon) among their four regular ones. In the church modes the device used was the substitution of B rotundum for B quadratum. The rules for organum and descant excluded the tritone, and its revenge was that this exclusion prevented them from prospering. Then a settlement was made: the treatises of mediaeval theorists are an endless chain of attempts to accommodate the "mi contra fa"; solmization is the attempt to take in the unwelcome guest with impunity. Finally, the tritone became the pet of harmony, through the outstanding importance given to all chord formations serving as dominants, through the harmony of Tristan and the chromaticism that followed in its wake, and even through such flimsily based devices as the whole-tone system that
flourished about the turn of the last century. For us, who have now learned the position of the tritone within the family of intervals, and the grounds of its claim to that position, it has lost its terrors. Yet even for us it remains a civilized demon—"der Geist, der stets verneint": the spirit that ever denies.

7

Significance of the Intervals

The inclusion of the tritone completes Series 2. To refute once and for all the superficial observation that might be made—to the effect that Series 1 and Series 2 are so much alike, except for a slight difference as concerns the thirds and sixths, that the setting up of two series is superfluous—let us once more state briefly the difference between them.

Series 1 consists of tones, in relation to a progenitor tone from which they derive their tonal position. Series 2 consists of intervals, without relation to a progenitor tone. (Instead of taking c¹ as the point of departure for our interval series, we could perfectly well have taken a different tone as the basis of each interval, without disturbing the orderly procedure of our investigation in the slightest; whereas in the construction of Series 1 that would not have been possible.) In the following example:

\[ \frac{g}{c} \]

the effect of Series 1 is such that the g¹, as fifth of the c¹, assumes a preferred status. The a¹, as major sixth of c¹, is less closely related, and the e¹, as major third, even less closely. (As we shall see later, Series 1 will not be used for the analysis of such small tonal groups; it will be reserved for higher purposes, and is instanced here only for the sake of comparison.) Series 2, on the other hand, tells us that the skip of a fifth (g¹-c¹) is stronger in harmonic effect than the skip of a fourth (e¹-a¹), which latter, however, is stronger than the third c¹-e¹ or the second a¹-g¹.

The value-order laid down in Series 2 brings us close to the question of the consonance or dissonance of intervals. The interval-pairs do not indicate by a gap of any kind that there is any point at which the consonances stop and the dissonances begin. The two concepts have never been completely explained, and for a thousand years the definitions have varied. At first thirds were dissonant; later they became consonant. A distinction was made between perfect and imperfect consonances. The wide use of seventh-chords has made the major second and the minor seventh almost consonant to our ears. The situation of the fourth has never been thoroughly cleared up. Theorists, basing their reasoning on acoustical phenomena, have repeatedly come to conclusions wholly at variance with those of practical musicians.

Our investigation dissipates the fog that has hitherto prevailed. We know that no point can be determined at which "consonance" passes over into "dissonance". We can afford to let these terms stand for the extreme boundaries of the satisfying and unsatisfying effect of intervals and chords. The consonant intervals would then appear at the beginning of Series 2 and the dissonant at the end. But the rate at which the consonance of the intervals near the beginning decreases and the dissonance of those near the end increases cannot be determined exactly. Between the octave as the most perfect and the major seventh as the least perfect intervals, there is a series of interval-pairs which decrease in euphony in proportion as their distance from the octave and their proximity to the major seventh increases. The tritone belongs neither to the realm of euphony nor to that of cacophony; here again, as a unique interval, it remains outside our classification.

We have constructed Series 2 on the basis of the combination-tone curves, in the order of increasing complexity. The history of Western music has followed the same path through the centuries in its recognition of the values of the harmonic intervals. The ear at first recognized only single, monophonic lines, consisting of nothing but fundamental tones (tones numbered 1 in the overtone series). In the course of time, proceeding from interval to interval, it discovered the secret of composites consisting of two or more tones, the secret contained in the combination tones. Singing in
octaves occurred before the earliest beginnings of polyphony, as the natural consequence of the participation of voices of different registers. Successions of parallel fifths and fourths were the first polyphonic devices; and gradually the value of thirds and sixths became apparent. The tritone was—here, too—an exception: it appeared comparatively early, as a component of independent harmonic formations. This would seem curious if we did not know that unimpaired successions of triads seemed to the musical ear, even in the earliest polyphony, a too unalloyed pleasure. Composers met the ear's desire for a more intense sound by introducing the tritone in its mildest form: the first inversion of a diminished triad. The intervals between the pairs of tones contained in this chord are only, apart from the tritone, the minor third and the major sixth; the harsher seconds and sevenths are still lacking. They slipped into harmonic combinations late in the development, and then only by the side-entrance of melodic function (passing tones), until at the end of the seventeenth century the ear had learned to accept them, too, as independent intervals, usable for harmonic purposes. The tritone lies at the top of the inverted diminished triad; the important bass tone is free to move, unhampered by the tritone. Even today, minor seconds and major sevenths have not attained full equality with the other harmonic intervals; and a thousand years of familiarity will not achieve it for them.

For intervals are not like clay, which receives an impression and faithfully preserves it until the next one comes along and effaces it; they are elastic, rather, like steel, and although they vary in hardness, none of them is completely pliable. If we spoke earlier of breaking the will of the tones, this must have meant that we must see to it that the force that is latent in the intervals must be prevented from simply acting freely as it chooses—not that we could by main force stamp the raw material into any shape, without regard to its natural elasticity. Under wise treatment, the tonal material can be easily bent and welded. But if too great a strain is put upon it, or if it is not handled in accordance with the laws of its own nature, it will break like any other building material, and the music constructed from it will be useless.

Harmonic and Melodic Value of the Intervals

Every tonal movement arises from the combined working of harmonic and melodic forces—to ignore the rhythmic element for the moment. Harmony is the more robust of the two elements. It has its own tendencies and it is hard to force. There are many possible harmonic combinations, and the gradations between them are innumerable. The very quantity of the material commands the composer's thorough consideration, and "inspiration" and "invention" can be effective only on the basis of adequate technical knowledge. The novice will hardly succeed in traversing the harmonic territory, which abounds in a wealth of the most manifold phenomena. Melody is less brittle. Many a dilettante, who has no conception of the craft of the composer, gives birth to pleasant melodic ideas. The melodic material is easier to conquer, being of limited extent, and light and airy as compared with the harmonic. But it is also more deceptive. In no field are taste, musical culture, and genuine inclination or the lack of it more important than in melody.

Harmony and melody are complementary elements. Neither is strong enough to stand alone; each needs the other for its full unfolding. Melody sets the sluggish harmonic masses in motion, for no harmonic progression can be made except through melody—that is, by traversing the intervals. Harmony, on the other hand, connects and organizes the waves of melody.

Since intervals are the stuff of music, every interval must have harmonic and melodic characteristics. Series 2 shows the distribution of these characteristics clearly:
Harmonic force is strongest in the intervals at the beginning of the series, and diminishes towards the end, while melodic force is distributed in just the opposite order. The strongest, most unambiguous interval, after the octave, which is unique, is the fifth, while the most beautiful is the major third, on account of the triad formed by it with its combination tones. From this point on, the harmonic effect decreases, until it nearly disappears in the minor second and major seventh. These two intervals are of almost exclusively melodic significance, since they form leading-tones. They can receive any considerable harmonic significance only through the simultaneous sounding of other intervals. The simplest melodic step of the minor second is followed, reading from right to left (after the minor seventh), by the strongest and most beautiful melodic interval, the major second. Just as the most beautiful harmonic interval was not at the very beginning of the series, so the chief melodic one does not lie at the very end.

Series 2, exposed to the free play of harmonic and melodic forces, now reveals clearly the weakness of those intervals which have the less favorable disposition of their combination tones, i.e., the inversions of the more favorably arranged ones: they offer less resistance. Harmonic force, which begins at the left, is almost helpless against the melodic strength of the seconds, whereas it is not without effect on the sevenths; on the other hand, melodic force, proceeding from the right, is helpless against the strong third, fifth, and octave. When these intervals occur melodically, that is, one tone after another, they organize even the most fluent line into harmonic groups. Their inversions yield more easily. Strong harmonic intervals exert a powerful attraction, while their inversions become more easily the object of attraction, and so are more likely to perform a melodic function. Thus the major seventh proceeds to the octave, the minor seventh to the sixth, the sixth to the fifth, and, at the left-hand side of the figure, the fourth yields to the attraction of the third. In the sixths, the two forces about balance. Harmonic force is not strong enough always to vanquish the tendency to melodic development—the step to the fifth—, and on the other hand, sixths are not so strong melodically that they must necessarily have their melodic consequence.

An understanding of these things makes our admiration for mediaeval musical theory greater than ever, for, with all the limitations of its field, and all its clinging to a heritage at odds with musical practice, it always showed an astonishingly sure instinct in all matters pertaining to the intervals. It knew nothing of the power of attraction of the strong intervals, based on the combination tones, and yet it rejected the harmonic interval of the fourth; and it looked with disfavor on melodic leaps of a sixth, not to mention the use of the harmonic intervals of the second and seventh. We are less timid today. We have learned, particularly, to handle sixths either harmonically or melodically, according to the need of the moment, though even today we avoid the harmonic interval of the fourth in places where force and definiteness of expression are desired.

The tritone has no definite significance, either harmonic or melodic. In order to determine its position, we need a third tone. This third tone may sound simultaneously with the tritone,

\[\text{Insert music notation here}\]

in which case the tritone is harmonically determined. Or else the tritone may form a part of a group of three successive tones.

\[\text{Insert music notation here}\]

When such groups are not mere broken chords (which would be of purely harmonic significance), and when no special means are employed to make the tritone the most important part of the group, it becomes melodically subordinate. One of its two tones becomes the neighboring tone of an interval that is harmonically unambiguous, which then purges the tritone of its indefiniteness.
The Conventional Theory of Harmony

There are four points in which the conventional theory of harmony appears too narrow a system for the determination and construction of chords:

1. The basic principle for the construction of chords is the superposition of thirds.
2. Chords are considered invertible.
3. By raising or lowering tones of the diatonic scales the chord-supply of a key may be enriched.
4. Chords are susceptible of various interpretations.

As to point 1:

Trad of all species arise from the superposition of thirds, and by the addition of further thirds seventh-chords and ninth-chords are produced. These groups of intervals may be changed, by the rearrangement of their layers, into chords of different degrees of tension. By this simple means, only a small selection from among the possible tonal combinations is made accessible—a selection which includes, to be sure, the best and most useful combinations. But music is caught in a net of which the warp and woof are scales with their inelastic tonal functions and chords with their inversions. Chords that cannot be traced back to a construction in thirds are unexplainable in conventional harmonic theory.

To explain the foregoing simple succession of three-tone chords, which is certainly not at all startling today, the academic theory of harmony has to employ the strangest devices. It may call them appoggiatura-chords or suspension-chords. But here it forgets that an essential part of the appoggiatura or suspension effect is resolution. As long as only the "dissonant" chord is present, and not its resolution, the conditions of an appoggiatura are not fulfilled, and

the chords must be looked upon as independent entities. Or it makes the ridiculous assertion that the chords are incomplete, or that they are substitutes for other chords. But who is to decide in each case what parts of the chords are lacking, or for what other chords these are substituting?

As to point 2:

Simple three- and four-tone chords can be rearranged so that their inversions are recognizable as other forms of an original position. That is no longer easy even with ninth chords, and the conventional theory of harmony, in order not to have to burst the bonds of its own rules, chops off parts of these chords in order to fit them into its bed of Procrustes—the inversion system. But the majority of chords, especially those not built up exclusively in thirds, cannot be inverted:

The foregoing formations would lose their character and their sense if their members were rearranged. And we cannot even invert them according to the rules of harmony, since we do not know to what root-tone they are to be related.

As to point 3:

The tonal relations to a basic tone are not exhausted in the tones that belong to the scale of a key. In order to be able to include chords containing tones foreign to the scale of a key without abandoning that key, resort was had to the concept of alteration. Originally conceived to justify a few very common departures from the simplest tonality (such as the lowered sixth degree, and the Neapolitan sixth chord), this idea was expanded to shelter everything else that did not easily fit into the tonal structure, and the result was that such uncertainty and ambiguity were introduced into the system that the only rule that remained valid was: "Any chord can occur in any key." That is the end of the diatonic system; we are now on chromatic ground. In the diatonic system, however, the newly added chords are looked on as subordinate harmonies, almost
as unwelcome intruders, whereas in the chromatic system they are considered from the first as independent members of the tonal system.

As to point 4:

If so definite a phenomenon as the dominant seventh chord (taken as a single example of the ambiguity which every chord possesses in harmonic theory) may be interpreted according to function and notation as being either in fundamental or in six-five or in four-three position,

![Chord Diagram](image)

the system is wrong. Of course it would be foolish to say that this chord has the same harmonic significance in all three forms, simply because it sounds the same. Similarly A takes on a different function in the domain of C than in that of F; and what we concede to the individual tone we cannot deny to the chord. Thus we see that in the first of the three resolutions the $g^1$ is related to the $c^1$—is in fact its closest relative, as Series 1 shows us. In the second progression it is less closely related to the following root, of which it is the minor second. Naturally, this more distant relationship cannot produce so strong a harmonic step as occurred in the first instance. The third progression, in accordance with the relation of the roots of the two chords, which is what governs, is between the other two in strength: the $g^1$ ($f^1$) is the major third of the following $d^4$ ($e^b$). From this example we see that the susceptibility of chords to various interpretations is not rooted in sound at all, but springs from the conflict between the acoustic phenomenon and its notation. On the keyboard there is no such ambiguity. Whether a triad is written $b^6$-$b^6$-$a^6$ or $b^7$-$d^x$-$f^x$, it is always played on the keys $c$-$e$-$g$, and always sounds so. If we had a tempered notation, there would be only perfect, major, and minor intervals. Augmented and diminished and still more extreme categories would disappear, except in the case of the tritone, which would be the only interval to retain the ambiguity that is indicated by the terms diminished and augmented, and could never be expressed in terms of the normal measures of other intervals. If it is possible to regulate sound to the point where the fine interval-gradations disappear between the keys of a tempered keyboard instrument, it should be simpler still to introduce a solution along similar lines into so purely external a medium as notation. Whether this will ever come about, and to what extent it would be possible to reform notation so that it not only would have one symbol for each of the twelve tones, but also would include all the other improvements that are urgently required, we need not consider here. So long as we continue to use the double notation in sharps and flats, we must of course insist on the most logical and consistent notation of musical phenomena, just as in reproducing the spoken language in writing or in print we stick to the traditional spellings, and continue to use the spelling “sh”, for example, while other systems of writing, such as the Cyrillic, or the phonetic, employ much simpler and clearer symbols.

Our somewhat complicated system of musical notation has the advantage of giving the singer or the player (especially of untempered instruments) in most cases a clear impression of the melodic or harmonic intentions of the composer. For analysis of the sound itself, on the other hand, it is not only worthless but actually a hindrance. For in such analysis our thesis must be that all intervals and chords are perceived, independently of their notation, as the ear first hears them, without reference to what has gone before or what comes after. The ear does not hesitate, in the course of this perception, between making all the necessary calculations of minute interval-differences, on the one hand, and, on the other, applying to each chord or interval the measurements derived from the simplest proportions of the overtone series. It always adopts the latter course, and hears every interval, even such as do not actually fit, as being of about the size of one of the intervals that we know from our two series. An interval whose tones stand only roughly in the proportions $5:6$ is always heard by the ear as a minor third, whether it is written and intended by the composer as an augmented second, a minor third, or a doubly diminished fourth. Aural
analysis thus takes account of no diminished or augmented intervals except the tritone; it hears all other intervals as forms of the intervals derived from the first six tones of the overtone series.

Now this thesis will seem to many musicians an aberration based on crass materialism. But when they examine their objections closely, they will find that the only support for these objections is in notation—and notation, as we have said, is not to be touched. Apart from their love of correct notation, however, they are by no means so fastidious, for they mostly do not hesitate to use the piano in their teaching of harmony, and that instrument takes no heed of their desire for functional accuracy. They should find food for thought also in the fact that in listening to music on untempered instruments (choruses, string quartets, orchestras, etc.) they would always be faced with the question of just which of the various interval-sizes to apply in each case, unless they knew the notation or their ears were kind enough to take care of the question independently. Even the hypersensitive ear goes through this same process of normalization in the perception of intervals, and it is well that this is so, for to an ear that analyzed every harmonic phenomenon with complete accuracy we should not be able to offer any usable tonal system. We should stand helpless before an incomprehensible world of tone. Thus we may recognize in our ability to accept complex intervals as versions of their nearest simple equivalents a friendly gift of nature that makes life bearable for the musical ear, as does for the spirit the ability to forget, and for the body the capacity for encouraging oneself to pain.

2. We must substitute a more all-embracing principle for that of the invertibility of chords.
3. We must abandon the thesis that chords are susceptible of a variety of interpretations.

Although all the chords that may be used in music must be covered by our new system in a clear and easily understandable order, it will not completely upset the theses of accepted harmonic theory. Despite the required basic changes, we shall make no such alterations within the relatively small domain covered by the chord analysis of the familiar theory of harmony that a stranger, wandering into our new structure, would be entirely lost. The ground-plan of the old building remains; it has simply been incorporated into a much larger one. The new structure must thus be regarded as a great and timely extension.

As to point 1:

We define a chord as a group of at least three different tones sounding simultaneously. Two tones do not form a chord, no matter how often they are doubled in any number of octaves; they form only an interval. The principle which is to replace that of the superposition of thirds as the basis for chord erection we derive from Series 2, and from the root effect connected with one tone of every interval. This principle will be clearest without a great deal of explanation and description if we approach it indirectly: We shall examine the nature of various kinds of chords, in order to deduce from them the means of synthesis.

At the beginning and end of Series 2, separated from the pairs of intervals, we have the octave and the tritone. The octave has no significance for chord analysis, since all it can do is to increase the weight of one tone of an interval, by doubling, without making any essential change in the content of the interval. The tritone, on the other hand, stamps chords so strongly with its own character that they acquire something of both its indefiniteness and its character of motion towards a goal. There thus arises an essential difference between chords containing a tritone and those without one; and our sense of the stability of chords and intervals thus divides the entire chordal material into two groups: Group A in-

10

Chord Analysis

(See the table at the end of this book)

The requirements of a new system of chord analysis follow from our criticism of the conventional theory of harmony.
1. Construction in thirds must no longer be the basic rule for the erection of chords.

[94]

[95]
includes all chords that have no tritone; Group B includes all chords containing a tritone.

If we appraise the intervals of Series 2 according to the degrees of relationship of Series 1, the five pairs of intervals will divide into two classes: those consisting of the first-generation descendants of the progenitor tone (fifths, fourths, thirds, and sixths) and those formed from the “grandchildren” (seconds and sevenths). This classification enables us to make a subdivision of the chords in Groups A and B. For if we construct chords using the intervals belonging to the first of these classifications only, it follows that such chords, owing to the simplicity and purity of their constituents, must form one division, the chords of which will be simpler and purer than those containing seconds or sevenths, which form the second division. About these we shall have more to say later.

Now as to the third factor that must be taken into consideration in our judgement of chords: the root, and its position in the chord.

Chords consist of intervals, and since in each interval one of the tones is the root and dominates the other, the interval-roots try to bring other tones under their control, and to exert their dominance in the chord as well as in their own intervals. Every chord, then, with a few exceptions to be mentioned later, has a root. To find it, we must find the best interval of the chord, appraised according to the values of Series 2: the fifth is the best, the major seventh the weakest of harmonic intervals (except for the tritone). For our calculation, we must take into account every interval in the chord. A major triad thus consists of a fifth, a major third, and a minor third. Here we see the difference between our method and that of the conventional theory of harmony, which relates the chord factors to the bass tone, a process which makes inversions possible. But at the same time it reckons with the intervals of the uninvolved,

fundamental position of the chord, so that the root remains the same in all inversions, and the other tones of the original position also retain their original functions in the inverted position. This double reckoning is inaccurate; it must be used only comparatively, if it is not to result in misunderstandings. We say, on the other hand: If there is a fifth in the chord, then the lower tone of the fifth is the root of the chord. Similarly, the lower tone of a third or a seventh (in the absence of any better interval) is the root of the chord. Conversely, if a fourth, or a sixth, or a second is the best interval of a chord, then its upper tone is the root of the chord. Doubled tones count only once; we use the lowest one for our reckoning. If the chord contains two or more equal intervals, and these are the best intervals, the root of the lower one is the root of the chord.

It makes no difference whether the tone that completes the best interval lies in the same octave or one or more octaves higher (in fifths, thirds, and sevenths) or, on the other hand, one or more octaves lower (in fourths, sixths, and seconds). In those occasional instances in which the compass of the whole chord is so great that the distance between the two tones of the root-determining interval permits the formation of “dissonant” combination tones such as were mentioned earlier, in the discussion of separate intervals (pp. 72–4), we have the choice either of making new rules or of simply treating the widely extended intervals like their closer prototypes.
Here, too, I hold it unnecessary to set up special rules to govern these few exceptions, instead of applying to the latter the rules that hold in the great majority of cases. We may therefore without hesitation treat extended intervals like those of Fig. 66 as fifths, fourths, etc., and accordingly assume the roots of these chords to be C, c\textsuperscript{3}, G\textsubscript{1}, C, and B\textsubscript{b}. 

\[ \text{\includegraphics{chord_diagram}} \]

In such unusual chord arrangements as

\[ \text{\includegraphics{chord_diagram}} \]

it would usually be better to take melodic influences (such as will be discussed later) into account, rather than to rely exclusively upon harmonic analysis. These chords would then become subordinate to others more easily analyzed, so that either the roots which our rules would lead us to deduce (b\textsuperscript{2} and f\textsuperscript{2}, for the chords of Fig. 67) would be confirmed by their context, or else the more effective roots of the predominant chords would make the analysis of the formations here notated unnecessary.

As to point 2:

In the conventional theory of harmony, the inversion of a chord never has the same strong and definite effect as the chord had in its original position. For in the fundamental position the root and bass tone are the same; the root, already the strongest tone of the chord, is further strengthened by its position at the bottom of the chord. In inversions the two forces are separated; the root is now in an upper part of the chord, and opposing its strength to that of the bass. Strictly speaking, it is not the rearrangement of the tones of a chord that constitutes an inversion, but the transposition of its root into an upper part. Hitherto the fact that one chord (an inversion) had to be related to another chord, of different structure (the original position, in which the root and the bass tone were the same), has prevented a comprehensive use of the principle of root-transposition. By freeing this principle from its fetters, we gain not only a wider view over the domain of numerous chords not hitherto covered by harmonic theory, but also a new criterion for the appraisal of chords. All chords in which the bass tone and the root are not identical are subordinate to chords whose other characteristics (root and chord-group qualities) they share, but in which the root and bass tone do coincide. Here, too, we do not care whether the interval which determines the root lies in the closest position in the chord, or whether its tones are spread out: over one or more octaves. There is, it is true, a difference in the sound and in the value of chords in which the root is emphasized by the close proximity of the tones of the intervals which determine it, as compared with those in which the root is weakened by being widely separated from its partner. But if we were to take account of these subtlest differences we should not be able to erect any practical system, since each tonal combination would have to have its own individual niche. The division here proposed may, however, despite the sacrifice of excessive subdivisions of sufficient accuracy, be accepted as a basis for the complete understanding of all chords, as will be seen when the picture is complete.

There is a kind of rearrangement of chord-tones which is not to be regarded as inversion, since it does not affect the root tone—which remains stationary—but simply transposes its complementary tone (perhaps with other chord tones) into a different octave. This is a change of what the accepted theory of harmony calls position ("close", "open", "mixed", "position of the octave", "fifth", "third", etc.). The ranking of the chords in the following example is the same whether they appear in form A or form B,
since it follows from what has been said that the increased distance between the root and its complement, despite the slight change in effect which results from it, does not affect the values we assign to the chords. Chords which, owing to their simple structure, possess only a mild tension are not greatly altered by such changes. Chords containing many tones, on the other hand, lose their particular character when they undergo such changes of position. Where the boundary lies between these two types of chords can be decided only in each individual instance.

As to point 3:
The division of chords into two main groups (A and B), the members of which are then further ranked according to their component intervals and the position of their roots, does away with all ambiguity. It does not, of course, abolish the harmonic uncertainty of the tritone. But anyone who considers this a failing should balance the uncertainty of a few chords against the inaccuracy of a system in which any chord may have a different meaning from that which the ear assigns to it.

As a matter of experience it is established that the tritone, when combined with other intervals to form a chord, subordinates itself to the best interval of Series 2. The intervals of the first two pairs (fifth and fourth, major third and minor sixth) do away with its uncertainty, but yield readily to its tendency towards a resolution. Thus it happens that in tritone chords containing these intervals the root is just as strong as it is in the chords of Group A, but stability is nevertheless lacking.

The intervals of the next pair (minor third and major sixth) have less strength to combat the uncertainty of the tritone and thus to make of it a clear harmonic combination:

Thus a chord which apart from the tritone possesses only a minor third or a major sixth remains as ambiguous as the tritone itself. Just as in the tritone itself, one of the tones of such a chord will be called the root representative. The contextual chord-succession determines which of the tones performs this function. There are only four such chords: the diminished triad with its two inversions and the diminished seventh chord.

Among the chords which have no tritone, also, there are two of which the interpretation depends on the context:

and which in consequence have no root, but only a root representative: the augmented triad and the chord composed of two superposed fourths.

Subdivision of the Chord-Groups

Within each of the two main Groups, A (without tritone) and B (with tritone), three subdivisions may be made, according to the principles already discussed. We shall label them with Roman numerals, so that Group A contains the sub-groups I, III, and V, while Group B contains II, IV, and VI.

Sub-group I of Group A contains chords having no seconds or sevenths, and in its first section (I₁) only those in which the root and the bass tone coincide—in which, that is, the best interval is
based on the bottom tone. There are but two chords that fulfill these conditions: the major and minor triads. These noblest of all chords constitute a section in themselves. They alone are completely independent, capable of being used for conclusions, and of being connected with any other chords. The chords of the next section (I₂) stand a little lower in the scale of values. These are the chords in which the root is not the lowest tone: the inversions of the major and minor triads. On account of the high position of the root, they are not independent enough to form satisfying conclusions; but otherwise they perform in somewhat weaker fashion the same functions as those of the preceding section. All the chords of these two sections are at most three-voiced; any additional tones can only be doublings of tones already present. These chords exhaust the possibilities of combining the intervals consisting of the tones most closely related to the progenitor tone (the "sons") in Series I.

The corresponding sub-group of Group B (II.), contains the chords of three or more voices in which the tritone is subordinate to stronger intervals. The requirement that the chords must contain no seconds or sevenths cannot be maintained here, for the presence of the tritone always (except in the diminished triad and its inversions) involves seconds or sevenths. Yet in this sub-group we shall limit ourselves to major seconds and minor sevenths, as the less sharp representatives of their species. The mildest form of the intensification brought about by the presence of the tritone is the minor seventh, in a chord from which the major second, as the stronger and sharper of the two intervals, still remains excluded. In this section (IIa) we shall thus find only the two most important chords with tritone: the complete dominant seventh, and the same chord without its fifth. The chords in which the major second as well as the minor seventh may appear fall into three sections. The first (IIb₁) includes those chords in which the root and the bass tone are the same: the strong dominant chords which are the nest simplest after the dominant seventh, and which in their structure lean heavily upon the chords of their neighboring section (I.), the triads. The second section (IIb₂) contains the chords in which the root is not at the bottom: the inversions of simple dominant chords and similar structures. Common to all the chords of sub-group II thus far named is the fact that they contain only one tritone. The chords of the third section (IIb₃) are similar in every respect but this: they contain two tritones. These chords are not included in the foregoing sections because their sound is so strongly colored by the tritones; yet they are not so intensified as to require assignment to sub-group IV.

Sub-group III of Group A contains chords of any number of tones which are extended by the addition of seconds or sevenths. These are a rough and unpolished race. The best of them are those with three or four tones, which either contain one of the chords of sub-group I, or at least in some of their tones approach this unattainable prototype very closely. And the chords that lack minor seconds and major sevenths (i.e., those limiting themselves to major seconds and minor sevenths) are of a higher class than the very sharp and grating ones which contain these intervals. None of the chords of sub-group III are independent; all of them depend very much on the course of melody; and they cannot be connected with all other chords. They include the secondary seventh-chords with their inversions. The first section again contains only those in which the root is in the bass; the second section contains those in which the root is in a higher part.

Sub-group IV contains a strange set of piquant, coarse, and highly colored chords. All the chords that serve the most intensified expression, that make a noise, that irritate, stir the emotions, excite strong aversion—all are at home here. The chords of this group can have any number of tritones, and the number of minor seconds and major sevenths is likewise unlimited. It would be unreasonable to expect chords of such strongly marked individuality to lend themselves without resistance to all chord-successions, as do triads and the simpler tritone chords. They are often very intractable, especially when they are used in progressions involving chords from various and rapidly changing sub-groups. The best of them are the easiest to handle—those that consist of only a few tones and that resemble chords of the simpler sub-groups.

Sub-groups V and VI are small. They contain the above men-
tioned uncertain chords—chords consisting of several superposed intervals of the same Size. The first chord of sub-group V consists of two major thirds and an augmented fifth. The augmented fifth can be counted a minor sixth, in accordance with what we have already established, and thus the constituents of the chord belong to the same pair of intervals, and the root cannot be definitely determined. The chord built up in fourths, which belongs to group V, may occur in forms in which its root can be determined. It is uncertain only in its closest position (see the table), or when its highest tone is doubled above or its lowest below. Any other doublings produce a fifth as the best interval of the chord, which would place it in sub-group III. The same is true even when it consists of only three tones, expanded or contracted by the octave transposition of one of them. If we add further fourths above, we had better assume the presence of a root, for the choice of possible root representatives becomes too great. Accordingly we shall treat all chords consisting of three or more superposed fourths as having the root of their lowest fourth as the chord-root. Two superposed fifths do not belong to group V but to group III; likewise two superposed major or minor seconds. Chords consisting of two or more superposed minor thirds constitute group VI.

For handling the chords of Group B (those with tritone), it is not enough to know their roots. If we are to be able to make convincing chord-progressions, we must treat the tritone as their most important constituent. We find the root by the familiar method. But, in addition, one of the members of the tritone must serve as the guide-tone. To find the guide-tone, the following rules apply:

1. That tone belonging to one or more tritones in the chord which stands in the best relationship to the root (measured by the interval-values of Series 2) is to be considered the guide-tone:

   ![Diagram](image)

   In doubtful cases—as for example, when a choice must be made between two tones which lie above and below the root and are equally related to it—let that tone be taken as guide-tone which, itself a part of a tritone, leads best to the root of the next chord (if that chord has no tritone) or to the guide-tone of the next chord (if it contains a tritone).

2. When there is only one tritone in the chord, and the root forms a part of this tritone, the other tone forming the tritone is to be considered the guide-tone.

   ![Diagram](image)

When isolated intervals appear between one chord and the next, they are to be regarded as belonging to that group to which their own nature would assign them. The fifth and the thirds belong to I, the fourth and the sixths to I₂, the seconds to III₂, the sevenths to III, the tritone to VI.

This system of appraising chords and intervals results in a classification of all chords. There is no combination of intervals which does not fit into some division of our system. Chords which a theorist would analyze only in his nightmares, and which any self-respecting counterpoint book would not tolerate, can now be easily explained.

The system is as comprehensive as it can be, in view of the possible variety of chords. Nevertheless, even by this system, there will always remain a certain number of exceptional chords which cannot be interpreted with complete satisfaction. These include those which consist of so many different tones that the individual units of which the structure is composed hardly count, as well as those which, although they contain only a few tones, are so spread out that their constituent tones can only with difficulty be perceived as constituting harmonic intervals. But it is a question whether a system of investigation which aims to make clear the harmonic side of tonal combinations should be applied to chords which, like the first-named, are effective chiefly through their intensity, their mass, or their energy, or, like the second group, result simply from the isolated effect of single tones or lines. An investi-
The Value of Chords

The complete material of the conventional theory of harmony is contained in our sub-groups I, II, and VI, except for an isolated chord here and there belonging to groups III and IV. And, of course, all possible chords may occur under conventional harmonic theory, but it accepts them only as structures resulting from strong melodic tendencies, and it interprets all tones that complicate chords beyond the familiar limits of triads and seventh chords as passing tones, suspensions, appoggiature, etc. When one of the chords belonging to our groups III or IV shows an urge for independent existence, and cannot be explained as consisting of appoggiature or suspensions or passing tones, it is considered simply as non-existent. There is no room in a well-ordered household for such rabbles; they had better be chased from the door, before one is tempted to examine them more closely.

In another way, too, the familiar theory of harmony prevents chords from the free unfolding of their vital urge. For it proclaims as the highest harmonic law the relationship of tones and chords in a key. The diatonic scale with its limited possibilities determines the position and rank of the chords, which are the mere satellites of this power. The chord must blindly subordinate itself, and attention be paid to its individual character only as the key allows. This theory of harmony is like an employer who keeps a small number of gifted and versatile artisans at work. They are tied to him for better or for worse, and he has always kept them so dependent upon him that they are no longer capable of making their own decisions. Therefore he places a supervisor over them, who plans and thinks for them. The work of such men will never rise above a certain quality, since the directing personality is not omniscient and not equally well prepared at all times and for every contingency, since work which is not free cannot develop beyond a certain point, and finally since although the versatile and highly trained workmen are masters of many trades, the very quantity of their skills prevents them from the development of new and rationalized methods and the acquisition of specialized knowledge.

Our enterprise works along different lines. It has a far greater number of workmen at its disposal, and their work varies greatly in its value to the whole. From the special craftsman with the highest ability to the know-nothing, and from the most industrious worker to the drone, we can call upon people of all grades of ability. We can therefore put at each place a workman whose abilities correspond to the functions he must perform, and who will accordingly perform them faster, better, and more reliably than a man impeded by his very versatility. On the other hand, it is extravagant to waste the abilities of superior workmen at jobs which, though indispensable, can be performed by know-nothings and drones who could do nothing else. Our work thus becomes a competition among the strongest forces, which accomplishes more than the laborious execution of the arbitrary plan of any supervisor. But in order that our effort shall not flow off into all sorts of side-channels, there is a group of workmen of higher rank who produce the individual parts devised by specialists, and bring them together according to a plan worked out by a director of operations in accordance with the needs of the market and the capacity of his plant. We have, accordingly, the most thoroughgoing specialization in the lowest levels of our production process, directed from above, firmly, but with an intensity that springs from a clear consciousness of purpose and the ability to fulfill it.

Let us translate this into musical terms. It means that the key and its body of chords is not the natural basis of tonal activity. What Nature provides is the intervals. The juxtaposition of intervals, or of chords, which are the extensions of intervals, gives rise to the key. We are no longer the prisoners of the key. Rather, we now have a free hand to give the tonal relations whatever aspect we deem fitting. The different harmonic tensions which we need for this purpose are indicated by the ranking of the interval-values. If the intervals are of different values, then the chords constructed
of them must differ correspondingly. A closer investigation of our chord table will confirm this statement.

From the considerations which impelled us in setting up our table it follows as a matter of course that the chords having no tritone (Group A) are of higher rank than those in which the tritone occurs (Group B). This general rule is however modified by the division of the chords into sub-groups with the result that sub-group II contains chords of higher value than those of subgroup III, even though the latter belong to the higher Group A, and similarly sub-group IV is higher than V. The tonal value of the sub-groups thus diminishes from I, containing the purest combinations, through the simple tritone chords of II, down to the uncertain chords of the lowest group, VI. Within the individual groups I–IV there are finer degrees of distinction, so that the higher numbered sub-sections come after the lower in the scale of values. Thus IIa is of higher rank than IIb₁, which in turn outranks IIb₂ and IIb₃; but we must not forget that IIb₂ is higher than III₁. Thus in chord-successions a move from lower-numbered to higher-numbered chords is a step down; or, to put it in general terms, based on the visual appearance of our table: Every step downwards or to the right means a decrease in tonal value, and every step upwards or to the left an increase. (Chord-progressions toward a member of groups V or VI may represent an exception to this law; this possibility will be discussed later.) We have already stated that further distinctions are possible among the chords of one and the same group, but that the systematization of such differences would lead to an atomic view of our chordal material.

We may state the result of our investigations into the qualities of simultaneous tone-combinations as follows: In contrast to the conventional theory of harmony, in which all tones and tone-combinations are ranked according to their relation to an a priori tonal scheme, and thus have only relative values, our system attributes a fixed value to each. Between those of highest and those of lowest value we recognize a great number of gradations, each of which has a constant value. How tonal groupings result from the conjunction and contrast of these values is the subject of the next chapter.

CHAPTER IV

Harmony

1

Movement in Chord-Successions

We have seen that the individual tone is useful for musical purposes only as a result of the interval between it and another tone. Similarly, a chord, which is an aggregation of tones, has musical significance only when the appearance of another harmonic aggregation creates a space between them. The spanning of this gap—chord-connection—is the beginning of all harmonic reality. Thus in the realm of chords the procedure is the same, on a higher level, as that through which the simplest of all tonal building units acquired its significance: the creation of tension by the juxtaposition of two entities.

Three forces are at work in chord-connection: rhythmic, melodic, and harmonic. Each of them works in two directions. Rhythm determines the duration of the chords, and groups them by division into stressed and unstressed members of the structure. Melody in voice-leading regulates linear expansion, and in the two-voice framework sets the pitch limits. In the placing of the harmonic center of gravity and in the regulation of relationships we see harmonic energy at work.

We can leave the function of rhythm out of consideration here, but not because it is unimportant. Without rhythm—that is, without relationships in time—neither of the other forces could operate. It is rather the all-pervading force of the primeval element of rhythm that allows us to take it for granted, and to simplify our
work by ignoring it. Furthermore, all questions of rhythm, as well as the formal characteristics of composition which spring from it, are still so largely unexplained that it seems impossible at the present time to include rhythm as an integral part of a system of teaching the craft of composition. In this work, which does not aim at the complete scientific explanation of the deepest impulses that underlie musical writing, but rather seeks to be of practical use, we shall limit ourselves to the examination of the other forces. Whereas rhythm can find expression without relation to the tones and chords—in fact, without relation to sound itself—these other elements, in order to make themselves felt, must behave according to the laws which we have come to know in Series 1 and Series 2. They thus form a coherent group, different from and counter-balancing the rhythmic element. We are limiting our field of investigation by excluding from it on the one hand the primary musical elements—pitch, intensity, and timbre—and rhythm, and on the other hand the forces of dynamics, phrasing, and agogics, which of course affect chord-connections but do not change them.

The special characteristics of melodic structures will be discussed in Chapter V. Still, we must treat the subject of voice-leading at this point in our discussion at least sufficiently to explain its workings in harmonic connections. In one type of musical writing, the form and content of the piece is determined by melodic force: this is contrapuntal writing. When, on the other hand, the chords and their connections are what give the piece its character, melody is of less significance.

Between the extreme of linear writing and a texture so thoroughly chordal in purpose there are innumerable species of the mutual interpenetration of melodic and harmonic forces. What determines our assigning of a composition to one or the other domain is not so much the external appearance as the basic attitude of the piece. In contrapuntal writing the composer takes the idea of movement as his point of departure, and the chords come about as the product of the play of line (although this product itself must have its own logic)—they form the adhesive which binds these lines together. In chordal writing, the composer sets out with the opposite idea: he breaks up the inert chord masses and breathes life into them by dividing their components among moving voices. The binding together of the flowing lines may be so firm, and the breaking up of the chord masses may set so much movement free, that a music in which the most alive linear movement is combined with the most logical harmonic progression can hardly be classed as primarily one thing or the other. In an idiom employing really independent voices, every tone of one chord moves to a tone of the next, while in a more compact idiom (such as in keyboard writing) no such melodic progression of the individual tones is sought, and it is rather chords that are juxtaposed, in whole or in part. Thus harmonic masses are set into flowing motion by melodic means of progression, on the one hand by the aggregation of individual movements of tones, and on the other by the shifting of entire chord-structures (see also Chapter III, Section 8).

In well balanced progressions involving really independent voices, all melodic steps are of great (though not of equally great) importance. It is not true, as many a theorizing aesthete would have us believe, that the voices are absolutely free to move as they choose, and that the chordal aspect may then be left to assume whatever shape it will. To leave the harmonic dimension, which after rhythm and melody is the most important element of music, to chance—would this not be like planning only the horizontal parts of a building? Whether the architect is building a tower or a shed, he cannot escape vertical as well as horizontal elements. He can emphasize one element, and thus subordinate the other, but he cannot wholly exclude either one. In music, too, no matter how much attention is focused on the melodic lines, the harmonic aspect cannot be ignored. If it has no logical relation to the linear texture, and if it is not in itself logically developed, the music is unpalatable.

There is, of course, another linear style of writing from which, despite the convincing nature of the harmonic progression, we derive no satisfaction. Here independence has been pushed too far: every voice has such a strong and independent life of its own that the net result is a maze of activity that is difficult to understand;
rhythm and harmony can no longer impose unity on the various melodic personalities which insist on going their own way. Just as one cannot handle six different kinds of things at the same time, so one cannot follow a large number of independent voices; one's attention is torn hither and thither, and it takes account at each instant only of the most prominent point from among the tangled lines, subordinating the rest. Even two very independent and conflicting lines are hard to follow, unless they are bound together by a fairly simple harmonic foundation. When there are three voices, none of them is completely free in its spatial (melodic) aspect; therefore in skilful three-part writing one of the three voices is always subordinated to the other two. The significance of the voices, their prominence or lack of it, may change within time units less than one beat long. In contrapuntal writing for more than three voices, the relative prominence given to the voices needs even more care; here the sustaining of individual tones, the parallel coupling of voices, and the emphasis of the harmonic content bring about numerous gradations, of which the fugues of Bach furnish the most perfect examples.

Contrapuntal writing does not satisfy all demands. It is found lacking particularly by those players and bearers who seek in music more sweetness and gracefulness, as well as a greater display of force and a more immediate effect than the cool strength and logic of independent lines can produce. Writing which is primarily chordal and homophonic sacrifices the multiplicity of lines, and can accordingly allow to its small linear ingredient, which rests upon the chords as the wave does upon the waters, a freer and more sensuous development. What is important here is the exact fitting of the main harmonic components, and the strength of these easily carries the weight of the less important element; in fact, it carries much incidental and often even much, superfluous material as well. The play of line may become so weakened that it can hardly be observed at all; music in which this happens runs the risk of utter futility, or of the same incomprehensibility that attends counterpoint pushed to dogmatic lengths. Incomprehensibility is the result also of too rapid changes of chord, of continual use of the sharpest combinations (those belonging to sub-groups III and IV), and of inaccurate or obscure harmonic progressions. The danger of becoming unintelligible is greater in contrapuntal writing; the chordal style is more liable to sink into shallow insignificance.

2

The Two-Voice Framework

In both the contrapuntal and the chordal idioms, harmonic development, which is tied to melodic movement, takes place within an external spatial frame—a scaffolding which gives the chords the necessary contour. This framework is constructed by the bass voice and the most important of the upper voices. The bass line may be clearly melodic in character, as in contrapuntal writing, or it may hardly rise above connecting the main points of harmonic support, but it will always have, as the lowest voice and the foundation of the structure, decisive importance for the development of the harmony. The next most important line may in contrapuntal writing be entrusted to any one of the upper voices; or, since the voices perform functions of constantly varying importance, it may move about from voice to voice. In the chordal idiom, it will always be found in the "theme" or the "melody", or whatever one wishes to call the linear formation that floats above the chords. Usually this is the highest stratum in the tonal composite; less often it is embedded in the middle of the chords. If the melodic aspect is so obscured by the rhythm or the harmony that one can hardly speak of any thematic life, then the upper voice that results from the chord-successions takes its place. If the bass voice holds an organ point, then the next higher moving voice constitutes the lower half of the framework, although care must be taken to see that the organ point really leaves the harmony free to develop, and does not have a disturbing significance of its own. For when the organ point becomes a constituent of the harmony (even though only in passing), it loses its organ-point function and is reckoned as part of the harmony. Similarly, when the upper voice holds a tone for some time, the next lower voice becomes the upper member of the framework.
If writing in several voices is to sound clear and intelligible, the contours of its two-voice framework must be cleanly designed and cogently organized. The bass voice must make with the upper voice, irrespective of the difference in register between them, a good, intelligible piece of two-part writing, without the necessity of anything's being added. The two voices must not get in each other's way, as can easily happen if each is made to bear too much melodic weight; rather must they be contrasted and balanced one against the other, in their shape and in their time-values. This two-part framework is no mere scaffolding to assist the composer in his work; it is a living member of the body of the musical work. Thus it must not be limited to the form of a meaningless two-part counterpoint exercise, note against note. Yet it must also not assume such importance as to reduce the other elements of the piece to insignificance, for despite its importance it is only one part of the tonal structure.

The intervals formed by the two voices must be carefully planned. Thirds and sixths are pleasant intervals, but to construct a two-part texture mainly of them would be to bore the listener with continual sweetness. Seconds and sevenths add strength and tension to two-part writing; yet their continuous use would dull the ear and make it insensible to the subtler charms of the more satisfactory intervals. Thus a combination of euphony and sharpness of sound must be found, appropriate to the nature and purpose of the composition. Tensions and relaxations must alternate. But there is no place here in the Theoretical Part of this work for specific rules governing two-part writing; such rules will be found in the later volumes.

The progression of the two-voice framework is wholly independent of the other tones of the chords. To be sure, these other tones belong to the whole tonal picture just as much as the outlines themselves, but they have no more influence on the latter than the form of the spleen or the liver has on the external appearance of a man. Already in examining the nature of the individual chord we saw that the tones that filled it up, and their close or extended position, did not have the same significance as the position of the chord in space determined by the position of its root. In chord-progressions, then, since they are composed of individual chords, these filling-in tones are also of secondary importance. The path of a chord in space is affected only very little by the inner tones; its shape is determined rather by melodic progression, principally in the form of the two-voice framework. It is in the harmonic relations of the chords, and especially in the shifting of the center of harmonic gravity, that the inner voices play their full rôle.

In the polyphony of all ages and styles this two-voice framework will be found, tracing the spatial boundaries of the harmony. As the contour of the chords it acts as a constant reminder not to allow spots of harmonic color to become so gorgeous as to obscure the drawing itself; it is at the same time, however, like a rudimentary organ that remains in the human body as a heritage from some evolutionary ancestor, an honorable legacy from the dawn of polyphony, a relic of which we can no more free ourselves than we can of such atavistic parts of the body.

3

Harmonic Fluctuation

A solid object—a brick, for example—can be pushed or pulled in such a way that its under side remains in uninterrupted contact with the surface on which it rests. But it can also be turned over on its side, or even moved violently enough to turn on one of its corners. In this case we have added to change of position a turning of the object on its own axis, resulting from a shift in its center of gravity. The brick then touches the ground with first one surface and then another. The first of these types of movement corresponds to the harmonic progression in which the relations of all the roots to their respective chords is governed by the same principle—as when none but chords belonging to a single sub-group (I, or I₂, III₃, etc.) are used. The second type corresponds to the shift of harmonic gravity; for this we shall henceforth use the term harmonic fluctuation.

If we look at the sub-groups of our chord-table—say, at group...
value: our brick is placed together with other identical bricks, and receives its particular significance from its relative position in the building. In the connection of chords of identical structure there is no harmonic fluctuation; there are only harmonic relations which vary and together with the rhythmic pulse regulate the tonal movement and build forms out of it.

The foregoing may be made clearer through the following examples.* In the first example we see six chords all belonging to Group A:

\[
\begin{array}{c}
\text{Degree of Tension:} \\
1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 1
\end{array}
\]

The first and last of these chords, belonging to section I., are the best and most satisfying, while between them a harmonic development takes place which passes through two chords of I., one of III., and one of I., again. Thus there is an increase of tension from the first chord to the fourth, which is then resolved. The harmonic fluctuation is here not very sudden. The greatest gap is between I. and III., and this gap is smoothly bridged by the I. chords. There is a harmonic crescendo and diminuendo that is indissolubly connected with the nature of this progression; it cannot be altered by the performer. It is thus different from purely dynamic increases and decreases, the control of which always rests with the singer or player.

The fluctuation in the next example is less smooth:

\[
\begin{array}{c}
\text{Degree of Tension:} \\
1 \quad 3 \quad 2 \quad 4 \quad 5 \quad 1
\end{array}
\]

* The drawings are intended to give an approximate picture of the changing tension in the musical examples. Beneath the chords of I., which are altogether lacking in tension, the upper and lower lines of the diagram come together, while the point of greatest tension corresponds to the widest separation of these lines.
Between the first chord and the second, the gradient, so to speak, is very steep. The highly tense chords of group III give the rest of this progression its character, while the fluctuation among them is not very wide, rising and falling between III₁ and III₂. Of all these chords, the fourth is the sharpest, its minor second (or ninth) e⁻⁴ exceeding in this respect the major seconds and minor sevenths of its neighbors. Between the next to the last chord and the last one there is again a considerable change of level, which gives the effect of a considerable relaxation of tension. The next two examples contam chords from Group B (with tritone), the first example restricting itself to the harmless ones of group II, while the second one employs the more biting combinations of group IV. The nature of the harmonic fluctuation will doubtless be clear to the reader with the help of the numbering and the diagrams.

For anyone with some knowledge of the devices of the technique of composition, consideration of the harmonic fluctuation introduces no new difficulty, particularly since the theory of harmony takes account of it, even though only to a slight degree: the division of chords into those in root position and those in inversion is nothing else. The theory of harmony does not provide, of course, any key to the construction of complicated designs of fluctuating tension; for this we must employ our detailed knowledge of chord-values. Anyone who possesses such knowledge can create harmonic structures of the most daring thrust and tension without having to rely on the uncertain method of trying each individual combination out by ear—a process which soon becomes more a guessing game in pursuit of concealed possibilities than a form of creative work. We thus add to the time-honored practices of harmony—voice-leading, and the production of tonal relations—the observation of the rise and fall of harmonic tension as an exact and completely reliable procedure. Since this procedure adds to the accuracy of our planning and execution of harmonic structures, the composer may unhesitatingly accept the added work which this increased material brings with it. The secret of good arrangement of this rise and fall is completely open to him in our table of the chord-values.

The only rule we must observe which involves consideration of individual cases is to remember that the indeterminate chords of groups V and VI introduce an element of uncertainty into harmonic developments. When one is dealing with chords belonging to the other sub-groups, one stands on firm ground; but the introduction of the indeterminate chords is like a step into mud and quicksand. Usually a progression to such a chord represents a decrease in value and accordingly an increase in tension, as our chord-table shows. But sometimes the effect of the use of chords of groups V and VI is such that the whole progression takes on a wavering unsteadiness. The ear still perceives changes of value and tension, but it cannot determine the exact degrees of these changes. In such progressions, therefore, care must be exercised. A single step into uncertainty may be very pleasant, for variety's sake, but a progression consisting exclusively of uncertain chords is always of poor effect. In such a progression we lose all sense of direction; we seem to be tossed aimlessly hither and yon on an endless series of waves until the ear becomes confused to the point of actual physical discomfort. As a counterpoise to the stable and tensionless chords of group I, a chord from group V or VI may be useful; it can almost always be successfully juxtaposed even against chords from group II. But in using it with chords from groups III and IV, care must be exercised. In the midst of such chords, a chord of group V or VI often puts us completely off the track; it seems to cause the whole chord-structure to collapse. Probably the vagueness which results from the complex, over-sharp profiles of the chords of groups III and IV is unsatisfactory when
associated with the opposite type of vagueness, which results from the washed-out ambiguity of those of groups V and VI. Progressions of this type must accordingly be handled with extreme care, if they are not to be entirely inconsistent with the rest of the harmonic development.

I can imagine that to a reader who is thoroughly entangled in earlier ways of harmonic thinking the measurement of the harmonic fluctuation will seem an unnecessary addition to the composer's task. And not improbably even some readers who have accepted all the innovations and extensions suggested so far will look on this one as mere hair-splitting, instead of as a logical part of our system. Perhaps both types of readers will be convinced of its necessity by examining the following progression:

\[ \text{Diagram of a harmonic progression} \]

The theory of harmony has no place for any of these chords except the first and last. It admits only that the example is in a poor sort of C major. But even our reckoning of the roots tells us nothing more than that all the chords have the same root, c\textsuperscript{1}. Since, however, they are, despite that fact, quite different from one another, we must expand our analysis by the addition of another criterion. The mere movement of the tones tells us only that against a constant center, consisting of two voices sustaining their tones, two lines are set in contrary motion (the upper one in parallel fourths), reaching their widest compass in the fourth chord. In the two-voice framework we see a unison expanding through a major third and a minor sixth until it too reaches its widest span, a minor seventh, in the fourth chord: and then, after the minor third, the indeterminate tritone before the end introducing an element of uncertainty into the progression. Only in the harmonic fluctuation have we an explanation for the varying harmonic tension of different chords upon the same root. We see there that after a sharp ascent from I, to III\textsubscript{3}, the third chord brings a further slight increase in tension, which is somewhat relaxed in the fourth chord, containing no minor second. The high point of harmonic tension is in the next to the last chord. Thus the harmonic fluctuation has a quite different development from that of the voice-leading or of the two-voice framework. In this simple example we see the interlocking play of the structural elements quite clearly. The more carefully these elements are balanced, the more convincing, the more interesting, and the more attractive a harmonic progression will be. The purpose of the progressions, the intensity with which they are to affect the hearer, must always determine the coinciding or contrasting designs of all the different elements.

4

Movement in Chord-Connection, Expressed in Root-Progresions

In harmonic fluctuation we see how chords compare with one another as tonal masses of particular forms and densities. We know that the chords are placed in reciprocal relations by means of voice-leading. We need no reminder that in these relations even the most incidental tonal constituent cannot be left to chance. The composer must find for every chord and every tone the treatment that will best reconcile his artistic intention with the nature of his material. For the quick appraisal of chord-progressions, without which fluent work is inconceivable, a sort of abbreviated reckoning is employed to indicate the value of a progression and to show its direction (about which the harmonic fluctuation gives us no information). For this purpose, we make use of the roots of the chords, and in our study of this subject we shall restrict ourselves to the simplest relations: those between the roots of two adjacent chords. These two roots, once they have been extracted from their chords, we shall now consider apart from the tonal masses in which they originated, simply as two tones of given pitches without chordal relations. As such, they form an interval which has all the char-
characteristics we observed in Chapter III. Here, as in that chapter, a third is a strongly harmonic interval, while a second is essentially melodic; in short, we stand once again before the whole series of interval-values. This juxtaposition of roots derived from chords is a reliable means of judging the value of a chord-progression, equally useful for the analysis of progressions already in existence and for the construction of new ones.

Let us examine the chord-progressions within Group A, which because of their independence and complete certainty will furnish the clearest illustrations of what has been said. The simplest progressions are those involving only the chords of sub-group I. In the following example we see the roots of two adjacent chords first at the distance of a fifth, then of a fourth, and so on up to the major seventh (minor second).

A progression based on the interval of a fifth between its roots naturally has a surer foundation than one based on a minor sixth: this is the strongest of all chord-progressions. If we keep the values of the chords equal (as we have determined to do here, confining ourselves to the chords of group I), then the next best progression after that based on a fifth is that based on a fourth. Then follow in the familiar order progressions based on root-progressions of a third and a sixth, in which the softness of the root-progression is not only repeated but multiplied in the movement of the tonal masses of the chords. The melodic step of a second is similarly confirmed in the chord-progression based on it. The experience we have had with Series 2 teaches us that a chord-progression based on a root-progression of a tritone will be the least valuable of all.

The nature of root-calculation should already be clear from the few examples given. It serves the same purpose in the realm of tones, roughly speaking, as logarithms do in the realm of numbers: the reckoning is done with small exponents, which represent the quantities sought. The accumulation of melodic steps in the chord-progressions, the product of multiplied melodic tensions, can be reduced to the harmless addition and subtraction of single tones. But apart from the fact that our tonal materials are numerically limited, reckoning with roots has one disadvantage as compared with that with logarithms: chords, which are the sums of the tones of which they are composed, are not definite quantities, for over the same root a great variety of chord structures is possible. Here an investigation of the two-voice framework and the harmonic fluctuation will clear up all ambiguity.

The ever-obtrusive tritone is not satisfied with affecting the individual chord, or with its influence upon chord-roots: it also has a bearing upon the tonal sums that are juxtaposed in chord-progressions. Progressions like the following

and similar ones always retain, despite the smoothest voice-leading, a certain unwieldiness, because the tritone is wedged in between them. That explains why the familiar progression of the tonic and dominant triads is satisfying when the dominant is a major triad, whereas when a minor triad on the dominant is connected with a major tonic the progression is less smooth on account of the tritone between the thirds of the two triads.
In the progressions from a minor tonic to a major dominant and from a minor tonic to a minor dominant (of course we are speaking only of progressions involving chords of group I), there is no tritone.

The strict antithesis of the unwieldy progressions obstructed by the tritone wedge is that in which all the tones move in minor seconds. This chromatic voice-leading produces the smoothest and most flowing progressions; it acts like a magic formula to make every imaginable chord-progression usable. The simultaneous movement of all voices over the same distance, though in different directions, brings the melodic step of the minor second so strongly into the foreground that the ear relegates the harmonic activity indicated by the root-progression to the background. This kind of universal joint cannot be used everywhere, on account of its soft and sliding effect, and it is particularly inappropriate in a style which in general makes very sparing use of chromaticism. Furthermore, its spell is weakened the moment the chromatic movement is not shared by all the voices. Yet even when only some of the voices move by half-tone steps, the chromatic influence is so strong that progressions which would otherwise be difficult to handle, on account of the position and context of the chords, may be made smooth by this means. This explains why in Figure 81 Examples b and d are smoother than the others: each contains two half-steps. In these progressions, the conspicuousness of the tritone is diminished by the chromatic voice-leading. Thus we experience once again in the realm of the simplest chord-relations what we observed in Figure 78, where the lack of root-progression was compensated for by the clearly defined harmonic fluctuation: the various forces at work in harmony may be so played off against one another that the sharpness of one element is made up for by the smoothness of another, and the weaknesses of one by the extra strength of another.

In the well-rounded progressions among chords of group I, lacking in tension as they are, such compensations for one force by another will take place only to a limited extent. But they become extremely significant when we introduce the sharper sounds of group III. The strong tendencies of the latter, arising from the fact of their containing numerous seconds or sevenths, require more careful treatment. Since it is not always appropriate to let these chords work themselves out in undiminished sharpness of sound, they can be adapted to their environment by means of smooth voice-leading, by gradualness of harmonic fluctuation, and by a smooth two-voice framework.

In reckoning roots, it makes no difference which octave they occur in. The differences in their position are of course of decisive importance to the harmonic fluctuation, but for the reckoning of roots a rough procedure suffices: we transpose all the roots, so far as possible, into the same octave, so that the intervals between them are always small.

With an understanding of root-calculation, harmonic fluctuation, and the two-voice framework, all chord-progressions using the materials of groups I and III can be easily handled. Formations which have always been very refractory, and the successful use of which has always been possible only by a constant process of trial and error, or by simple arbitrary decision, may now be handled with complete knowledge; they offer no more resistance to treatment than their more tractable comrades of group I.

A progression of which one member belongs to group I or III and the other to group V offers certain difficulties, because of the fact that the chord belonging to group V has no root. Since any one of its three tones may act as root representative, we may choose which one we wish to connect with the root of the other chord. In general, that one will be chosen which is the best connected to the roots of the chords preceding and following it. This procedure seems to smack of arbitrariness, but when we come to the analysis of more extended progressions we shall see that this is an illusion. The impression of arbitrary procedure will be heightened when we treat progressions involving two or more chords of group V. For in a progression of one of these chords to another, one has one's choice among six possible root representatives, and thus one can arrange
the root-progression as one pleases. But in a wider view the number of possibilities becomes much smaller, so that the one best suited to our scale of values and to the goal of the progressions is easily to be found. The advantage of these chords—their ambiguity—is fully preserved in this treatment, for they contain in their very texture an indefinite, opalescent quality; yet they can no longer escape being drawn into an orderly sequence. We shall handle them with care, and define them at least sufficiently to make them fit the mould which their context of definite chords leaves open to them.

5

**Progressions Involving Tritone Chords**

The addition of chords from Group B raises the number of possibilities of progression enormously. When the independent chords of Group A are connected with chords of Group B, the tritone, which seeks resolution, and which gives the chords of Group B their character, sets up fields of force to which the chords of Group A offer more or less resistance according to their individual natures. Whenever a tritone chord is followed by a chord of Group A, the tritone is thereby resolved, and the pure sounds of group I, lending themselves willingly to this attraction, produce a feeling of complete relaxation after tension. Chords of group III following a tritone chord also resolve the tritone, it is true, but because of their own considerable tension (though they are free of tritones) the resolution they offer is not complete. Similarly, the progression of a tritone chord to a chord of group V may be only partially satisfactory, because where tension leaves, uncertainty enters. Progressions in the opposite direction—from a chord of Group A to a tritone chord—whip up the sound from rest to tension, and the more complicated the second chord, the greater the feeling of tension.

To gain a clear picture of the nature and value of all these progressions, let us compare the roots of the two chords in each case, as we have done in the progressions of Group A chords already discussed, and thus obtain a reduced and easily grasped image of the harmonic change. The resolution of the tritone, which we must also examine if we are to investigate the progression thoroughly, makes it necessary for us to add to our calculation. For this purpose we shall employ the guide-tone previously mentioned. In all progressions of a chord of Group B to a chord of Group A, the guide-tone of the B chord must move by a good interval to the root of the A chord if the resolution is to be satisfactory. The simplest resolutions occur when this takes place by the step of a second, or when the guide-tone remains stationary, being identical with the root of the A chord. In the second case, the difference in tension between the two chords can be but slight, since the holding over of an important factor cannot be more impressive than its motion by the step of a second. The reckoning of the guide-tone does not stamp any progressions as unusable, any more than did our other means of investigation based on the differing values of the intervals. Progressions of tritone chords in which the guide-tone proceeds by a good interval (the definitions of “good” and “bad” being derived from Series 2) to the root of the chord of resolution have, then, an advantage over those in which it proceeds by a less good interval. And this fact will enable the composer, in handling these often clumsy chords, to place exactly the right chord at the right place for his purposes.

The following examples show the application of the guide-tone principle:
Figure 83 shows progressions of chords of group II to chords of group I; the enharmonic identification of various guide-tones and roots in Examples d, f, and g, should, after what has been said, meet with objection only from those who are fanatics for correct writing. All these simple progressions may be produced without calculation of the guide-tone, simply with the help of the root-progressions. In the following examples (progressions of chords of group II to chords of group III) we could not make any accurate judgement of the value of the progressions without a conscious understanding of the treatment of the guide-tone.

In c and d of Figure 84, the roots progress by tritones. Now, when one tries these progressions out one will observe that c sounds quite satisfying, while d is less convincing. This is because the voice-leading in c is chromatic throughout, the resultant smoothness being emphasized by the holding through of the tone b♭, while the whole-tone step and the skip in d lay bare the tritone in the root-progression.

In Figure 85 tritone chords of group II are connected with the indeterminate chords of group V. The assumption of different root representatives makes possible in each case three different interpretations of the guide-tone progression.

More complicated progressions, in which the first chord is one of group IV, are shown in Figures 86–88 (IV–I in 86, IV–III in 87, IV–V in 88). The progressions using the indeterminate chords of group V permit, as always, a choice of root representatives. In 88a, the root-progression f♯-e avoids the tritone skip f♯-c, although the tritone is still present in the progression of the guide-tone (a♯-e). The root-progression f♯-g♯ yields a good progression of the guide-tone, too, and is accordingly the best. It is almost impossible to judge the connection of the very sharp chords of group IV with those of group V—the augmented triad and the chord in fourths—without reference to any other chords preceding or following. The value of such progressions can be judged only from the wider harmonic context.

When several chords of Group B follow one another without interruption, the tritone remains unresolved. Instead of a resolution, each chord presents a new tritone, which keeps the harmony in approximately equal, though perhaps differently focused, tension. The succession of tension and relaxation which is indispensable to musical structure may be produced in such progressions by the harmonic fluctuation or by family relations among the chords. Not until the entrance of a chord of Group A is the tritone resolved as above described. Successions of chords within Group B are treated, as far as the roots are concerned, like the chord-progressions already discussed. The guide-tone of the tritone in the first chord moves to the guide-tone of the second. The interval traversed in this suc-
cession is again the measure of the value of the progression (in terms of the known interval-values), although only secondarily so, since in such progressions the root-succession is of primary importance. Although the interval of the root-succession and the interval of the guide-tone succession should have values appropriate to the expressive value of the chord-progression itself, the two successions are dependent on each other only in relation to the two-voice harmonic interval based upon each separate root. The two lines of the root-progression and the guide-tone progression need not combine to make correct two-part writing. If each of the lines is logical in itself, and if at each point in their joint progression they form a clear and intelligible harmonic interval, it makes no difference whether they obey the rules of two-part writing or form the crudest infractions of those rules. Not until the resolution into a chord of Group A must the balance be restored by the progression of both root and guide-tone to the root of the chord of resolution.

The chords of group II and many of those of group IV have one particular characteristic. In a succession of two such tritone chords of which the roots in turn are separated by the distance of a tritone, there is a tritone also between the guide-tones of the two chords, and the tritone included in the first chord is also contained in the second:

This chain of tritones links these two chords so closely together that they seem almost like fractional parts of the same chord; they thus perform good service when a close but highly tensed progression is needed, but they are quite out of place when a strong root-progression is desired.

Progressions within groups II or IV, or those from II to IV are thus easy to appraise:

If the progression is from a chord of group II or group IV to one of group VI, the root of the first chord moves to the most convenient tone of the indeterminate VI chord, as it does in the progressions from chords of Group A to those of group VI, already discussed.

The guide-tone of the first chord, too, goes to any convenient tone of the indeterminate chord, whether it is the same one to which the root proceeds or not. Here, too, it is only in a progression of two chords torn from its context that the choice is completely free. If the chord of group VI is between two other chords, the latter do not leave many possibilities open—whether because the guide-tones of the preceding and following chords require a particular tone between them, or because the following chord is one of Group A, of which the root is a common goal for both the guide-tone and the root of the preceding chord, and must be reached in the smoothest possible way from the last definite guide-tone over the assumed guide-tone as a bridge. In progressions within group VI, the calculation of the root representatives suffices.
Richard Wagner
Tristan und Isolde, Prelude

Harmonic and Melodic Analysis

1. Fluctuation
2. Two-Voice Framework
3. Degree-Progression
4. Tonality

[210]
The Prelude to Tristan is one of the finest examples of the elaboration of a two-voice framework. The observer of the intervals formed by the outside lines of the harmony will be astonished to see how intervals of varying tension are juxtaposed. The procedure is illustrated beginning with the very first chord: the interval of a minor third (written as an augmented second) is followed by a major third, which represents a decrease of tension; the tension is then sharply increased again in the tritone on the first eighth of the third measure, only to be resolved completely in the fifth which follows. In this admirable way the tensional development of the framework is calculated from beginning to end, as the section here notated illustrates.

No less remarkable is the handling of the harmonic fluctuation and of the degree-progression. The distribution of the harmonic tension produces a beautifully varied succession of sharp and mild chords—chords of Group A and chords of Group B. Yet this ebb and flow dispenses almost entirely with chords of the highest tension—those of group IV. In the degree-progression, both the roots and the guide-tones are indicated. The first measure with its upbeat is treated as a broken chord, because the ear relates the two tones; likewise the fifth, eighth, and ninth measures. In measure 9, the d⁴ of measure 8 has to be taken into account, and thus beneath the a⁴ of the ninth measure there is a root, d⁴. The six-four chord in measure 18 is so passing in character that one is justified in taking its bass tone as its root, just as in a six-four chord preceding the dominant in a cadence. The same is true in measure 32.

In the analysis of the tonality it should be noticed that those roots upon which tritone chords are built must be regarded as dominants of tonics lying a fifth below. Thus the tonal center of the first three measures is a, and of measures 5–7, c. If we set out the centers of the various tonalities in succession, we obtain the following series:

```
10 8 6 4 2 3 5 7
```

from which it appears, in view of the repetitions of A and its support by its most closely related tones, that A is indisputably the tonal center of the whole; and this is still further confirmed in the later development of the piece.

One cannot expect, in so wonderfully constructed a harmonic organism, to find an equal perfection of melody. It is impossible to balance the two elements exactly; one of them must always predominate. Here melody yields first place to harmony. It confines itself for the most part to steps of a second and broken-chord formations. Thus nothing remarkable in the way of either melody degree-progression or step-progression can arise. The continual stepwise motion yields too little harmonic result for the degree-progression, and this stepwise motion is itself the step-progression, which is not built on large lines. But in the place where melody assumes somewhat greater importance (measures 25–32) I have added the melodic analysis.