# Microtonal Pitch Systems in Some Recent Music

**Warren Burt,**
Visiting Lecturer in Computer Music,
Australian Centre for the Arts and Technology,
Canberra

**Abstract**

A survey of microtonal systems used by the author in some recent works for computer, most of which involve algorithms of one kind or another. Brief examples from the following works will be played and discussed: "Scenes from the Jungle of Intonational Injustice" (1993) (non-octave based equal temperaments); "Sines/Forks" (1993) (harmonic series based just intonation); "Pi and the Square Root of 2" (1994-95) (non-just, non-equal tempered scale with Setharean matching of timbre and scale); "3 Movements for Movement Controlled Sounds" mvt. 3 (1995) (inharmonic (random frequencies) spectra with scale of (random) tiny intervals to produce beating); for Roman Verostko (1995) (juxtaposed equal temperaments in both tuning and timbre); "Remembering Griffes II" (1995) (Yasserian partitioning of 19-tone equal temperament). Reference to the author's recent work with Wilsonian combination-product sets will also be made. Illustrated with many charts and graphs and examples from cassette tape.

**Introduction**

Although I've been working with microtonality since at least the early 70s, a real expansion of my interest came about with my 1985-86 residency with the CSIRO's Applied Physics Department, and with the availability of easily retunable digital synthesizers in the mid-80s. If not for these new resources, I would probably have abandoned pitch totally in my music. Since then, almost every piece I have written has used a different tuning system (or several at once) in a different way. I am involved in an exploration of an extremely diverse set of musical resources. One key aspect of how I've been using these tunings is to find some aspect of the tuning which informs the structure of the piece using it. This paper sketchily outlines tuning ideas in 6 recent works.

**1. *SCENES FROM THE JUNGLE OF INTONATIONAL INJUSTICE* (1993)**
for computer controlled microtonal piano and wind samples.

The central idea in this piece is a re-evaluation of what ¤"equal temperament" means. Following on work by Brian ¤McLaren in evaluating the possibilities of non-octave based ¤equal temperament scales, I came up with the following ¤definition: an equal tempered scale is any scale that can be ¤expressed by the formula the Nth root of X, or is any scale ¤made up of a chain of any single interval, such that no ¤octave reduction of the resulting chain of intervals is made.

An example of the first kind of scale is the 12th root of 2, normal 12-tone tuning, where the octave, the ratio 2:1, or 1200 cents, is divided up into 12 parts of 100 cents each. Another example is the 13th root of 3, where the ratio 3:1, or 1902 cents (from the fundamental to the 3rd partial in the harmonic series), is divided up into 13 parts of 146.3 cents each. Note that the interval of the octave does not exist in this scale. For this reason these scales are sometimes referred to as "non-octave" scales.

An example of the second kind of scale would be one made up of a chain of 16/15 intervals, a just intonation minor second, 112 cents. A chain of 11 of these intervals gives a total interval of 1232 cents - wider than an octave by 32 cents. Another example is the tuning that Gary Morrison proposes - a tuning of 88 cents per step. This tuning produces very flat major thirds (4 x 88 = 352 cents, with 386 cents being the 5/4 just major third), but almost perfectly in tune major 10ths (18 x 88 = 1584 cents, with 1586 cents being the 5/2 just major 10th). Implicit in this scale, therefore, is the idea of voicing if intervals implying tonality are used.

"Scenes" is made of two independent lines, one of piano samples, the other of wind samples. The piano samples play through a series of scales made in the first, the nth root of x, way. The wind samples use a series of scales each made by stacking a single just intonation interval. The idea of changing scales with each new section of the piece also suggested that each section would also have a new tempo and type of musical gesture. Further, the fact that the scales would be derived differently for each of the lines suggested that the two lines would progress at different rates. The ratio of 3:2 was used to relate the tempi of the piano and winds. Since this is the ratio of the perfect 5th, this was my one concession in this piece to ideas of tonality and traditional structure.

There are 63 different scales used in the piano part. These are made by a permutation cycle of 9 n's and 7 x's (given the formula the nth root of x). The first 18 of 63 of these tunings is shown in chart 1. Note that here all the x's are even, which means that at times, an octave based scale is generated. For example the 10th root of 4, the opening piano scale, is equivalent to the 5th root of 2.

**Chart 1:**

First 18 (of 63) tunings for piano part of SCENES

| Form: | $\sqrt[10]{4}$ | $\sqrt[12]{6}$ | $\sqrt[14]{8}$ | $\sqrt[15]{9}$ | $\sqrt[16]{10}$ | $\sqrt[18]{12}$ |
|---|---|---|---|---|---|---|
| Cents: | 240 | 259 | 257 | 249 | 239 | 228 |

| Form: | $\sqrt[20]{14}$ | $\sqrt[21]{4}$ | $\sqrt[22]{6}$ | $\sqrt[10]{8}$ | $\sqrt[12]{9}$ | $\sqrt[14]{10}$ |
|---|---|---|---|---|---|---|
| Cents: | 228 | 114 | 141 | 360 | 317 | 285 |

| Form: | $\sqrt[15]{12}$ | $\sqrt[16]{14}$ | $\sqrt[18]{4}$ | $\sqrt[20]{6}$ | $\sqrt[21]{8}$ | $\sqrt[22]{9}$ |
|---|---|---|---|---|---|---|
| Cents: | 287 | 286 | 133 | 155 | 171 | 173 |

The wind part uses a series of 21 different scales each made from stacking one just-intonation interval, which intervals are, in fact, the intervals of Harry Partch's 43-note just intonation scale. (That this use of these intervals is significantly different from Partch's is hopefully obvious.)

These 21 intervals, which gradually get wider, are then used in retrograde to make one large-scale aspect of the structure of the piece: the winds begin by playing a scale based on 22 cent microtones, gradually work their way to a scale based on 583 cent augmented fourths, and then move slowly back to the microtonal scale of the beginning. The sizes of the intervals of the scales and their order is shown in chart 2.

**Chart 2:**

Step size of wind scales. Sections 1 - 21. (reverse for sections 22-42.)

| Sect: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cents step | 22 | 53 | 85 | 112 | 151 | 165 | 182 | 204 | 231 | 267 | 294 | 316 | 347 | 386 |

| Sect: | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|
| Cents: | 418 | 435 | 471 | 498 | 519 | 551 | 583 |

This kind of structure was used 1) to give each section of the piece its own unique harmonic colour, and 2) to have a dazzling and (in the nicest sense) disorienting succession of these colours which also manifested some aspect of a simple structure. As stated before the succession of tunings then suggested the different gestures that exist from section to section of the piece. The piece was made with Sound Globs algorithmic composing software and various synthesizers and samplers.

Sound example 1: Piano texture 1 from "Scenes" - tuning: 240 cents/step.
Sound example 2: Piano texture 12 from "Scenes" - tuning: 285 cents/step.
Sound example 3: Wind texture 1 from "Scenes" - tuning: 22 cents/step.
Sound example 4: Wind texture 4 from "Scenes" - tuning: 112 cents/step.
Sound example 5: Opening 1 minute of "Scenes"

**2. *SINES/FORKS* (1993-94, rev. 1996)**
for tuning forks and computer-controlled just-intonation sine waves.

In this piece, two just-intonation (tuning by means of small interval ratios) scales, related by a 3:2 (the perfect 5th) are used. The tuning forks (made in 1985 at the CSIRO in Melbourne, and in 1993 at RMIT) produce a pure sine wave after the initial .6 second attack, and use a just scale based on 1/1 = G = 784 HZ. The computer produces complexes of sine waves, using a just intonation scale of harmonics 32- 64 and subharmonics 32-64 based on a fundamental of 32/32 = 1/1 = C = 261.63 Hz.

The algorithmic composing program Drummer is used to select which pitches and rhythms the sine waves will play. 50 different textures of these are defined. Twenty-five of these are "fast," and twenty-five are "slow" versions of the same. The forks are swung in the air, producing doppler and other phasing effects. Additionally, the forks scale has a number of very small intervals which produce beats, which provide another level of timbral and rhythmic interest. Swinging the forks past a sensor (a Buchla Lightning) tells the computer program to select a different texture of pitches and rhythms. The performer cannot predict which of the 50 textures the computer will select. In this way, an improvisational relationship is set up between the tuning forks and the electronics.

**Chart 3:**

Forks scale, based on 1/1 = G = 784 Hz

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Ratio: | 1/1 | 28/27 | 16/15 | 25/21 | 6/5 | 2409/2000 | 9/7 |
| Cents: | 0 | 63 | 112 | 302 | 316 | 322 | 435 |
| Hertz: | 784 | 813.0 | 836.3 | 933.3 | 940.8 | 944.3 | 1008.0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Ratio: | 4/3 | 7/5 | 10/7 | 1003/700 | 297/200 | 747/500 | 3/2 |
| Cents: | 498 | 583 | 617 | 623 | 684 | 695 | 702 |
| Hertz: | 1045.3 | 1097.6 | 1120 | 1123.4 | 1164.24 | 1171.2 | 1176 |

| | | | | |
|---|---|---|---|---|
| Ratio: | 14/9 | 8/5 | 9/5 | 2/1 |
| Cents: | 765 | 814 | 1018 | 1200 |
| Hertz: | 1219.5 | 1254.5 | 1411.2 | 1568 |

**Chart 4:**

Just intonation scale of Harmonics 32-64 based on 32/32 =
(1/1) = C = 261.63 Hz. (invert for subharmonics)

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Harm: | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 |
| Cents: | 0 | 53 | 105 | 155 | 204 | 251 | 298 | 342 | 386 | 429 | 471 | 512 | 551 | 590 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Harm: | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 |
| Cents: | 628 | 666 | 702 | 738 | 773 | 807 | 841 | 874 | 906 | 938 | 969 | 999 | 1030 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Harm: | 59 | 60 | 61 | 62 | 63 | 64 |
| Cents: | 1059 | 1088 | 1117 | 1145 | 1173 | 1200 |

This kind of tuning was used because with both acoustic and electronically produced sine waves, the resulting harmonies were absolutely gorgeous!

Sound example 6: Scale of tuning forks, ascending.
Sound example 7: Example of "fast" sine waves texture, using harmonics 8 - 16 and subharmonics 16 - 8.
Sound example 8: "Slow" sine waves texture, using harmonics 20 - 11, and subharmonics 11 - 20, over 2 octaves.
Sound example 9: Fragment of a live performance of "Sines/Forks"

### 3. *PI AND THE SQUARE ROOT OF 2* (1995)
an installation for computer controlled sampler, graphics, text, etc.

This piece was an investigation into numerology. There is a mathematical distinction between the digits of pi, which are called a "transcendental" series, and the digits of the square root of 2, which are called an "algebraic" series. The difference is in how the numbers are calculated. Both series produce what appear, however, to be a random series of integers. It is speculated that the pi series is the most random object in the universe. I wanted to hear if there were any hearable structural differences between these two series. Since these series are most usually expressed in base 10, that is, with digits 0 - 9, this suggested to me making scales of 10 notes to play the pitches represented by the digits 0 - 9. In the piece, pitch, rhythm, and loudness, are all derived from the series of digits of pi and the square root of two. Tuning and timbre, additionally, are derived from the ratios of pi and the square root of two. This matching of tuning and timbre was suggested by the work of William Sethares, and will be discussed in more detail later in this paper.

The approximate ratio of pi is 31415/20000, which equals 782 cents, a very flat minor 6th. The approximate ratio of the square root of two is 1414/1000, which equals 600 cents, the augmented fourth. A scale is constructed out of the difference between these. That is, if pi = 782 cents, and root 2 = 600 cents, 782 - 600 = 182 cents. (Coincidentally, this is the just interval of 10/9, the smaller major second.)

If intervals are added (in a way analogous to the way Pythagorean tuning adds its fifths), we get a chain of pi intervals of 782 cents, 364 cents 1146 cents, etc. Subtracting 182 cents from each of these gives us its "root 2" partner. Chart 5 shows the derivation of the scale.

**Chart 5:**

Derivation of the scale of Pi and the Square Root of Two.

pi = 782 cents
2 pi = 1564 cents (-1200 to put in octave = 364)
3 pi = 2346 cents (-1200 to put in an octave = 1146)

Pi - root 2 = 182 cents.
2pi (364 cents) - 182 cents = 182 cents.
3pi (1146 cents) - 182 cents = 964 cents.
etc.

| Degree: | Cents: | Derivation: |
|---|---|---|
| 0 | 0 | 0 Starting Point |
| 1 | 128 | 5pi - (pi-root2) |
| 2 | 182 | 2pi - (pi-root2) |
| 3 | 364 | 2pi |
| 4 | 546 | 4pi - (pi-root2) |
| 5 | 600 | root2 |
| 6 | 728 | 4pi |
| 7 | 782 | pi |
| 8 | 964 | 3pi - (pi-root2) |
| 9 | 1146 | 3pi |

Note that in this scale, 5pi is not used. If it was, it would be 310 cents. To use either it or 5pi-(pi-root2) or to use 0 cents, the starting point, was an arbitrary decision, although recently I've been more inclined, if I use this scale again, to leave out 0 cents and include 5 pi.

The timbres used in the piece are also derived from this scale. Two timbres, each with a fundamental and 9 partials, are made. Each of the partials is, in its relevant octave, tuned to one of the pitches of the scale. The algorithmic composing program "Kinetic Music Machine" was used to make this piece.

This tuning was used to explore the very pretty sonic world created by a rigorous application of an admittedly arbitrary numerology.

Sound example 10: Scale of "Pi/Root 2" with 2 different timbres.
Sound example 11: Fragment of "Pi/Root 2"

## 4: MOVEMENT 3 from THREE MOVEMENTS FOR MOVEMENT CONTROLLED SOUNDS (1995)

for Buchla Lightning, EPS sampler, and movement.

In this piece, the performer's movement thru space is used to play a sound of many changing inharmonic partials (made with additive synthesis). This sound is played using a scale of many tiny intervals. The performer's movements make a semi- random selection of pitches, where spatial position is mapped to a selection of musical ranges (movement to stage right selects lower pitches, movement to stage left selects higher ones).

Numbers chosen off the top of my head were used for the frequencies of the inharmonic partials of the changing tone. No record of these frequencies was kept. Similarly, in keeping with the arbitrary nature of the inharmonic partials of the tone, the pitches were also chosen spontaneously, but with the knowledge that when these changing inharmonic spectra were played with a series of very narrow beating intervals, the result would be beautiful, shimmering, and at times almost vocal-like sounds.

**Chart 6:**

Two keyboard octaves of the scale from "Movement 3" using Ensoniq notation. Acoustically this covers about a minor 7th. The scale extends for two more octaves in either direction in a similar manner.

| Key | ETpitch | Deviation(cents) |
|---|---|---|
| C3 (midi 48) | B3 | +50 |
| C#3 | B3 | +61 |
| D3 | B3 | +73 |
| D#3 | C4 | +47 |
| E3 | C4 | +56 |
| F3 | C4 | +69 |
| F#3 | C4 | +87 |
| G3 | D4 | +61 |
| G#3 | D4 | +78 |
| A3 | D4 | +90 |
| A#3 | E4 | +17 |
| B3 | E4 | +25 |
| C4 | E4 | +33 |
| C#4 | E4 | +48 |
| D4 | F#4 | +40 |
| D#4 | F#4 | +53 |
| E4 | F#4 | +65 |
| F4 | F#4 | +76 |
| F#4 | F#4 | +89 |
| G4 | G4 | +3 |
| G#4 | G4 | +19 |
| A4 | A4 | +0 |

| | | |
|---|---|---|
| A#4 | A4 | +11 |
| B4 | A4 | +23 |
| C5 | A4 | +97 |

Sound example 12: Basic tone of "Movement 3" on C3, Midi 48.
Sound example 13: Some clusters with the basic tone ascending from C3.
Sound example 14: Fragment of "Movement 3."

## 5: FOR ROMAN VEROSTKO

for computer generated sounds on tape. (1995)

The main idea in this piece was to create as exact a sonic analogy as I could to Roman Verostko's "Scarabs," a computer graphics program which generates an infinite variety of symmetrical computer drawings reminiscent of various artificial life-forms. The piece could be realized with any tunings or timbres, but additionally, I decided to explore William Sethares' ideas about matching tunings and timbres in this piece. This was done to provide a very different sense of timbre and harmonic colour than 12-tone tuning would . On their own, the tuning ideas also have merit worth exploring.

Sethares maintains that if the partials of a tone are in tune with the pitches of a given scale, then intervals played with that timbre will have a greater sense of "consonance" than will intervals played with timbres where the partials are not tuned to the pitches of that scale. For this piece, four additive synthesis tones were created, each of which had its partials tuned to the pitches of a different equal-tempered scale: 16, 17, 18 and 19 tones per octave. Each of these timbres can then appear with any of the four scales. That is, a scale of 19 tones/octave can be played by a tone with its partials tuned to 19 tone/octave, or with any of the three other tones. This gives 16 possible combinations of tuning and timbre, which, in the order given in Chart 7, form the harmonic structure of the piece:

### Chart 7:

Harmonic structure of "For Roman Verostko" based on combinations of tuning and timbre.

```
Section: 1  2  3  4  5  6  7  8  9  10 11 12 13 14 15 16
Scale:   16 17 18 19 16 17 18 19 16 17 18 19 16 17 18 19
Timbre:  16--------> 17--------> 18--------> 19-------->
```

(Each section consists of 3 symmetrical algorithmically generated phrases. The algorithmic composing software "Kinetic Music Machine" was used for this piece.)

Chart 8 gives an excerpt from a chart made by William Sethares as to which pitches in which equal tempered scale most closely approximate a given harmonic. Chart 9 shows the frequencies used in one of the timbres of the piece, the tone with its partials tuned to 19 tones/octave.

### Chart 8:

Equal tempered scale timbre construction (from a chart by William Sethares)

*Structures of each timbre:*

num of steps per oct. — partial number (pitch number of scale to use)

| tuning: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 16: | 0 | 16 | 25 | 32 | 37 | 41 | 45 | 48 | 51 | 53 | 55 | 57 |
| 17: | 0 | 17 | 27 | 34 | 39 | 44 | 48 | 51 | 54 | 56 | 59 | 61 |
| 18: | 0 | 18 | 29 | 36 | 42 | 47 | 51 | 54 | 57 | 60 | 62 | 65 |
| 19: | 0 | 19 | 30 | 38 | 44 | 49 | 53 | 57 | 60 | 63 | 66 | 68 |

**Chart 9:**

Example of frequencies used in one tone: Tone with partials tuned to pitches of 19 tones/oct.

```
Fundamental = partial 1 = 261.6255
              partial 2 = 523.2510
              partial 3 = 781.6108
              partial 4 = 1046.502
              partial 5 = 1302.5733
              partial 6 = 1563.2217
              partial 7 = 1808.8199
              partial 8 = 2093.004
              partial 9 = 2335.0764
              partial 10 = 2605.1466
              partial 11 = 2906.4524
              partial 12 = 3126.4435
```

Sound example 15: Tone with partials tuned in 16-tone tuning.
Sound example 16: Tone with partials tuned in 17-tone tuning.
Sound example 17: Tone with partials tuned in 18-tone tuning.
Sound example 18: Tone with partials tuned in 19-tone tuning.
Sound example 19: Arpeggio, then chord, 16-tone scale, 16-tone timbre.
Sound example 20: Arpeggio, then chord, 16-tone scale, 17-tone timbre.
Sound example 21: Arpeggio, then chord, 16-tone scale, 18-tone timbre.
Sound example 22: Arpeggio, then chord, 16-tone scale, 19-tone timbre.
Sound example 23: Fragment of "For Roman Verostko."

## 6. REMEMBERING GRIFFES II: AN EVOCATION, SPECULATING, AS IF. (1995)

for Buchla Lightning, computer, synthesizer.

In this piece, I was using algorithmic composing programs (Godfried-Michael Koenig's "Project 1," and John Dunn's "Kinetic Music Machine" to speculate about what kind of music the American modernist musical pioneer Charles Tomlinson Griffes (1885-1920) might have written if he had lived into the 1960s, instead of dying in 1920. Joseph Yasser's ideas as to how to divide the 19 tone equal tempered scale into "diatonic" and "chromatic" modes, which were developed in the 1920s and 30s, seemed particularly relevant and useful. One could have a series of chromatic pitch sets which were similar in structure, but which would be related "modally" to each other in the way that the "white key" modes on the piano are. Since Griffes had used "white key" modes in his impressionist works, my speculation was this: had he encountered Yasser's ideas, and extended his harmonic thinking into the 19 tone scale, he would have been attracted to the idea of 19's "chromatic modes" as a way of combining his modal thinking with his increasingly dense chromaticism.

In Yasser's theory, the 19 tone equal tempered scale is derived from a chain of (flattened - 694.7 cents) fifths. The first 12 elements of this chain form a 12 note prime set, while the last 7 notes of this chain form a 7 note secondary set. The 12 note set can function very similarly to the 12 tone "chromatic" scale, while the 7 note set resembles greatly the 7 note "diatonic" scale of 12-tone tuning.

Further, the 12 note prime set can be divided up into a 7 note "white key" set and a 5 note "black key" set. (The structure of the 7 note secondary set is identical to the 7 note "white key" subset of the 12 note prime set.) So within the 19 tones you can have a plethora of "chromatic," "diatonic," and "pentatonic" sets, all of which can have complete modulatory freedom.

In this piece, 7 different 12 note sets are used, each with their fundamentals being one of the 7 note secondary set. In each of sections 1 to 7 a different scale is used, while sections 8 to 14 reverse the order of these scales. Chart 10 shows in extremely condensed form Yassers derivation of structure in his scales. Chart 11 shows the order of scales used in the piece.

## Chart 10:

Derivation of Yasser's partitioning of the 19 tone scale.

Going around the circle of (694.7cent) fifths - that is, advancing 11 scale steps at a time:

*Scale degrees*

```
0  11  3  14  6  17  9 || 1  12  4  15  7 (12 note prime set)
18  10  2  13  5  16  8  (7 note secondary set)
```

*12 note scale in linear order:*

```
Scale degrees: 0  1  3  4  6  7  9  11  12  14  15  17
Steps between:  1  2  1  2  1  2  2   1   2   1   2   2
```

```
Broken into 7 "white" + 5 "black"
"White" 0  3  6  9  11   14   17
         3  3  3  2   3    3    2 (compare with steps in 12t)
"Black"   1  4  7        12   15
           3  3  5         3    5   (compare with steps in 12t)
```

*Structure of the 7 note "secondary" set:*

```
Scale degrees: 2  5  8  10  13  16  18
Steps between:  3  3  2   3   3   2   3  (a diff. "mode")
```

## Chart 11:

Order of scales in "Remembering Griffes II." Each 12 note "prime" set begins on one of the pitches of the 7 note "secondary" set.

| Sects: | 1,14 | 2,13 | 3,12 | 4,11 | 5,10 | 6,9 | 7,8 |
|---|---|---|---|---|---|---|---|
| Pitch | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| nums: | 17 | 17 | 17 | 17 | 18 | 17 | -18- |
| | 16 | 16 | 15 | 16 | 16 | -16- | 16 |
| | 14 | 14 | 14 | 14 | 15 | 14 | 14 |
| | 13 | 12 | 12 | 13 | 13 | 12 | 13 |
| | 11 | 11 | 11 | 11 | -12- | 11 | 11 |
| | 9 | 9 | 9 | -10- | 10 | 9 | 10 |
| | 8 | 8 | -8- | 8 | 8 | 8 | 8 |
| | 6 | 6 | 6 | 6 | 7 | 6 | 6 |
| | 5 | -5- | 4 | 5 | 5 | 4 | 5 |
| | 3 | 3 | 3 | 3 | 4 | 3 | 3 |
| -fdn- | -2- | 1 | 1 | 2 | 2 | 1 | 2 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(Note that this structure contains a MISTKAE! Sections 5 and10 should use a scale based on tone 13 (0 1 3 5 6 8 9 11 13 14 16 17 0) and not 12, as shown here. But the piece is finished, and it's too late to change it. Too bad!)

Sound example 24: 19 tone scale with piano timbre
Sound example 25: 13 note set excerpted from that.
Sound example 26: 7 note diatonic set, then 5 note pentatonic set derived from 12 note scale of ex. 25, then repeated 7 note diatonic set, with mistakes.
Sound example 27: Fragment from "Remembering Griffes II"

## Conclusion

Clearly, although this work is exhaustive, it is only scratching the surface of what can be done with these many new tunings. Most recently my researches have been directing me to the incredibly fertile work of Ervin Wilson. A first exploration of one of his many methods of generating scales, the "combination-product" method, is the subject of a recent two-part paper of mine, "Adventures in Scale Generation Along a Wilsonian Path," currently being published in "1/1," the journal of the Just Intonation Network. In this paper, I discuss the combination-product method of generating scales, along with two recent compositions, "Vingt Enflures Sur l'Enfant Melvin" and "Three Cat Laxative Sonatas," made using them. Prepublication copies of this paper, which covers material not covered here, can be obtained from me on request.

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