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Long Distance Composing for Computer Controlled Microtonal Acoustic Instruments.

Keywords

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Abstract

From mid 2004 until early 2005, I was involved in a project to compose works for a series of computer-controlled acoustic instruments, some of which are microtonal, built by Godfried Willem Raes and associates at the Logos Foundation in Gent, Belgium. However, I was in Wollongong. I composed for these works by long distance, using the internet, in a slow, non-real time manner. Further, I composed the music for these instruments using a series of over two dozen mathematical functions that I implemented for John Dunn's ArtWonk and SoftStep Windows algorithmic composing environments. The pieces then, are the product of two intercontinental collaborations, one with the software, another with the instruments. This paper briefly examines the instruments, the method of internet collaboration that occurred, the functions implemented in the software to compose the pieces, and the pieces themselves. It also looks in somewhat more depth at harmonic structures implemented in one of the pieces, and details of its composition.

Introduction

From mid 2004 until early 2005, I was involved in a project to compose works for a series of computer-controlled acoustic instruments, some of which are microtonal, built by Godfried Willem Raes and associates at the Logos Foundation in Gent, Belgium. However, I was in Wollongong. I composed for these works by long distance, using the internet, in a slow, non-real time manner. Further, I composed the music for these instruments using a series of over two dozen mathematical functions that I implemented for John Dunn's ArtWonk and

SoftStep Windows algorithmic composing environments. The pieces then, are the product of two intercontinental collaborations, one with the software, another with the instruments.

The Logos Foundation

The Logos Foundation is a research institution and concert-giving organization in Gent, Belgium, founded in 1969 by Godfried Willem Raes and Moniek Darge. It is funded by the Flemish state government and the City of Gent. Over 65 concerts of new music are given there each year. Since the mid-90s, one of the projects of Godfried Willem Raes has been designing and building computer controlled acoustic instruments. This began with a player piano mechanism, designed in collaboration with Trimpin, and has progressed from there to include percussion, organ, wind, brass and unique instruments. Some of the more unique instruments include Flex, a computer-controlled musical saw; Belly, an automated carillon; and So, a computer-controlled tuba. All the instruments are controlled by Midi, each on a unique Midi channel, and ensembles of the instruments can be set up. Godfried has been inviting composers to work with the instruments for several years. After a visit to the Logos Studio in December 2003, when Godfried gave us a demonstration of the instruments, my curiosity was aroused, and I began thinking about composing for this ensemble. I was most intrigued with two of the microtonal instruments, Puff, a unique quarter-tone organ which has a separate bellows for each pipe; and Tubi, a quarter-tone tubulong. I was also interested in Belly, the computer-controlled carillon, which uses a collection of found-object signal bells instead of tuned musical

bells, and I was also attracted to the computer-controlled piano.

The Working Method

The working method on the pieces was as follows: I would make a Midi-file of test sequences I would like to hear on an instrument. I would then email the Midi-file to Kristof Lauwers, Godfried's assistant at Logos. He would play the sequence, scaling parameters in the Midi-file, if need be, record it, and put an mp3 file of the recording on his website where I could download it. My test sequences included single notes at a variety of pitches and loudness levels so that I could load the timbres into a sampler and work directly with them. On finishing a sketch, I would email that to Kristof, who would then again place the recording on his website for me to download. If there were things I wanted revised, I would make corrections and we would repeat the cycle until the results satisfied me. Following this procedure, we have made six pieces.

Of course, if I had been in Gent, I could have worked interactively with the instruments, but part of Godfried's invitation involved seeing what would happen when these instruments were worked with at a distance. I have been working interactively in electronic music for over 30 years, where hearing the exact sound being produced in real time was an intrinsic part of the process, and in some ways, this seems like returning to the days of writing instrumental music, and then waiting to hear it performed. The difference is that in this case, the performers are robots, the score and recordings are sent over the internet, and I'm not constrained by the limitations of human performers (just the limitations and idiosyncrasies of the machines). I'm not a devotee of "cyberculture", so I don't think the addition of internet usage to any process makes all that much difference. I can see that the technological tools of this collaboration may be different, but structurally, I'm still a composer sending out a score for performance. And here I should convey a huge thank you to Kristof Lauwers, who has been as cheerful, helpful and responsive a collaborator as one could ask for.

The question might arise, are the performances on the instruments any different than with samples of the instruments? The answer is a resounding yes. Not only are the sounds of the acoustic instru-

ments more subtle than performances made with samples of them, but there is a space - a kind of room reverberation - around each individual sound, which just doesn't happen with sampled sounds. And then, the sampler is just too perfect - each of these instruments is experimental, and sometimes the response of the instruments is not 100% predictable. In my case, all of the instruments I chose to work with had idiosyncrasies of their own. For example Belly is made of found object bells, each with their own dynamic range. A midi velocity that would produce a mezzo forte on one bell would produce a deafeningly loud sound on another. The aluminium tubes of Tubi are mounted vertically and have a bit of "give" and swing to them, so sometimes, in rapid passages, an attack might be missed. And Puff has an individual bellows for each note, providing a rapid puff of air quite unlike that supplied to a normal organ - very delicate control of dynamics is called for to avoid (or get) overblowing. And the controllers for Puff click rather loudly, meaning that the mechanical noise of the instrument is an inherent part of writing for it. In fact, the instrument in which sampled sounds come closest to the sound of its acoustic counterpart is the piano, and even there, the sound of Logos' Kawai KG1 grand piano is considerably more mellow and rounded than any sampled piano I've encountered. Although the use of samples of the instruments gives me a good idea of what the piece will sound like, there are always surprises when I hear the mp3 file of the performance on the intended instruments.

New Composing Functions for ArtWonk

While working on these pieces, I was also involved with beta-testing and development of Fort Worth, Texas based John Dunn's latest algorithmic composing environment, ArtWonk. http://algoart.com For ArtWonk, I was developing a package of probability distributions and other semi-random function generators, as well as testing out various image reading aspects of the program. These functions were used in the pieces as I developed them. Later, I took the package and rewrote it so that it would also work in Dunn's earlier composing environment SoftStep, which I used to compose some of the later pieces in the series. These distributions were added to the already rich set of fractal, chaotic and random resources of the programs to provide a larger set of random resources than just the few standard distributions described by, for exam-

ple, Dodge and Jerse (Dodge and Jerse 1997) that seem to be available in a number of programs. For those interested the distributions developed include the Beta, Borel, Bradford (2 versions), Burr, Cauchy, Exponential, Extreme LB, Gaussian, Generalized Logistic, Gumbel, Laplace (Bilex), Linear, Pareto, Reciprocal, Triangle, and Weibull distributions; the tENT, Sine, and Logistic and Henon (2D) attractors; the Lehmer Function, a Shift Register Feedback function, and a 4 Variable Iterated Function System (IFS).

The Compositions

As stated earlier, six pieces have been written to date:
1) Belly, for Belly, a computer-controlled found-object carillon (dur: 7:33)
2) tENT aTTRACTOR for tENT No. 1, for computer-controlled piano (dur: 3:06)
3) Homage to Wyschnegradsky, for Tubi, a computer-controlled quarter-tone tubulong (dur: 5:24)
4) Probable Occurrences - in Layers, for computer-controlled piano (dur: 5:38)
5) Lehmer's Kookaburra, for Puff, a computer-controlled quarter-tone organ rank (dur: 3:48)
6) Beneath the Slopes of Mt. Corrimal, for Tubi, Puff, Belly and Piano (dur: 12:50)
Each of these pieces will now be described briefly.

1) Belly

While testing out the new graphics reading capabilities of ArtWonk 2.0, I made a series of small graphics with Mirek's Cellebration, a freeware cellular automata explorer. I wondered about the possibility of using these diagrams in the manner of drum-machine grids, where a black background would be silence, and a "live" square generated by the automata would be a sound. I decided to make a piece for Belly in this manner. A series of three CA diagrams, each 34 pixels high (one pixel for each of the 34 bells of Belly) by 146 pixels wide, was made, and these were used, forward and backward, and at different tempi, in making the various sections of the piece. All 34 bells were used, and I tried to scale velocities so that the dynamic response of each bell, which varies wildly from bell to bell, would produce as evenly balanced a mezzo-forte as possible. Unfortunately, Belly has some bells which even at their softest, only speak at ff. As a result, a Belly piece which uses all 34 bells, in which the probability of any bell occurring is about equal, will be loud. Very loud. In fact, even with velocity scaling, my piece, because of the ways the bells combine, is so loud that Kristof and Godfried decided that it can only be played outdoors, when

be played outdoors, when Logos does its summer outdoor events. As someone who crusades against ear-shredding volume in electronic and improvised music, I find this slightly embarrassing, albeit amusing, and I look forward to getting a recording of the piece in its proper outdoor setting. For now, I'll be contented with a recording of the piece made in an empty concert hall, with Kristof safely in the next room. However, in late May, I heard from Kristof that Belly had indeed been performed in an outdoor street theatre festival in Bruges, and that outdoors, it sounded very effective. I still await a recording of it in an outdoor environment.

2) tENT aTTRACTOR for tENT no. 1

One of the first families of functions I investigated while creating my package of random composing routines were the one-dimensional attractors. Of these, the Logistic attractor, also known as the Bifurcation diagram, or the Feigenbaum attractor is the best known. Much simpler than the Logistic attractor, and with much wilder results at some settings is the Tent attractor. (Peak and Frame, 1994) Like the Logistic attractor, this is a feedback equation, where the results of each generation are fed back into the equation as parameters for the next generation. The Tent equation is $x(next) = s *(0.5 - Abs[x(current) - 0.5])$, where s is a value between 1 and 2 and Abs indicates the absolute value of the expression in brackets. For values of S between 1 and 1.414 (the square root of two), regular or irregular alternations between two or more values occur. For values between 1.414 and 2.0, many kinds of random-like sequences, but with repeating elements are found, and when used, for example, to control pitch, produce melodies with many exciting patterns in them. In making the piece, I used two simultaneous versions of the Tent attractor, each controlling a piano over a separate pitch range, and at different tempi (in order to get polyrhythms), and dynamics. A button was placed on the control screen, and when this button was pushed, new values of S and X were generated, as well as new dynamics. Pushing this button can potentially throw the equation into completely different behaviour, producing a radically different kind of melody. With grim wartime humour, I labelled this button "Regime Change." Finally, I doubled each of the two voices in fourths - this gave a much richer harmonic sound and resulted in a wide variety of chords between the voices. The use of doubling in fourths or fifths also seems to change the piano timbre in slight ways, producing a timbre with more depth, to my ears at least. Finally, in homage to my friend, the Pittsburgh based multi-artform explorer tENTATIVELY, a cONVENIENCE, I changed the typography of the Tent at-

tractor to match the inverse typography of his name. It is now the tENT aTTRACTOR, and I am pleased to report that tENT does, indeed, find the output of this equation quite attractive.

3) Tubi and modal quarter-tone harmony

I next composed a piece for Tubi, the quarter-tone tuned tubulong. I would like to examine this piece in more detail. Tubi is made of aluminium tubes which are suspended upright and struck by solenoids. The instrument covers three octaves of pitches up from C = 525 Hz (Midi 72) in quarter tones. Midi notes 72-108 control the normal 12 tone pitches, while Midi notes 36 to 71 control the quarter-tone shifted pitches. A quite wide range volume and repetition rate can be obtained from this instrument. In this piece, I wanted to use 7 note subsets of the quarter tone scale as my basic harmonic material. The starting scale was a 7 note Moment of Symmetry scale made with a generating interval of 11 quarter-tone steps. This interval is 550 cents, which is only 1 cent flat of the 11/8 eleventh harmonic interval. This scale can be notated as follows. Scale step size is on the top line, while scale degrees are on the bottom line:

Step size (in scale steps) 7 2 2 7 2 2 2

Scale degrees (fdn = 0) 0 7 9 11 18 20 22 24

This 7 note mode, which has a vaguely "gamelanish" sound, can have 7 rotations. This is similar to the organization of the "white key" modes on the piano. When these seven modes are added together, they make a 13 note mode in 24 tone equal temperament. Surprisingly (or maybe not so surprisingly), this mode itself has Moment of Symmetry properties, that is, it has two and only two sizes of scale degrees. Not all 7 note MOS modes in 24-tone equal temperament exhibit this property. For example, the 7 note MOS mode of 4 3 4 3 4 3 3 steps can also be rotated to produce 7 modes, and the sum of these modes is a 13 tone mode, but the mode does not have MOS characteristics. Here is the 13 tone mode:

Step size 2 2 2 1 2 2 2 2 2 1 2 2 2

Scale deg. 0 2 4 6 7 9 11 13 15 17 18 20 22 24

This mode is one of the modes the Russian-French microtonal pioneer Ivan Wyschnegradsky called "diatonisee" modes. In his composition Vingt-quatre préludes dans tous les tons de l'échelle chromatique diatonisée à 13 sons, opus 22 (1934) [24 Preludes in all the tones of the chromatic scale diatonicized in 13 notes, opus 22] for two pianos in quarter-tones, he extracted a 13 tone mode from

the 24 tone scale, and made a series of etudes, each one using a rotation of that basic 13 tone mode as its pitch set. Wyschnegradsky was unaware of the term Moment of Symmetry, but his diatonicization technique, which refers to having a "diatonic-like" structure in the 13 tone modes he uses, (that is having only 2 step sizes in one's scale), is as clear and elegant an example of MOS thinking as exists in almost any microtonal music. The fact that my 7 note modes added up to his 13 note mode, and that both were MOS scales, seemed a neat enough coincidence to base a piece upon it.

Here is a chart of the 7 modes used in the piece. Each mode covers a 3 octave range. The numbers are scale degrees in a 24 tone scale (scale degrees 0-24 are the first octave, with 3x24 = 72 being the scale degrees for a three octave scale listing), beginning with the lowest note of tubi (Scale degree 0 = C = Midi 72 = 525Hz).

Mode 1: 0 7 9 11 18 20 22 24 31 33 35 42 44 46 48 55 57 59 66 68 70 72

Mode 2: 0 2 9 11 13 20 22 24 26 33 35 37 44 46 48 50 57 59 61 68 70 72

Mode 3: 0 2 4 11 13 15 22 24 26 28 35 37 39 46 48 50 52 59 61 63 70 72

Mode 4: 0 2 4 6 13 15 17 24 26 28 30 37 39 41 48 50 52 54 61 63 65 72

Mode 5: 0 2 4 11 13 15 17 24 26 28 35 37 39 41 48 50 52 59 61 63 65 72

Mode 6: 0 2 9 11 13 15 22 24 26 33 35 37 39 46 48 50 57 59 61 63 70 72

Mode 7: 0 7 9 11 13 15 20 24 31 33 35 37 44 46 48 55 57 59 61 68 70 72

Note that in this arrangement of the modes, no more than two pitches per octave change as one advances from mode to mode. If we used this ordering of the modes, we could get a sense of "modulation" which had minimal change from mode to mode, and a maximum amount of "common tones" for each change. The pitches which change as one advances from mode to mode are shown by underlining.

The piece was made with an interactive algorithmic process. I made a patch in SoftStep that would allow me to perform interactively, with samples for now, and eventually, with the real instrument in Gent. The patch allowed me three levels of control. First, I could change modes freely. There are seven "snapshots", each of which contains the pitch numbers for a particular mode. By changing snapshots, I can change which mode the music is playing in. Second, I can change the structure of the music, and the polyrhythms that are occuring. The piece consists of three voices, two of

which are monophonic, and a third plays dyads. This third line plays two voices, the second of which is always 5 modal degrees higher than the first. These "modal degrees" will, of course, be of different sizes, depending on which mode is being used, but conceptually at least, I think of this line as being doubled in "fifths" - fifths of some kind, at any rate.

Each line is made by controlling its pitch, duration and loudness with numbers chosen from 21 possible linear or triangle distribution random number distributions. These are probability distributions made in the simplest possible manner. A left-linear distribution (my term) is one that has more low values than high values. It's made by taking the lowest of N different random numbers chosen. There are 7 different distributions like this, ranging from the lowest of 2 random numbers, to the lowest of 8 random numbers. There are also 7 different right-linear distributions (again, my term), which, conversely, are simply the highest of N different random numbers, and have many more high values than low values. Finally, there are 8 triangle distributions, which have more values in the middle than at either end. These are made by taking the average of N different random numbers. There are 3 voices, and each uses three different probability distributions, one each for pitch, duration and loudness. The outputs of the functions are scaled to desired levels and then applied to the required parameter. There are 9 distributions required for this. When a Strobe button is pushed, a Multi-Rand module (which gives a unique result for each output chosen) with 9 different outputs selects 9 different numbers from a possible 21, and one of these 9 different results is used to select a probability distribution for each of the required 9 parameters (3 parameters each in 3 different voices). Pushing the Strobe button for probability distributions can produce radically differing musical textures, because not only is there a possibility for changing the steepness of the slope of a particular distribution, but also, the very ranges in which things are happening. For example, if a left-linear 2x distribution were controlling the pitch of a voice, there would be more low values than high in the pitches chosen. But if that were then to switch to a right-linear 8x distribution, suddenly there would be mostly very high pitches only, with only a very few occasional lower pitches happening unpredictably. If one considers rhythmic character, and loudness choices, and how a change of distribution would change those, one will realize that pushing the probability choice Strobe button can indeed radically change the musical texture happening.

Rhythm, or tempo is subject to a similar change. Each set of durations generated is multiplied by a number between 3 and 11. This transposes the rhythm of that voice into a tempo 1/n times as slow as the original tempo. Each time the Strobe button for rhythm is pushed, 3 different random numbers between 3 and 11 are chosen. One of each of these numbers is used as the multiplier for one of the three voices, resulting in the three lines having tempi related by whole number ratios. Polytemporal relations such as 8:10:7, or 3:7:11, or 5:3:4 are all possible, as are all the other combinations of 3 out of 8 elements. Given that a range of almost 2 temporal octaves (11:3) is covered by this control, it will be realized that the rhythm Strobe button can produce quite a radical change in musical texture as well.

The third and final layer of control is selecting how many voices are actually playing at any one time. Each of the four Midi Out modules used (one each for voices 1 and 2, and two for voice 3) has a disable/enable button on it, which can be clicked with the mouse, turning that voice off or on. In performance, I can select if I'm having anywhere from 0 to 4 voices occurring.

To make the piece, I practiced using the SoftStep patch controlling a sampler with tubi samples in it. I tried to get a sense of when it was appropriate to change modes, and a sense of when I should change probability distributions and polyrhythms, as well as selecting which lines, and which voices were sounding. When I thought I had developed an interesting form, I recorded my performance into a Midi file, sent that file to Kristof Lauwers, who recorded it and sent the result back to me. In this way, I was able to preserve as much as possible of my preferred interactive method of composing, despite the distance involved. I am very happy with the results, which combine a sense of polyrhythmic complexity with a sense of modal, developmental harmony that I enjoy.

4) Probable Occurrences, In Layers

The next piece was made for the piano. It was actually not composed in the first instance for Logos, but for a Disklavier concert that Jerome Joy was organizing for the Paris Autumn Festival. However, after the performance in Paris, the piece worked equally well with the piano at Logos. In this piece, I wanted to explore the idea of different voices progressing in different registers at different tempi. My desire for a complex sound reached its peak here. In Probable Occurrences - In Layers, I made an ArtWonk patch with four layers, each of which was in a different pitch register (low, mid-

low, mid-high, and high), and each of which was controlled with a different random function. The lowest register was controlled with the tENT aTTRACTOR. In fact four different tENT aTTRACTORs were used, one each for pitch, duration, velocities, and length of notes. Each of the other voices similarly used four instances of its function to control the different parameters. The mid-low register was controlled by the Burr Distribution, a random distribution with more low values than high values, but one which is highly shapable and controllable. The mid-high register was controlled by the Pareto Distribution, another distribution with more low values than high, but one with a totally different shape than the Burr Distribution. The highest register was controlled by the Sine Attractor, another one-dimensional non-linear attractor with results that are somewhere between the wildness of the tENT aTTRACTOR, and the more sedate randomness of the Logistic Attractor.

Further, each of these four voices are also controlled by two function generators, which produce slowly descending ramp waves. These control the overall tempo and loudness of each voice, so each voice also has within it a gradual acceleration and crescendo, which then snaps back to a slower, softer articulation at the end of the ramp cycle. For performance, there are only 4 controls: on-off buttons for each of the four voices. A successful performance results from the judicious choice of which registers to combine in real-time, bringing out different polyrhythmic and multi-registral textures as the performance progresses. A midi-file recording of one such performance produced the final piece.

5) Lehmer's Kookaburra

The Lehmer equation is a simple recursive function that is often used as the basis for pseudo-random number generation in much computer hardware and software. (Battey, 2004; Ames, 1992) However, depending on its input parameters, it can produce anything from a single repeating value to simple and not-so-simple repeating patterns, all the way up to equally weighted randomness. This piece was designed to exploit the changeable nature of this equation, producing musical patterns that ranged from simple repetition through repeating riffs to totally unpredictable random gestures. Two independent voices are used, one covering the entire pitch range of Puff, the other only the top half. Pitch, duration, and velocity of each note is controlled by the Lehmer equation. Additionally there are six very slowly moving independent ascending ramp generators. These control the parameters of the Lehmer equation, the range of durations used, and the range of pitches used. This

produces lines which snap to fast tempi and then gradually decelerate, while covering a wider and wider pitch range. Each of the two lines' tempi and pitch ranges change at different rates, however, so that nothing in the piece is ever in sync, but in a process of constant change and evolution, although characteristic gestures, such as snapping back from a wide pitch range to a single repeating pitch are heard frequently throughout the piece. The sound of Puff is quite jolly, and when rapidly repeated pitches are played in some registers, the result has some resemblance to the laugh of a kookaburra. Our house in Wollongong is surrounded by kookaburras, we hear their laughter all through the day, and far into the night. As a result, in addition to the structuring, there's also a lot of kookaburra that's gotten into this short piece.

6) Beneath the Slopes of Mt. Corrimal

After working with all these instruments individually, it was only natural to want to combine them. Many of the ideas used in the previous pieces went into "Beneath the Slopes of Mt. Corrimal," the final piece of the series. SoftStep has a probability module which enables you to draw any probability distribution you desire. Rather than the elegant found objects of the distributions I developed for the Algorithmic Arts programs, I decided here to sculpt distributions to the ensemble needs of each section of the piece. The piece is sectional, and in each section a different probability distribution controls the pitch, duration, tempi and velocities of (the softer bells of) Belly, Tubi and Puff. The Multi-Rand generator, which produces a series of unique (non-duplicated) outputs, was used for the piano. The piano played six voice chords, while the other instruments played single line melodies, each in a different tempo. In some of the sections, psycho-acoustic effects are explored. For example, in one of these, the piano plays high clusters (around high C above the treble clef) while Tubi and Puff play quarter-tone melodies using all the quarter-tone pitches available within the cluster the piano is playing in 12-tone tuning. Meanwhile, Belly is playing its bells that come closest to this cluster. The result is a quite delicious swirling and beating sound, as the quarter-tones of Tubi and Puff beat against the different piano and Belly pitches. In other sections, Tubi and Puff play modal scales made from randomly permuting a set of intervals including 4 quarter-tones, 2 major thirds, and 1 major second, which coincidentally, are the intervals that constitute the ancient Greek enharmonic modes, while the piano plays clusters in a completely different register and Belly plays found-object pitches which are somewhat similar to the quarter-tone scales used by Tubi and Puff. In still other sections, the full range of all the in-

struments and a slow tempo combine to make a fragmented colourful set of isolated timbres. In these sections the primacy of pitch perception seems to disappear, and I, at least, hear things mainly in terms of timbre. Over the course of the piece, I feel that a quite satisfying variety of modes of listening are explored, and that the piece quite neatly sums up ideas which I explored in the other works.

Future Developments

Future developments of the Logos instruments include QT, a quarter-tone organ under construction, Melauton, a computer-controlled melodica, and Ake, a computer-controlled accordion. Future software developments include further compositional exploration of the more than two dozen random functions I developed for Algorithmic Arts. It is my hope that I can travel to Gent sometime in the next year to work directly with these instruments under control of the functions that I have developed.

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