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SUMMER 1996

Volume 9, Number 3

THE JOURNAL OF THE JUST INTONATION NETWORK

A PROJECT OF OTHER MUSIC, INC.

Adventures In Scale Generation Along a Wilsonian Path, Part 1

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In my article "An Interesting Group of Combination-Product Sets Produces Some Very Nice Dissonances" (**1/1** 9:1, March 1995), I said that John Chalmers and Ervin Wilson called the technique of multiplying sets of numbers against each other making a combination-product set. Shortly after the article appeared, I received a charming note from Erv Wilson, stating that what I had described was a cross set, rather than a combination-product set.

I was intrigued, and on a visit to Los Angeles in August '95, visited Erv and asked him to explain to me what the difference was. It was one of the most information packed two-hour visits I've ever had. Erv showed me his instruments, including a delicious glass marimba tuned to 31-tone equal temperament (I want to make one!), and talked in detail about his ideas. His amazingly quick mind frequently left me far behind, and I often had to ask him for clarifications and simplifications. I came to realize that what the tuning community badly needed was a "User's Guide to Erv Wilson," which would explain simply the basics of his theories, with examples, charts, and the like. Unfortunately, such an undertaking is too much for me at the moment. Perhaps assembling an anthology of writings about his theories might be a good place to start. (A good introduction to some aspects of Wilson's thinking is contained in Paul Rapoport's article "Just Shape, Nothing Central" in *Musicworks* #60, Fall 1994.)

Erv explained to me that the difference between a combination-product set and a cross set was defined in elementary set theory:

A combination-product set takes all the unique, non-overlapping pairs of one set, and multiplies them.

So the set 2, 3, 5, 7, 11 has the combination-product set of

2 × 3	3 × 5	5 × 7	7 × 11
2 × 5	3 × 7	5 × 11	
2 × 7	3 × 11		
2 × 11			

(note: 3 × 2 would duplicate 2 × 3, and you can't have, for example, 3 × 3, because there aren't two 3s in the source set.)

This gives ten resulting elements, a combination that Erv calls a *dekany*, from the Greek for "ten."

A cross set takes all the products of all the pairs of two sets. So when I multiply (as I did in my article) 2, 3, 5, 7, 11 by 5, 7, 11, the second 5, 7, 11, is considered a separate set.

Taking all the products of those two sets yields

5 × 2	7 × 2	11 × 2
5 × 3	7 × 3	11 × 3
5 × 5	7 × 7	11 × 11
5 × 7	7 × 11	
5 × 11		

(7 × 5 already exists as 5 × 7, as do 11 × 5 and 11 × 7.) This gives twelve resulting elements, which formed the scales I described in my article.

It is worth noting that nine elements of the combination-product set dekany, all but the 2 × 3, are in my

(text continued on page 6)

(*Adventures...*, *continued from page 1*)
twelve-element scale. So combination-product sets and cross sets are related, but not quite the same thing.[1]

The combination-product set described above takes every possible combination of two elements from a source set of five. This is called a 2/5 CPS. Many possible combination-product sets exist. For example, a 3/5 CPS also gives a set of ten elements, while a 2/6 CPS gives a set of fifteen elements. A 3/6 CPS gives a set of twenty elements, which Erv calls an *eikosany* (from *eikos*, the Greek for twenty), and the 4/8 CPS yields the seventy-element monstrosity called the *hebdomekontany* (derived from the Greek for seventy). With each element of the hebdomekontany being made of four of eight possible factors, the number of relationships contained in such a set is truly mind boggling. Before plunging into a labyrinth such as that, I decided to explore the more familiar shores of smaller sets of numbers.

It is worth pointing out here that any numbers can be used to make a CPS. In fact, they don't even need to be numbers —set theory deals with any kind of elements. Numbers just happen to be very useful for our purposes. (Although maybe some composer will prove me wrong by making a piece based on a 2/6 combination-product set of Jell-O, a trout, the score of a Schubert string quartet, a quart of garlic and cabbage flavored ice cream, a postage stamp, and a garden slug.)

It's also worth pointing out that a combination-product set will reflect the characteristics of its source set. That is, if your source set has the numbers three and two in it, any 2/n CPS made with it will have 3:2s in it. If you wish to avoid 3:2s in your resulting set, avoiding threes and twos in your source set is a good place to start.

For example, the 2/5 CPS of 2, 3, 4, 5, 6

2 × 3	3 × 4	4 × 5	5 × 6
2 × 4	3 × 5	4 × 6	
2 × 5	3 × 6		
2 × 6			

equals

6	12	20	30
8	15	24	
10	18		
12			

Taking six, for example, as our denominator (we could use any element in the set), and multiplying all denominators to place the ratios within an octave, we get:

6/6	12/12	20/12	30/24
8/6	15/12	24/24	
10/6	18/12		
12/12			

which reduces to

1/1	1/1	5/3	5/4
4/3	5/4	1/1	
5/3	3/2		
1/1			

Note the presence of 4/3 and 3/2 in the resulting set. Note also that because we had two pairs of elements in our source set that were related by a 2:1 (that is, 2 and 4, 3 and 6), we also have a number of 1/1s in our resulting dekany. If you want to get ten unique elements in your dekany, you'll need a source set that doesn't have factors of 2:1, 4:1, or the like. This is one reason for using sets of prime numbers as your source set.

On the same trip down the U.S. West Coast, I also visited Brian McLaren in Oregon. While I was there, he gave me a series of charts, computer realizations of 4/8 CPS hebdomekontanies he had made. These were printed out in cents, Yamaha TX81Z tuning units, Yamaha DX7 II tuning units, and Ensoniq format (that is, C + n cents, C# + n cents, and so on). His source sets were varied. One was the first eight primes (1, 3, 5, 7, 11, 13, 17, 19), another was every tenth prime (1, 31, 73, 127, 179, 233, 283, 353), another every other odd number (1, 5, 9, 13, 17, 21, 25, 29), and so on.

Following his lead, but wishing to keep things manageably small (someday I'll be ready for the relational Everest of the hebdomekontany), I decided to construct a number of dekanies using the ascending series of "primeth primes," tune them up on my EPS, and hear what they sounded like. I decided to do this also because I already knew that if my source sets had small-number consonant relationships in them, my dekanies would also. I knew what a set like, say, 1, 3, 5, 7, 11 would generate. I was curious to see what other sets would produce.

Taking the prime number series 2, 3, 5, 7, 11, 13, 17, 19...; take the second, third, fifth, seventh, and eleventh primes as one set. Take the third, fifth, seventh, eleventh, and thirteenth primes as another; the fifth, seventh, eleventh, thirteenth, and seventeenth primes as another, and so on. (I'm not counting 1 as a prime, and anyway, its presence is implied by having the element 2 in my source set.)

The first source set (the second, third, fifth, seventh, and eleventh primes starting with 2) is 3, 5, 11, 17, 31. The 2/5 CPS of that set is

3 × 5	5 × 11	11 × 17	17 × 31
3 × 11	5 × 17	11 × 31	
3 × 17	5 × 31		
3 × 31			

which gives

15	55	187	527
33	85	341	
51	155		
93			

Putting all these into a row gives:
15, 33, 51, 93, 55, 85, 155, 187, 341, 527.

Multiplying each of these by 2, again and again, until they all are within a 2:1 of the largest element, 527, gives us
480, 528, 816, 744, 880, 680, 620, 748, 682, 527, which, when put into ascending order, gives us
480, 527, 528, 620, 680, 682, 744, 748, 816, 880.

Treating each of these as a ratio, with 480 as the denominator, (that is, 480/480, 527/480, 528/480, 620/480, and so on) give us, in cents:
0, 162, 165, 443, 603, 608, 759, 768, 919, 1049

I was delighted with this resulting scale. Like the cross-product sets of my previous article, they gave some very small musical intervals, mixed with some larger ones. And I found, like Paul Rapoport did in his article, that these dekanies were delightfully asymmetric. (Some might say that these small intervals are "not musically useful." I disagree, as I find, for example, that a trill of 603 and 608 cents with a piano timbre is definitely different in sound than a trill on two pitches tuned identically.)

Here is a chart of five dekanies I made in this way. I've only used these in one piece so far, in a piece called *"Fugue" and Preludes, and Three Cat Laxative Sonatas (Music for Microtonal Piano Sounds, Part 5)*, which uses a composing program I wrote that generated symmetrical melodies, doubled with their inversions. In the first movement, "Fugue," the program controlled the EPS sampler with piano timbres, and the result was put through a long delay line with feedback, resulting in each melody being repeated about seven times at an interval of five seconds. This made a very fast, thick music; I used each of these dekanies for a thirty-second section of the piece. This is hardly a context that allows these scales to be heard individually, with their own colors, but in "Fugue," I was only looking for a sense of rapidly changing, dissonant harmonic color. However, I've been improvising every day with these tunings, so a piece that more intensively investigates their individual characters probably isn't far off.

Because these scales share overlapping elements in their source sets, they also have some overlapping members. This effect, though, is mitigated by my taking the lowest value element of each dekany as its "fundamental," and reckoning scale values in cents, from it, as if it were a 1/1.

Every Primeth Prime, Ascending Series, beginning on 2:

1: Source Set: 3 5 11 17 31
Dekany: 15 33 51 93 55 85 155 187 341 527
Cents: 0 162 165 443 603 608 759 768 919 1049

2: Source Set: 5 11 17 31 41
Dekany: 55 85 155 205 187 341 451 527 697 1271
Cents: 0 43 312 594 636 754 784 796 919 1078

3: Source Set: 11 17 31 41 59
Dekany: 187 341 451 649 527 697 1003 1271 1829 2419
Cents: 0 324 348 508 594 832 918 954 1040 1078

4: Source Set: 17 31 41 59 67
Dekany: 527 697 1003 1139 1271 1829 2077 2419 2747 3953
Cents: 0 134 238 324 458 484 954 1088 1114 1174

5: Source Set: 31 41 59 67 83
Dekany: 1271 1829 2077 2573 2419 2747 3403 3953 4897 5561
Cents: 0 21 134 155 505 630 764 850 1114 1135

With these last scales, we may be getting away from the idea that Just Intonation uses the "smallest integers consonant with a given aesthetic purpose," unless our aesthetic purpose is, perhaps, to hear the sort of sounds those large integers produce. But one may then come to like these pungent sounds. I like to think of these higher dekanies as jalapeño scales.

I also made three other sets of five dekanies like this. One set was based on similar ascending series of every "oddth" odd number, another on every primeth odd, and the last on every oddth prime. The scales all have a sort of "family relationship." They're all very dissonant, most have some very small (less than 30 cents) intervals, many have very large (more than 300 cents) intervals, and all are asymmetrical. They now form a vocabulary of scales that my daily improvisations are roaming through.

It is possible to expand a dekany, or other CPS, to create further resources. There are a number of ways of doing this. One is called stellation, and is explained in Paul Rapoport's article. Another would be to take every element in the CPS as both a numerator and a denominator in ratios with every other element on the set. This would result in a tonality diamond, such as Harry Partch or Augusto Novaro used, and would generate twice the number of scales as you have elements in your CPS. (Note that the tonality diamond is itself a cross set where the horizontal and vertical axes are inversionally related.) To explore this, I used the 2/5 CPS of the source set 2, 3, 5, 7, 11. This will yield scales with more low-integer, consonant ratios than the scales above, but the ratios of 11 and 7 will still provide some dissonance and piquancy.

The set: 2 3 5 7 11

The dekany:

2 × 3	3 × 5	5 × 7	7 × 11
2 × 5	3 × 7	5 × 11	
2 × 7	3 × 11		
2 × 11			

equals

6	15	35	77
10	21	55	
14	33		
22			

Placing these in a tonality diamond formation, and adjusting all terms so they fit within an octave, yields the results in Table 1.

Reading each column in both vertical and horizontal directions gives us twenty ten-element scales. Because certain ratios occur in several scales, however, there are only 51 unique tones within these twenty scales. So our original dekany can be expanded to a 51-element collection of ratios. Tables 2a and 2b show the scales resulting from the above, listed with both ratios and cents values. For convenience, I numbered the scales made from vertical rows 1V, 2V, etc., and the scales made from horizontal rows 1H, 2H, etc.

Again, I've made disks of piano timbres tuned to these scales for my EPS sampler, and have been improvising regularly with them. As with the dekanies shown earlier, I've only used these scales in one piece so far, the "Preludes" designed to follow (yes, follow) the "Fugue" mentioned earlier. The composing program I wrote was inspired by the work of computer graphic artist Roman Verostko. I made the program to make a sound analogy to his very elegant "scarab" program, which makes an endlessly varying series of small (3 inch square) symmetrical drawings. In my program, the computer generates a short motive of five notes. This motive is then made into a "gesture," consisting of the motive followed by its retrograde. Accompanying this in rhythmic unison is the inversion of the motive and its retrograde, at a randomly determined interval of transposition. Up to four of these little two-voice "gestures" are played simultaneously, each at its own tempo, to form a "phrase." Each phrase lasts less than ten sec-

Table 1

	x/6	x/10	x/14	x/22	x/15	x/21	x/33	x/35	x/55	x/77
6/x	1/1	6/5	12/7	12/11	8/5	8/7	16/11	48/35	96/55	96/77
10/x	5/3	1/1	10/7	20/11	4/3	40/21	40/33	8/7	16/11	80/77
14/x	7/6	7/5	1/1	14/11	28/15	4/3	56/33	8/5	56/55	16/11
22/x	11/6	11/10	11/7	1/1	22/15	22/21	4/3	44/35	8/5	8/7
15/x	5/4	3/2	15/14	15/11	1/1	10/7	20/11	12/7	12/11	120/77
21/x	7/4	21/20	3/2	21/11	7/5	1/1	14/11	6/5	84/55	12/11
33/x	11/8	33/20	33/28	3/2	11/10	11/7	1/1	66/35	6/5	12/7
35/x	35/24	7/4	5/4	35/22	7/6	5/3	35/33	1/1	14/11	20/11
55/x	55/48	11/8	55/28	5/4	11/6	55/42	5/3	11/7	1/1	10/7
77/x	77/48	77/40	11/8	7/4	77/60	11/6	7/6	11/10	7/5	1/1

Table 2a

1H: 6/x	1/1	12/11	8/7	6/5	96/77	48/35	16/11	8/5	12/7	96/55
Cents	0	151	231	316	382	547	649	814	933	964
2H: 10/x	1/1	80/77	8/7	40/33	4/3	10/7	16/11	5/3	20/11	40/21
Cents	0	66	231	333	498	617	649	884	1035	1116
3H: 14/x	1/1	56/55	7/6	14/11	4/3	7/5	16/11	8/5	56/33	28/15
Cents	0	31	267	418	498	583	649	814	916	1081
4H: 22/x	1/1	22/21	11/10	8/7	44/35	4/3	22/15	11/7	8/5	11/6
Cents	0	81	165	231	396	498	663	782	814	1049
5H: 15/x	1/1	15/14	12/11	5/4	15/11	10/7	3/2	120/77	12/7	20/11
Cents	0	119	151	386	537	617	702	768	933	1035
6H: 21/x	1/1	21/20	12/11	6/5	14/11	7/5	3/2	84/55	7/4	21/11
Cents	0	84	151	316	418	583	702	733	969	1119
7H: 33/x	1/1	11/10	33/28	6/5	11/8	3/2	11/7	33/20	12/7	66/35
Cents	0	165	284	316	551	702	782	867	933	1098
8H: 35/x	1/1	35/33	7/6	5/4	14/11	35/24	35/22	5/3	7/4	20/11
Cents	0	102	267	386	418	653	804	884	969	1035
9H: 55/x	1/1	55/48	5/4	55/42	11/8	10/7	11/7	5/3	11/6	55/28
Cents	0	236	386	467	551	617	782	884	1049	1169
10H: 77/x	1/1	11/10	7/6	77/60	11/8	7/5	77/48	7/4	11/6	77/40
Cents	0	165	267	432	551	583	818	969	1049	1134

Table 2b

1V: x/6	1/1	55/48	7/6	5/4	11/8	35/24	77/48	5/3	7/4	11/6
Cents	0	236	267	386	551	653	818	884	969	1049
2V: x/10	1/1	21/20	11/10	6/5	11/8	7/5	3/2	33/20	7/4	77/40
Cents	0	84	165	316	551	583	702	867	969	1134
3V: x/14	1/1	15/14	33/28	5/4	11/8	10/7	3/2	11/7	12/7	55/28
Cents	0	119	284	386	551	617	702	782	933	1169
4V: x/22	1/1	12/11	5/4	14/11	15/11	3/2	35/22	7/4	20/11	21/11
Cents	0	151	386	418	537	702	804	969	1035	1119
5V: x/15	1/1	11/10	7/6	77/60	4/3	7/5	22/15	8/5	11/6	28/15
Cents	0	165	267	432	498	583	663	814	1049	1081
6V: x/21	1/1	22/21	8/7	55/42	4/3	10/7	11/7	5/3	11/6	40/21
Cents	0	81	231	467	498	617	782	884	1049	1116
7V: x/33	1/1	35/33	7/6	40/33	14/11	4/3	16/11	5/3	56/33	20/11
Cents	0	102	267	333	418	498	649	884	916	1035
8V: x/35	1/1	11/10	8/7	6/5	44/35	48/35	11/7	8/5	12/7	66/35
Cents	0	165	231	316	396	547	782	814	933	1098
9V: x/55	1/1	56/55	12/11	6/5	14/11	7/5	16/11	84/55	8/5	96/55
Cents	0	31	151	316	418	583	649	733	814	964
10V: x/77	1/1	80/77	12/11	8/7	96/77	10/7	16/11	120/77	12/7	20/11
Cents	0	66	151	231	382	617	649	768	933	1035

onds, and is symmetrical as regards both rhythm and pitch. I use each of the scales above for three "phrases" before progressing on to the next.

As in "Fugue," this results, from one point of view, in compositional nonsense. Pitch symmetry is one of the best ways of defeating a sense of tonal motion and resolution, so any sense of resolution these scales might have is lost. And, as before, the scales change too rapidly to really allow one to hear the rich relationships each one possesses. However, I have no problem with creating nonsensical structures. I think they have as much to offer us aesthetically as so-called "sensible" structures. But seeing and making audible relationships inherent in a structure are among the joys of composition, so a series of pieces exposing these in a more easily comprehensible manner will probably result soon.

At this point, my head was swimming. A sensation of vertigo gripped me as I glanced into a yawning chasm of an ever expanding series of intonational resources. If any set of integers[2] can generate a CPS, and that CPS can be expanded with any number of techniques (stellation, tonality diamond, and so on) into a larger set of resources, then even sticking to lower prime numbers for one's source sets, there is an impossibly large number of scales that can be generated with these methods. And if one considers expanding a hebdomekontany by making a 70 × 70 tonality diamond with it, resulting in 140 different, but related 70-note sets.....

Here, I probably should have stopped. But one final question nagged at me. If the qualities of the tonality-diamond scales were determined by the relationships inherent in the source set, what would happen with a source set where, for example, all the elements were related by large dissonant intervals such as "out-of-tune" octaves and sevenths? Taking a source set starting with 53 and each subsequent prime closest to (2N) – 1 of the previous element in the set gave me the set 53, 103, 199, 397, 787. This source set has only large dissonant intervals inherent in it. The 2/5 CPS generates impossibly large numbers. For a Just Intonation predicated on small-integer ratios, this set is indeed heretical and a nonsense. However, secure in the knowledge that this set would generate sonic heresy, I decided to work the numbers out and see what happened.

The set: 53 103 199 397 787

The 2/5 CPS:

53 × 103	103 × 199	199 × 397	397 × 787
53 × 199	103 × 397	199 × 787	
53 × 397	103 × 787		
53 × 787			

gives:

5,459	20,497	79,003	312,439
10,547	40,891	156,613	
21,041	81,061		
41,711			

To avoid charts full of giant numbers (the ratio 312,439/156,613, in all its polydigital glory, for example), I will only present the chart of the ratios worked out to their values in cents (Table 3). It could be argued that nearest cents values are too crude to express the relationships between these large number intervals. That may be true, but for the sake of convenience (and to keep the heretical inaccuracy metaphor going), I'll use them here. Note that in this chart, if cents values fall below 1/1 = 0 cents, I've listed them as values of 1,100+ cents. This makes 1/1 the reference point for all the scales.

Here we have a set of twenty ten-note scales, with 51 unique elements, all of which fall within a 5/4 (or at least within 386 cents). At first glance, this would not seem to be musically useful, but in fact, this set immediately suggested a piece to me.

With all these small intervals, most of the relations between pitches would only produce beating. If a cluster of all ten notes of a scale is played simultaneously, a very complex beating sound, not quite a chord, but definitely not a single tone, results. If the tones of the cluster are added one by one, and then subtracted in the same way, the beating cluster grows and decays, creating a constantly changing series of rhythmic beats. Since there are twenty scales, this suggested a series of twenty of these swelling and decaying clusters, each one very subtly different from the one before.

I made a tone with only five partials, the frequencies of which were related to each other by the proportion 53:103:197:397:787. The relative loudness of the 5 partials was set to 80, 75, 55, 40, and 30 decibels. This gave a stable, non-beating tone, which I loaded into my sampler to play the above clusters.

Table 3

	x/5,459	x/10,547	x/21,041	x/41,711	x/20,497	x/40,891	x/81,061	x/79,003	x/156,613	x/312,439
5,459/x	0	60	64	80	110	114	129	174	189	193
10,547/x	1,140	0	4	20	50	54	69	114	129	133
21,041/x	1,136	1,196	0	16	46	50	65	110	125	129
41,711/x	1,120	1,180	1,184	0	30	34	49	94	109	113
20,497/x	1,090	1,150	1,154	1,170	0	4	19	64	79	83
40,891/x	1,086	1,146	1,150	1,166	1,196	0	15	60	75	79
81,061/x	1,071	1,131	1,135	1,151	1,181	1,185	0	45	60	64
79,003/x	1,026	1,086	1,090	1,106	1,136	1,140	1,155	0	15	19
156,613/x	1,011	1,071	1,075	1,091	1,121	1,125	1,140	1,185	0	4
312,439/x	1,007	1,067	1,071	1,087	1,117	1,121	1,136	1,181	1,196	0

Just before I began all this work, I had attended a performance, by pianist Michael Kieran Harvey, of Messiaen's vast and awesome *Vingt Regards Sur L'Enfant Jesus* (Twenty Adorations of the Infant Jesus). While thrilled to hear one of the compositional masterpieces of the twentieth century beautifully played, something nagged at me. Somehow, I felt a bit put off by what struck me as a wee bit of spiritual pretentiousness on Messiaen's part. Perhaps, I thought, *Vingt Regards* was not an "oeuvre fini," but part of an ongoing dialog. Having all these scales in sets of twenty was a nice coincidence. I could easily compose a set of twenty etudes using piano timbres, each one using one of these scales. But that seemed too conceptually glib, and structurally trite. But having twenty different sets of beating tone clusters made with a kind of intonational heresy was, on the other hand, irresistible.

The resulting compositional heresy is a twenty minute tape or live-sampler piece called *Vingt Enflures Sur L'Enfant Melvin*. "Enflures" means swelling, as in a bruise. My clusters throbbed like a bruise, swelling and decaying as notes were added and taken away. "Melvin" was a generic name commonly used in *Mad Magazine* in the 1950s for an annoying, slightly ridiculous child. I'm sure that if there's a composers' afterlife, Messiaen is going to clobber me for this, but, like Till Eulenspiegel, I'll just have to take my chances.

More seriously, however, on repeated listenings to *Vingt Enflures*, (and I kept it around as a piece, and not just a bit of research or a sketch, because I actually liked how it sounded) I am struck with how it does something I like very much: it lives on an aesthetic "edge." It's a piece that, for me, blurs boundaries. It has a drone-like nature, but the pitches change too rapidly for one to really settle into it as if it were truly a drone. On the other hand, it doesn't change enough for many people to consider it truly "musical." It's a piece that, to me, seems to live on a kind of perceptual border between "sound" and "music." If all my wanderings in this Wilsonian[3] universe have led me to this boundary, a place where I can actually learn something about the nature of my perception, then I think that these wanderings were well worth the effort. **1/1**

Notes:

1. On examining my article, I also noted that its charts, which both David Doty and I had proofread several times, still appeared with errors! The best laid plans...

In the chart on page 8, "first six minor keys (2 minor–13 minor)" at the bottom of column 5m, the number 3128 with 30 adjacent to it appears. These numbers shouldn't be there at all. Likewise in the 4th chart, on page 9, "second six minor keys (17 minor–37 minor)" under 23m in the cents column (6th from the top) the number 2021 appears with 786 adjacent to it. These two numbers also shouldn't be there.

2. In fact, as John Chalmers points out, the elements of a source set for a CPS don't even need to be integers. They could just as easily be complex, or imaginary numbers.

3. With his typical modesty, and scrupulous concern for historical accuracy, Erv Wilson, in a recent letter (January, 1996) , points out that knowledge of a multplicative basis for musical scales goes back (at least) to Leonhard Euler's 1739 *Tentamen novae theoriae musicae*. See Adriaan Fokker, *New Music with 31 Tones*, p. 20, Verlag fur systematische Musikwissenschaft, Bonn, 1975.

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