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An Interesting Group of Combination-Product Sets Produce Some Very Nice Dissonances

by Warren Burt

Back in 1988, while working on a piece with sound poet Chris Mann, I invented a set of scales that used prime numbers and got more and more dissonant the farther you went into the series. Perhaps some of you will also find them useful. I invented them partly to explore the concept of modulation by common tones; however, in a number of compositions where I've actually set up such modulations using these scales, I don't hear them as modulations in the tonal sense. Rather, they just seem to endlessly snake up the series of sets, rather like Wagner's "endlessly modulating" harmonies. Be that as it may, I enjoy playing with these scales, both singly and in sets, so for those that might want to also play, and especially to experience the delightfully grinding dissonances that occur in the farther reaches of the series, here they are. I acknowledge the bias a number of contributors to **1/1** have for low-prime just harmonies, and for ideas of tonality they like to explore with them. For my own tastes, however, I've found over the past four or five years that I'm more interested and more pleased by dissonant sounds. For example, I've come to regard 41:30 not as an interval with its own merits—it still sounds like an out-of-tune 4:3 to me—but I've come to regard 4:3, 41:30, and many others as "flavors" of fourths, each of which has its own pleasures and piquancies.

I first noticed that if you took a set of five prime numbers and multiplied all the elements of that by the last three elements of the series, you got a set of twelve unique elements. (I later learned that Ervin Wilson and John Chalmers called this technique making a combination-product set.) This suggested to me a way of generating sets of twelve pitches with which to play around on a black-white keyboard. For example, if we take the series (2, 3, 5, 7, 11) and multiply each element by 5, 7, and 11, we get the following set of numbers:

(2 × 5)	10	(5 × 5)	25
(2 × 7)	14	(5 × 7)	35 (also 7 × 5)
(2 × 11)	22	(5 × 11)	55 (also 11 × 5)
(3 × 5)	15	(7 × 7)	49
(3 × 7)	21	(7 × 11)	77 (also 11 × 7)
(3 × 11)	33	(11 × 11)	121

If we put them all into the same octave we get:
66, 70, 77, 80, 84, 88, 98, 100, 110, 112, 120, 121, 132. Notice the plethora of simple just intervals (88:66 = 4:3, etc.) in this set. Also notice that already there are some very small dissonant intervals here (112:110 = 56:55).

If we go up the series of prime numbers, the next set would be made by multiplying each element of the series (3, 5, 7, 11, 13) by 7, 11, and 13. This will give us seven tones in common with the set that began on 2 and five tones that are different. (See the chart at the end of this for examples.)

Going up to the twelfth prime number, we get the set (37, 41, 43, 47, 53), with each element multiplied by 43, 47, and 53. This yields the set 1591, 1739, 1763, 1849, 1927, 1961, 2021, 2173, 2209, 2279, 2491, 2809, with 3182 added to complete the octave. This set has lots of nice crunchy, tiny intervals in it, but still has seven tones

(text continued on page 4)

"Just Intonation is the best intonation" —Lou Harrison

Example 1. Common tones in 2, 3, and 5 Major

in common with its predecessor in the series, the set of (31, 37, 41, 43, 47) each multiplied by 41, 43, and 47.

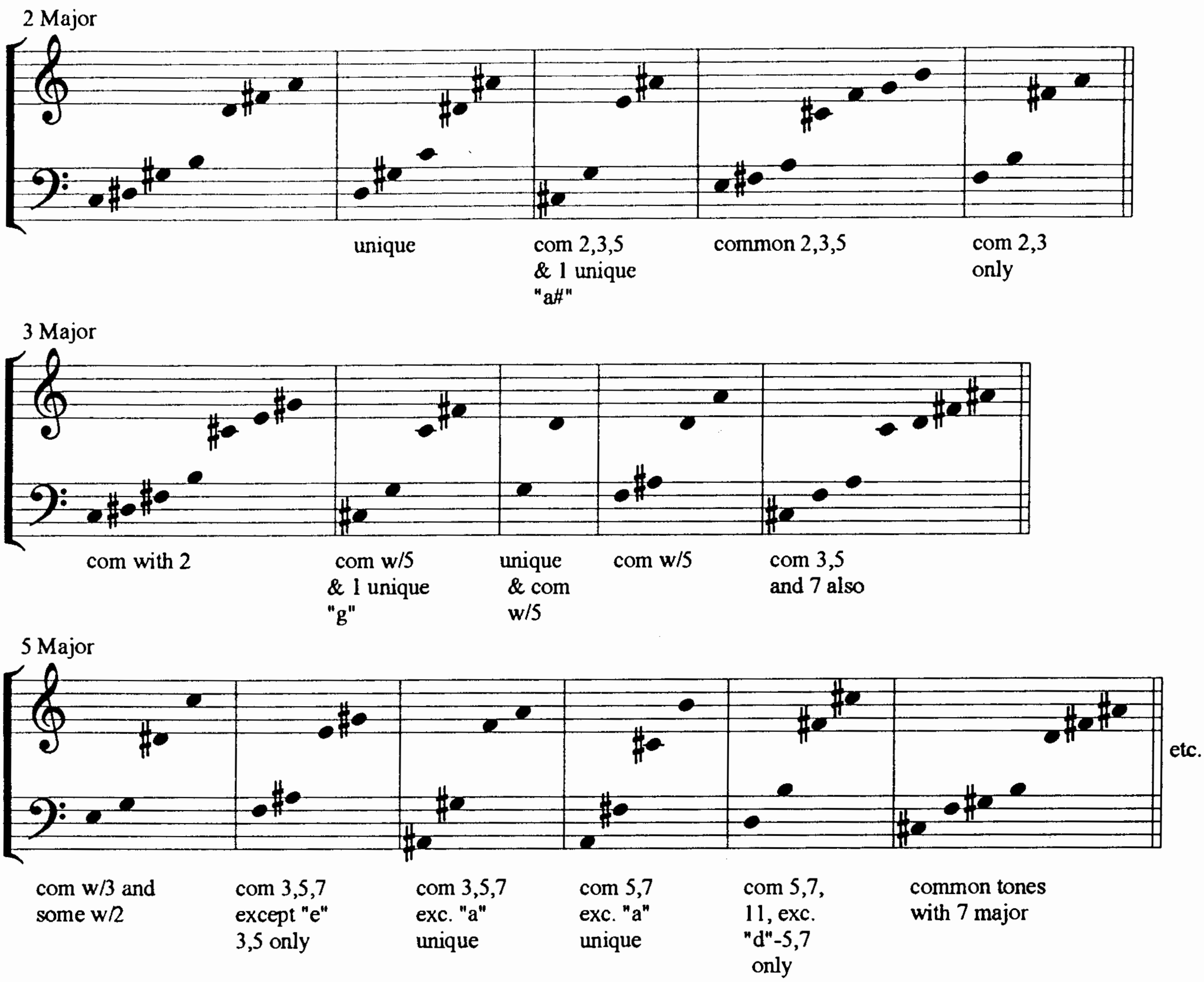
You can treat these numbers as either harmonics or subharmonics. This will give you two inversionally related scales. For convenience, I named the scales "major" if they were made of the numbers treated as harmonics, and "minor" if they were made of the numbers treated as subharmonics. I also named the scales after the lowest prime number they were made from. So, for example, the first scale treated as harmonics would be "2 major," whereas the last scale in the series treated as subharmonics would be "37 minor."

The tables on pages 7–9 show all 24 scales, twelve each of major and minor, made in this way. All the tones are scaled up to match the octave of the highest prime numbered scale, where 1591 is treated as the basis for all the scales and is regarded as 0 cents.

The prime product number for each pitch is given, followed by its cents value in the scale. The listing of cents values should help those who want to load them into their synthesizers' tuning registers. I have played these scales with both Yamaha TX81Z and Emu Proteus 1 synthesizers, and, using Denny Genovese's Microtonal MIDI Terminal program, have used them with a number of other synthesizers whose tuning registers don't allow the kinds of intervals used here.

I've used these scales in "of course" for voice and live electronics by myself and Chris Mann (1988); in "24 Chorales for Chris Mann," (1991) for solo computer improviser (using a composing program I wrote in the

Example 2. Harmonic structures—first three scales: "Three One-Way Journeys"

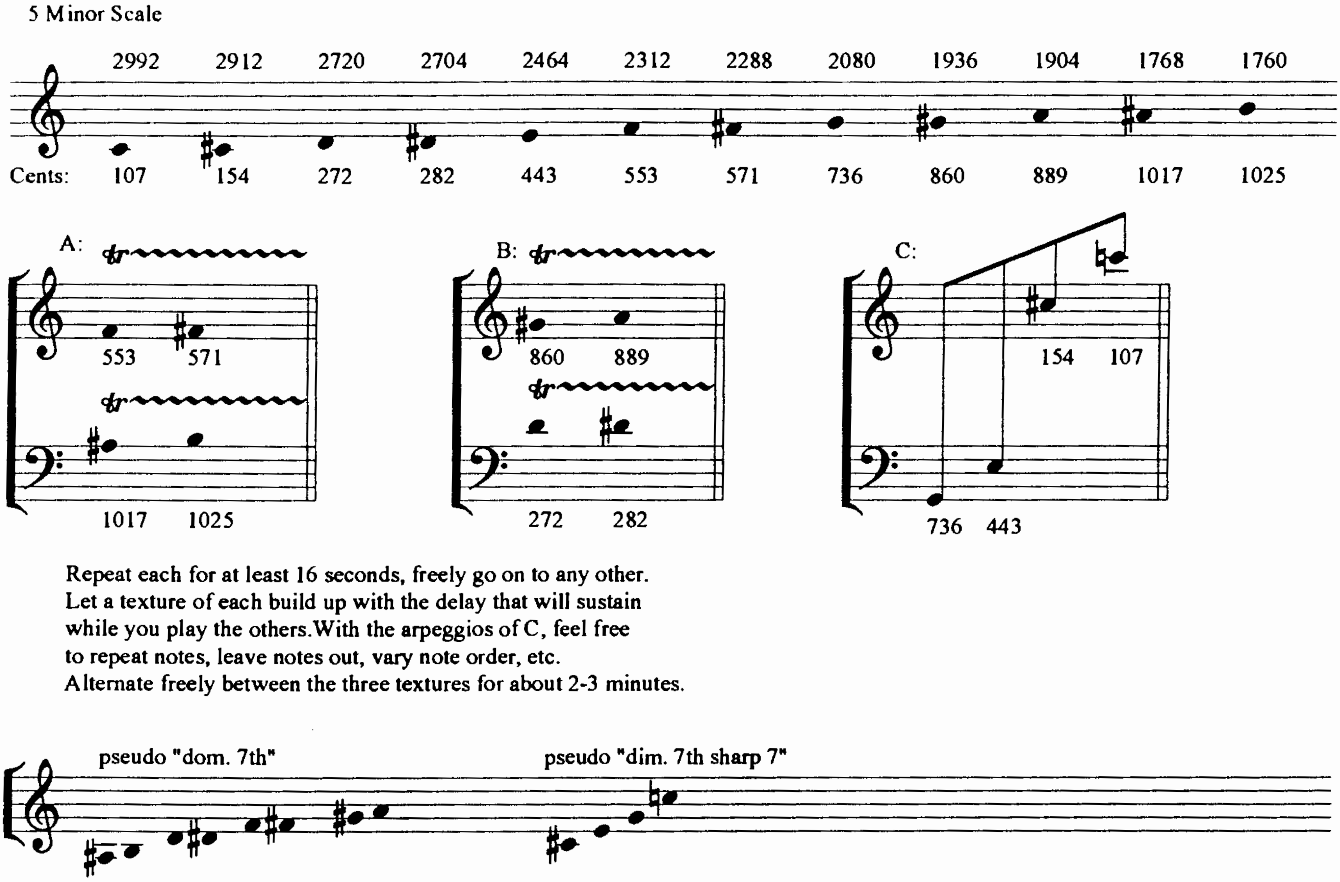


Ravel language); in "A Fig Tree (In a Post-Coltrane Environment, $1/f$ is Too Highly Correlated)" from "Chaotic Research Music" (1990), in which I also developed additive synthesis timbres where the partials were tuned to the members of the scale they were using (the timbres sounded bell-like—surprise!); and in "3 One-Way Journeys (Up a Prime-Numbered Spiral)" from "Music for Microtonal Piano Sounds, Part 1" (1992), which is a computer music on tape piece, and which is the piece where I developed the "modulating" chord progressions which don't actually sound like modulations. (Maybe I never really learned what modulation actually *was*.)

Example 1 shows how three of the scales were mapped to a twelve-tone keyboard, and how they related. Numbers below the noteheads are the ratio-product numbers. The straight lines connect the notes in each scale that are the same pitch, even though they may be assigned to different keys in each scale. Below the listing of the scales, the second part of the example shows sets of common tones listed in musical notation. How these were used musically is shown in Example 2.

Example 2 shows the basic harmonic progression I used in the first three sections of all three movements of "3 One-Way Journeys (Up a Prime-Numbered Spiral)." In the first movement, the notes in each bar were played as a block chord, making a chorale. In the second movement, the notes in each bar formed the pitch set with which various random number routines (some using chaos equations) programmed in the "Ravel" language, made textures of random notes. In the third movement, the notes of each bar were used as pitch sets with which the algorithmic composing program "M" made a mono-

Example 3. Improvisation structure for 5-minor scale, electric piano timbre and eight-second delay with feedback (at least five or six repeats)



phonic melody.

The words under each bar show how the chord was derived from the way the different scales related. For example, bar two of the third line, "5 Major," consists of all the notes common to 3, 5, and 7 Major, plus the E, which is common to 3 and 5 Major only. The next bar consists of the notes common to 3, 5, and 7 Major in a different voicing, with the addition of A, which is a pitch unique to 5 Major. Each section in a scale makes a gradual transition from chords based on common tones with the previous scale, to chords based on tones common with the next scale in the sequence. In the complete piece, all twelve major scales are used, going in order from 2 major to 37 major.

Perhaps the reason this progression doesn't seem to modulate is that, in fact, it doesn't. In order for a sense of modulation to occur, perhaps more time would need to be spent establishing the identity of each "key," so that the contrasts between successive keys would be made more clear. As the progression stands now, the obsessive making of chords from tones common to adjacent scales in the sequence defeats a sense of harmonic change, but makes a sense of gradual advance through a large harmonic world consisting of all the pitch resources of all twelve major scales.

I've also used individual scales in several other pieces, such as my 1988 live electronics piece "Musical Chaology." This piece uses the scale of 19 minor as a pitch set from which my composing machine, "Aardvarks IV," makes four simultaneous heterophonic melodies. I've also used the scales occasionally in live keyboard improvisations. These later have sometimes turned out to be blues, even though the scales usually don't imply the blues, but all those tiny intervals have sometimes gotten me jamming as if they were blue notes. One example of an improvisational structure that wasn't a blues is given in Example 3.

This example is from a series of such structures I used

Prime Product Harmony—First Six Major Keys (2-Major–13-Major)

2M		3M		5M		7M		11M		13M	
number	cents	number	cents	number	cents	number	cents	number	cents	number	cents
3136	1175	3136	1175								
								3128	1170	3128	1170
										3016	1106
				2992	1093	2992	1093	2992	1093		
		2912	1047	2912	1047	2912	1047				
						2888	1032	2888	1032	2888	1032
2816	989										
				2720	928						
		2704	918	2704	918	2704	918				
2688	908	2688	908								
										2668	895
						2584	840	2584	840	2584	840
2560	824										
		2496	780								
2464	757	2464	757	2464	757						
								2392	706	2392	706
				2312	647	2312	647	2312	647		
		2288	629	2288	629	2288	629				
2240	592	2240	592								
										2204	564
						2128	504				
								2116	494	2116	494
2112	490	2112	490								
		2080	464	2080	464						
								2024	417		
						1976	375	1976	375	1976	375
										1972	372
1936	340	1936	340	1936	340						
1920	325										
				1904	311	1904	311				
1792	206										
				1768	183	1768	183	1768	183		
1760	175	1760	175	1760	175						
								1748	163	1748	163
										1682	96
						1672	86	1672	86		
1600	10										

Prime Product Harmony—Second Six Major Keys (17-Major–37-Major)

17M		19M		23M		29M		31M		37M	
number	cents	number	cents	number	cents	number	cents	number	cents	number	cents
						3182	1200	3182	1200	3182	1200
3128	1171										
				3034	1118	3034	1118	3034	1118		
								2914	1048		
2852	1010	2852	1010	2852	1010						
		2812	986								
										2809	984
		2738	940	2738	940	2738	940				
2668	895	2668	895								
						2666	894	2666	894		
				2542	811	2542	811	2542	811		
						2494	778				
										2491	787
				2378	696	2378	696				
2356	680	2356	680								
		2294	634	2294	634	2294	634				
										2279	622
								2209	568	2209	568
2204	564	2204	564								
										2173	540

Prime Product Harmony—Second Six Major Keys (Continued)

17M number	17M cents	19M number	19M cents	23M number	23M cents	29M number	29M cents	31M number	31M cents	37M number	37M cents
		2146	518	2146	518	2146	518				
2116	494										
2108	487										
								2021	414	2021	414
1972	371										
										1961	362
								1927	332	1927	332
1922	327	1922	327	1922	327						
				1886	295						
						1849	260	1849	260	1849	260
1798	212	1798	212	1798	212						
						1763	178	1763	178	1763	178
1748	163										
								1739	154	1739	154
		1702	117	1702	117						
1682	96	1682	96								
				1681	95	1681	95	1681	95		
						1591	0	1591	0	1591	0

Prime Product Harmony—First Six Minor Keys (2-minor–13-minor)

2m number	2m cents	3m number	3m cents	5m number	5m cents	7m number	7m cents	11m number	11m cents	13m number	13m cents
1600	1190										
						1672	1114	1672	1114		
										1682	1104
								1748	1037	1748	1037
1760	1025	1760	1025	1760	1025						
				1768	1017	1768	1017	1768	1017		
1792	994										
				1904	889	1904	889				
1920	875										
1936	860	1936	860	1936	860						
										1972	828
						1976	825	1976	825	1976	825
								2024	783		
		2080	736	2080	736						
2112	710	2112	710								
								2116	706	2116	706
						2128	697				
										2204	636
2240	608	2240	608								
		2288	571	2288	571	2288	571				
				2312	553	2312	553	2312	553		
								2392	494	2392	494
2464	443	2464	443	2464	443						
		2496	420								
2560	377										
						2584	360	2584	360	2584	360
										2668	305
2688	292	2688	292								
		2704	282	2704	282	2704	282				
				2720	272						
2816	212										
						2888	168	2888	168	2888	168
		2912	154	2912	154	2912	154				
				2992	107	2992	107	2992	107		
										3016	93
								3128	30	3128	30
3136	25	3136	25								

Prime Product Harmony—Second Six Minor Keys (17-minor–37-minor)

17m		19m		23m		29m		31m		37m	
number	cents	number	cents	number	cents	number	cents	number	cents	number	cents
						1591	1200	1591	1200	1591	1200
				1681	1105	1681	1105	1681	1105		
1682	1104	1682	1104								
		1702	1083	1702	1083						
								1739	1046	1739	1046
1748	1037										
						1763	1022	1763	1022	1763	1022
1798	988	1798	988	1798	988						
						1849	940	1849	940	1849	940
				1886	906						
1922	873	1922	873	1922	873						
								1927	868	1927	868
										1961	838
1972	828										
								2021	786	2021	786
2108	713										
2116	706										
		2146	682	2146	682	2146	682				
										2173	660
2204	636	2204	636								
								2209	632	2209	632
										2279	578
		2294	567	2294	567	2294	567				
2356	520	2356	520								
				2378	504	2378	504				
										2491	424
						2494	422				
				2542	389	2542	389	2542	389		
						2666	306	2666	306		
2668	305	2668	305								
		2738	260	2738	260	2738	260				
										2809	216
		2812	214								
2852	190	2852	190	2852	190						
								2914	153		
				3034	82	3034	82	3034	82		
3128	30										
						3182	0	3182	0	3182	0

Note: I've prepared these tables in this format so that if you copy them and paste them together, with major keys side by side and minor keys side by side, you can see the relationships among all the keys

recently as accompaniments for Australian choreographer Eva Karczag's 1994 Australian tour. In this simple structure, motives A and B, through the use of a long delay with feedback, establish a shimmering, beating, dissonant "pseudo-dominant seventh" chord in the middle register, while a different chord, a "pseudo-diminished seventh, sharp seven" chord is spread over the registers above and below the central chord. Making trills with very closely spaced notes is, of course, one of the simplest ways of treating these scales, reducing a twelve note set to, in this case, eight "pitches," four of which are unique, and four of which may be regarded as detuned beating pairs. In other improvisational structures of this kind, I try to establish the unique nature of each pitch of the set, and try not to treat them as ornaments of each other.

Naturally, with structures of this type, there is no sense of progression or tonality, but simply the juxtaposing of different chords, each of which is dissonant in itself, creating a harmonic world that is both static and dissonant, but which I feel is quite pleasing in sound.

I would be very curious to hear what sorts of structures other people would make with these scales. If anyone else does use these scales, I'd be interested in hearing from you. **1/1**