

**A CATALOG OF 12 NOTE SCALES MADE FROM ADDITIVE SEQUENCES OBTAINED FROM THE  
DIAGONALS OF NUMBER TRIANGLES  
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**PART 1: THE MT. MERU SCALES (generator 1, 1)**

Here's Pingala's Meru Prastala, or Pascal's Triangle (or a small bit of it)  
This triangle is generated by the seed 1,1.

```

      1
     1 1
    1 2 1
   1 3 3 1
  1 4 6 4 1
 1 5 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below. Note that because the triangle is symmetrical, there is no difference in the scales derived from left-leaning and right-leaning diagonals.

**Generator (1-1) Meru Scales**

**Meru (1-1) Scale A - 12 tones**

$$A_n = A_{n-2} + A_{n-1}$$

Seed string from triangle: 1, 1

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	305/256	303.199
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	89/64	570.880
7:	377/256	670.105
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	55/32	937.632
11:	233/128	1037.023
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 13/8 21/16 17/16 55/32 89/64 9/8 233/128 377/256 305/256

Subsets:

MOS 7 = 0 1 4 5 8 9 10 (C C# E F G# A Bb)

MOS 10 = all except 3 and 7 (no Eb or G)

**Meru (1-1) Scale B - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 1, 1, 1

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 1 1 1 2 3 4 6 9 13 19 28 41 60 88 129 189 277...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	129/128	13.473
2:	277/256	136.491
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	41/32	429.062
6:	11/8	551.318 undecimal semi-augmented fourth
7:	189/128	674.691
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 9/8 13/8 19/16 7/4 41/32 15/8 11/8 129/128 189/128 277/256

Subsets:

MOS 5: 0 3 4 8 9 (C D# E G# A)

MOS 7: 0 3 4 5 8 9 10 (C D# E F G# A Bb)

MOS 9: 0 3 4 5 6 8 9 10 11 (C D# E F F# G# A Bb B)

MOS 11: all except 2 (no D)

**Meru (1-1) Scale C - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 1, 0, 1

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$  : 1 0 1 1 1 2 2 3 4 5 7 9 12 16 21 28 37 49 65 86 114 151...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	9/8	203.910 major whole tone
3:	37/32	251.344 37th harmonic
4:	151/128	286.086
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	43/32	511.518
8:	3/2	701.955 perfect fifth
9:	49/32	737.652
10:	7/4	968.826 harmonic seventh
11:	57/32	999.468
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 9/8 21/16 37/32 49/32 65/64 43/32 57/32 151/128

Subsets:

MOS 5: 0 2 5 8 10 (C D F Ab Bb)

MOS 7: 0 2 3 5 6 8 10 (C D Eb F Gb Ab Bb)

MOS 12: all tones

**Meru (1-1) Scale D - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 1, 1, 1, 1

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 1 1 1 1 2 3 4 5 7 10 14 19 26 36 50 69 95 131 181...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	131/128	40.108
2:	69/64	130.229
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	181/128	599.815
7:	95/64	683.827
8:	3/2	701.955 perfect fifth
9:	25/16	772.627 classic augmented fifth
10:	13/8	840.528 tridecimal neutral sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 19/16 13/8 9/8 25/16 69/64 95/64 131/128 181/128

Subsets:

MOS 5: 0 4 5 8 11 (C E F G# B)

MOS 7: 0 3 4 5 8 10 11 (C D# E F G# Bb B)

MOS 9: 0 2 3 4 5 8 9 10 11 (C D D# E F G# A Bb B)

MOS 11: all except 6 (no F#)

**Meru (1-1) Scale E - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 1, 0, 0, 1

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 1 0 0 1 1 0 1 2 1 1 3 3 2 4 6 5 6 10 11 11 16 21 22 27 37 43 49 64 80 92 113 144

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	37/32	251.344 37th harmonic
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	43/32	511.518
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	49/32	737.652
10:	27/16	905.865 Pythagorean major sixth
11:	113/64	984.215
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 11/8 21/16 27/16 37/32 43/32 49/32 23/16 113/64 9/8

Subsets:

MOS 7: 0 2 3 4 6 8 10 (C D Eb E F# G# A#)

MOS 10: all except 1 and 11 (no C# or B)

**Meru (1-1) Scale F - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 1, 1, 1, 1, 1

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 1 1 1 1 1 2 3 4 5 6 8 11 15 20 26 34 45  
60 80 106 140 185 245...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	5/4	386.314 major third
4:	11/8	551.318 undecimal semi-augmented fourth
5:	45/32	590.224 diatonic tritone
6:	185/128	637.658
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	53/32	873.505
10:	15/8	1088.269 classic major seventh
11:	245/128	1123.966
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 11/8 15/8 13/8 17/16 45/32 53/32 35/32 185/128 245/128

Subsets:

MOS 5: 0 3 4 7 10 (C D# E G Bb)

MOS 7: 0 1 3 4 7 8 10 (C C# D# E G Ab Bb)

MOS 12: all tones

**Meru (1-1) Scale G - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 1, 0, 1, 0, 1

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 1 0 1 0 1 1 1 2 1 3 2 4 4 5 7 7 11 11 16  
18 23 29 34 45 52 68 81...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	81/64	407.820 Pythagorean major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 11/8 9/8 23/16 29/16 17/16 45/32 13/8 81/64

Subsets:

MOS 7: 0 2 3 5 7 8 10 (C D Eb F G Ab Bb)

MOS 10: all except 4 and 9 (no E or A)

**Meru (1-1) Scale H - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 1, 0, 0, 1, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 1 0 0 1 0 1 1 0 2 1 1 3 1 3 4 2 6 5 5 10  
7 11 15 12 21 22 23 36 34 44 58 57...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	7/4	968.826 harmonic seventh
9:	57/32	999.468
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 11/8 15/8 21/16 23/16 9/8 17/16 29/16 57/32

Subsets:

MOS 7: 0 3 4 5 7 8 11 (C D# E F G Ab B)

MOS 11: all except 9 (no A)

**Meru (1-1) Scale I - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 1, 0, 0, 0, 1

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 1 0 0 0 1 1 0 0 1 2 1 0 1 3 3 1 1 4 6 4  
2 5 10 10 6 7 15 20 16 13 22 35 36 29 35 57 71...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	71/64	179.697
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	13/8	840.528 tridecimal neutral sixth
8:	7/4	968.826 harmonic seventh
9:	57/32	999.468
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 15/8 13/8 11/8 35/32 9/8 29/16 57/32 71/64

Subsets:

MOS 5: 0 4 6 8 11 (C E F# G# B)

MOS 9: 0 1 3 4 5 6 7 8 11 (C C# Eb E F F# G Ab B)

**Meru (1-1) Scale J - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 1, 1, 1, 1, 1, 1

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 1 1 1 1 1 1 2 3 4 5 6 7 9 12 16 21 27 34  
43 55 71 92...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	71/64	179.697
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	43/32	511.518
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	27/16	905.865 Pythagorean major sixth
10:	55/32	937.632
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 9/8 21/16 27/16 17/16 43/32 55/32 71/64 23/16

Subsets:

MOS 5 = 0 3 4 8 11 (C D# E G# B)

MOS 8 = 0 1 3 4 5 8 9 11 (C C# D# E F G# A B)

MOS 11 = all except 7 (no G)

**Meru (1-1) Scale K - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 1, 0, 0, 0, 0, 1

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 1 0 0 0 0 1 1 0 0 0 1 2 1 0 0 1 3 3 1 0  
1 4 6 4 1 1 5 10 10 5 2 6 15 20 15 7 8 21 35 35 22 15 29 56 70 57 37 44  
85...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	37/32	251.344 37th harmonic
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	85/64	491.269
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	7/4	968.826 harmonic seventh
9:	57/32	999.468
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 15/8 7/4 21/16 35/32 11/8 29/16 57/32 37/32 85/64

Subsets:

MOS 5 = 0 3 7 8 11 (C Eb G Ab B)

MOS 6 = 0 3 4 7 8 11 (C D# E G Ab B)

MOS 11 = all except 5 (no F)

## PART 2: THE LUCAS HEIGHTS SCALES (generator 2, 1)

Here's the Lucas Triangle (or a small bit of it). The Lucas Triangle is generated by the seed 2, 1. This makes the triangle asymmetrical. Therefore, additive sequences derived from left-leaning and right-leaning diagonals will be different, as will the scales derived from them.

```

      1/2
      1 2
     1 3 2
    1 4 5 2
   1 5 7 9 2
  1 6 14 16 9 2
 1 7 20 30 25 11 2
1 8 27 50 55 36 13 2
1 9 35 78 105 91 49 15 2
1 10 44 114 183 196 140 64 17 2
```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

### Generator (2-1) Lucas Scales - Right Wing Versions

#### Lucas (2-1) Scale A Right - 12 tones

$$A_n = A_{n-2} + A_{n-1}$$

Seed string from triangle: 2, 1

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 2 1 3 4 7 11 18 29 47 76 123 199 322 521...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	521/512	30.167
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	161/128	397.100
5:	11/8	551.318 undecimal semi-augmented fourth
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	199/128	763.950
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	123/64	1131.017
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 11/8 9/8 29/16 47/32 19/16 123/64 199/128 161/128 521/512

Subsets:

MOS 7 = 0 2 5 6 7 9 10 (C D F F# G A Bb)

MOS 10 = all except 1 and 4 (no C# or E)

**Lucas (2-1) Scale B Right - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 2, 1, 1

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 2 1 1 3 4 5 8 12 17 25 37 54 79 116 170 249 365...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	37/32	251.344 37th harmonic
3:	79/64	364.537
4:	5/4	386.314 major third
5:	85/64	491.269
6:	365/256	614.103
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	27/16	905.865 Pythagorean major sixth
10:	29/16	1029.577 29th harmonic
11:	249/128	1152.002
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 17/16 25/16 37/32 27/16 79/64 29/16 89/64 249/128 365/256

Subsets:

MOS 5: 0 1 4 7 8 (C C# E G Ab)

MOS 7: 0 1 2 4 7 8 9 (C C# D E G Ab A)

MOS 9: 0 1 2 3 4 7 8 9 10 (C C# D Eb E G Ab A Bb)

MOS 11: all except 6 (no F#)

**Lucas (2-1) Scale C Right - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 2, 0, 1

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 2 0 1 2 1 3 3 4 6 7 10 13 17 23 30 40 53 70 93 123...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	5/4	386.314 major third
4:	23/16	628.274 23rd harmonic
5:	93/64	646.991
6:	3/2	701.955 perfect fifth
7:	13/8	840.528 tridecimal neutral sixth
8:	53/32	873.505
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	123/64	1131.017
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 17/16 23/16 15/8 53/32 35/32 93/64 123/64

Subsets:

MOS 5: 0 3 6 7 9 (C Eb F# G A)

MOS 7: 0 1 3 4 6 7 9 (C C# Eb E F# G A)

MOS 12: all tones



**Lucas (2-1) Scale D Right - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 2, 1, 1, 1

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 2 1 1 1 3 4 5 6 9 13 18 24 33 46 64 88  
121 167 231 319...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	9/8	203.910 major whole tone
3:	319/256	380.895
4:	5/4	386.314 major third
5:	167/128	460.445
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	231/128	1022.099
11:	121/64	1102.636
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 9/8 13/8 33/32 23/16 11/8 121/64 167/128 231/128 319/256

Subsets:

MOS 5: 0 2 4 8 9 (C D E G# A)  
MOS 7: 0 1 2 4 7 8 9 (C C# D E G G# A)  
MOS 9: 0 1 2 4 6 7 8 9 11 (C C# D E F# G G# A B)  
MOS 11: all except 3 (no Eb)

**Lucas (2-1) Scale E Right - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 2, 0, 0, 1

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 2 0 0 1 2 0 1 3 2 1 4 5 3 5 9 8 8 14 17  
16 22 31 33 38 53 64 71...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	71/64	179.697
4:	9/8	203.910 major whole tone
5:	19/16	297.513 19th harmonic
6:	5/4	386.314 major third
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	53/32	873.505
10:	7/4	968.826 harmonic seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 9/8 7/4 17/16 11/8 31/16 33/32 19/16 53/32 71/64

Subsets:

MOS 7: 0 2 4 6 7 8 10 (C D E F# G Ab Bb)  
MOS 10: all except 3 and 9 (no Eb or A)

**Lucas (2-1) Scale F Right - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 2, 1, 1, 1, 1

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 2 1 1 1 1 3 4 5 6 7 10 14 19 25 32 42 56  
75 100 132 174 230 305...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	75/64	274.582 classic augmented second
3:	19/16	297.513 19th harmonic
4:	305/256	303.199
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	87/64	531.532
8:	3/2	701.955 perfect fifth
9:	25/16	772.627 classic augmented fifth
10:	7/4	968.826 harmonic seventh
11:	115/64	1014.588
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 19/16 25/16 21/16 75/64 33/32 87/64 115/64 305/256

Subsets:

MOS 5: 0 3 5 8 10 (C Eb F Ab Bb)

MOS 7: 0 3 5 6 8 9 10 (C Eb F F# Ab A Bb)

MOS 12: all tones

**Lucas (2-1) Scale G Right - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 2, 0, 1, 0, 1

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 2 0 1 0 1 2 1 3 1 4 3 5 6 6 10 9 15 15  
21 25 30 40 45 61 70 91 110...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	45/32	590.224 diatonic tritone
6:	91/64	609.354
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	55/32	937.632
10:	15/8	1088.269 classic major seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 9/8 15/8 21/16 25/16 45/32 61/32 35/32 91/64 55/32

Subsets:

MOS 7: 0 2 3 4 7 8 10 (C D Eb E G Ab Bb)

MOS 10: all except 6 and 9 (no F# or A)

**Lucas (2-1) Scale H Right - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 2, 0, 0, 1, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 2 0 0 1 0 2 1 0 3 1 2 4 1 5 5 3 9 6 8 14  
9 17 20 17 31 29 34 51 46 65 80 80 116 126...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	17/16	104.955 17th harmonic
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	23/16	628.274 23rd harmonic
6:	3/2	701.955 perfect fifth
7:	51/32	806.910
8:	7/4	968.826 harmonic seventh
9:	29/16	1029.577 29th harmonic
10:	31/16	1145.036 31st harmonic
11:	63/32	1172.736 octave - septimal comma
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 9/8 7/4 17/16 31/16 29/16 51/32 23/16 65/64 63/32

Subsets:

MOS 7: 0 2 3 4 6 8 10 (C D Eb E F# Ab Bb)

MOS 11: all except 11 (no B)

**Lucas (2-1) Scale I Right - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 2, 0, 0, 0, 1

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 2 0 0 0 1 2 0 0 1 3 2 0 1 4 5 2 1 5 9 7  
3 6 14 16 10 9 20 30 26 19 29 50 56 45 48 79...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	19/16	297.513 19th harmonic
3:	79/64	364.537
4:	5/4	386.314 major third
5:	45/32	590.224 diatonic tritone
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 9/8 7/4 15/8 13/8 19/16 29/16 25/16 45/32 79/64

Subsets:

MOS 5: 0 4 6 9 11 (C E F# A B)

MOS 9: 0 2 4 6 7 8 9 10 11 (C D E F# G Ab A Bb B)

**Lucas (2-1) Scale J Right - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 2, 1, 1, 1, 1, 1

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 2 1 1 1 1 1 3 4 5 6 7 8 11 15 20 26 33  
41 52 67 87 113...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	67/64	79.307
3:	5/4	386.314 major third
4:	41/32	429.062
5:	87/64	531.532
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	113/64	984.215
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 11/8 15/8 13/8 33/32 41/32 67/64 87/64 113/64

Subsets:

MOS 5 = 0 3 6 7 9 (C Eb F# G A)

MOS 8 = 0 1 3 6 7 8 9 11 (C C# Eb F# G Ab A B)

MOS 11 = all except 10 (no Bb)

**Lucas (2-1) Scale K Right - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 2, 0, 0, 0, 0, 1

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 2 0 0 0 0 1 2 0 0 0 1 3 2 0 0 1 4 5 2 0  
1 5 9 7 2 1 6 14 16 9 3 7 20 30 25 12 10 27 50 55 37 22 37 77...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	37/32	251.344 37th harmonic
3:	77/64	320.144
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	27/16	905.865 Pythagorean major sixth
9:	55/32	937.632
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 9/8 7/4 15/8 25/16 27/16 55/32 37/32 11/8 77/64

Subsets:

MOS 5 = 0 1 4 6 10 (C C# E F# Bb)

MOS 6 = 0 1 4 6 10 11 (C C# E F# Bb B)

MOS 11 = all except 3 (no Eb)

## Generator (2,1) Lucas Scales - Left Wing Versions

### Lucas (2-1) Scale A Left - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 1, 2

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 1 2 3 5 8 13 21 34.....

**This is the Fibonacci sequence - it forms the same scale as Meru(1-1) Scale A. (Whether left wing or right wing is immaterial - the Meru(1-1) triangle is symmetrical.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	305/256	303.199
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	89/64	570.880
7:	377/256	670.105
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	55/32	937.632
11:	233/128	1037.023
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 13/8 21/16 17/16 55/32 89/64 9/8 233/128 377/256 305/256

Subsets:

MOS 7 = 0 1 4 5 8 9 10 (C C# E F G# A Bb)

MOS 10 = all except 3 and 7 (no Eb or G)

### Lucas (2-1) Scale B Left - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 1, 2, 2

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 1 2 2 3 5 7 10 15 22 32 47 69 101 148 217 318...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	69/64	130.229
2:	37/32	251.344 37th harmonic
3:	159/128	375.460
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	101/64	789.854
9:	217/128	913.861
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 15/8 11/8 47/16 69/64 101/64 37/16 217/128 159/128

Subsets:

MOS 5: 0 4 7 10 11 (C E G Bb B)

MOS 7: 0 4 5 6 7 10 11 (C E F F# G Bb B)

MOS 9: 0 1 4 5 6 7 8 10 11 (C C# E F F# G Ab Bb B)

MOS 11: all except 3 (no Eb)

**Lucas (2-1) Scale C Left - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 1, 0, 2

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 1 0 2 1 2 3 3 5 6 8 11 14 19 25 33 44 58 77 102 135...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	135/128	92.179 major chroma, major limma
3:	19/16	297.513 19th harmonic
4:	77/64	320.144
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	51/32	806.910
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 11/8 7/4 19/16 25/16 33/32 29/16 77/64 51/32 135/128

Subsets:

MOS 5: 0 5 6 7 10 (C F F# G Bb)

MOS 7: 0 3 5 6 7 8 10 (C Eb F F# G Ab Bb)

MOS 12: all tones

**Lucas (2-1) Scale D Left - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 1, 2, 2, 2

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 1 2 2 2 3 5 7 9 12 17 24 33 45 62 86 119 164 ...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	41/32	429.062
6:	43/32	511.518
7:	45/32	590.224 diatonic tritone
8:	3/2	701.955 perfect fifth
9:	7/4	968.826 harmonic seventh
10:	119/64	1073.781
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 9/8 17/16 33/32 45/32 31/16 43/32 119/64 41/32

Subsets:

MOS 5: 0 3 4 8 9 (C Eb E Ab A)

MOS 7: 0 1 2 3 4 8 9 (C C# D Eb E Ab A)

MOS 9: 0 1 2 3 4 7 8 9 11 (C C# D Eb E G Ab A B)

MOS 11: all except 5 (no F)

**Lucas (2-1) Scale E Left - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 1, 0, 0, 2

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 1 0 0 2 1 0 2 3 1 2 5 4 3 7 9 7 10 16 16  
17 26 32 33 43 58 65 76...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
3:	17/16	104.955 17th harmonic
4:	9/8	203.910 major whole tone
5:	19/16	297.513 19th harmonic
6:	5/4	386.314 major third
7:	43/32	511.518
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 9/8 17/16 13/8 33/32 43/32 29/16 65/64 19/16

Subsets:

MOS 7: 0 3 4 6 8 9 10 (C Eb E Gb Ab A Bb)

MOS 10: all except 1 and 5 (no C# or F)

**Lucas (2-1) Scale F Left - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 1, 2, 2, 2, 2

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 1 2 2 2 2 3 5 7 9 11 14 19 26 35 46 60  
79...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	79/64	364.537
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 9/8 11/8 19/16 13/8 35/32 23/16 15/8 79/64

Subsets:

MOS 5: 0 2 5 8 10 (C D F Ab Bb)

MOS 7: 0 2 3 5 6 8 10 (C D Eb F Gb Ab Bb)

MOS 12: all tones

**Lucas (2-1) Scale G Left - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 1, 0, 2, 0, 2

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 1 0 2 0 2 1 2 3 2 5 3 7 6 9 11 12 18 18  
27 29 39 47 57 74...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	37/32	251.344 37th harmonic
3:	39/32	342.483 39th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	27/16	905.865 Pythagorean major sixth
9:	7/4	968.826 harmonic seventh
10:	57/32	999.468
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 9/8 11/8 27/16 29/16 39/32 47/32 57/32 37/32

Subsets:

MOS 7: 0 1 4 5 7 8 9 (C C# E F G Ab A)

MOS 10: all except 2 and 10 (no D or Bb)

**Lucas (2-1) Scale H Left - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 1, 0, 0, 2, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 1 0 0 2 0 1 2 0 3 2 1 5 2 4 7 3 9 9 7 16  
12 16 25 19 32 37 35 57 56 67 94...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	37/32	251.344 37th harmonic
5:	19/16	297.513 19th harmonic
6:	5/4	386.314 major third
7:	47/32	665.507
8:	3/2	701.955 perfect fifth
9:	25/16	772.627 classic augmented fifth
10:	7/4	968.826 harmonic seventh
11:	57/32	999.468
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 9/8 25/16 19/16 37/32 35/32 57/32 67/64 47/32

Subsets:

MOS 7: 0 3 5 6 8 9 10 (C Eb F Gb Ab A Bb)

MOS 11: all except 7 (no G)



**Lucas (2-1) Scale I Left - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 1, 0, 0, 0, 2

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 1 0 0 0 2 1 0 0 2 3 1 0 2 5 4 11 2 7 9 5  
3 9 16 14 8 12 25 30 22 20 37 55 52 42...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	37/32	251.344 37th harmonic
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	55/32	937.632
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 11/8 7/4 9/8 25/16 15/8 37/32 55/32 13/8 21/16

Subsets:

MOS 5: 0 3 5 6 10 (C Eb F Gb Bb)

MOS 9: 0 1 2 3 5 6 7 10 11 (C C# D Eb F F# G Bb B)

**Lucas (2-1) Scale J Left - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 1 2 2 2 2 2 3 5 7 9 11 13 16 21 28 37 48 61 77  
98...

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 1 2 2 2 2 2...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	37/32	251.344 37th harmonic
3:	77/64	320.144
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	49/32	737.652
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 9/8 11/8 13/8 21/16 37/32 61/32 77/64 49/32

Subsets:

MOS 5 = 0 1 4 7 10 (C C# E G Bb)

MOS 8 = 0 1 4 5 6 7 9 10 (C C# E F F# G A Bb)

MOS 11 = all except 8 (no Ab)

**Lucas (2-1) Scale K Left - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 1, 0, 0, 0, 0, 2

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 1 0 0 0 0 2 1 0 0 0 2 3 1 0 0 2 5 4 1 0  
2 7 9 5 1 2 9 16 14 6 3 11 25 30 20 9 14 36 55 50 29 23 91...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	5/4	386.314 major third
3:	11/8	551.318 undecimal semi-augmented fourth
4:	91/64	609.354
5:	23/16	628.274 23rd harmonic
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	55/32	937.632
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 9/8 11/8 25/16 15/8 55/32 29/16 23/16 91/64

Subsets:

MOS 5 = 0 1 2 6 9 (C C# D F# A)

MOS 6 = 0 1 2 3 6 9 ( C C# D Eb F# A)

MOS 11 = all except 4 (no E)

### Part 3: The WYTHOFF SCALES (generator 3, 1)

Sloane's "On-Line Encyclopedia of Integer Sequences" states that "the 6th row in the Wythoff array begins with the 6th term of the sequence (14, 23, 37, 60, 97, 157, ...).  $a(n) = f(n-3) + f(n+2)$  for the Fibonacci numbers  $f(n) = f(n-1) + f(n-2)$ ;  $f(0) = 0$ ,  $f(1) = 1$ ." (Sloane 2005

<http://www.research.att.com/~njas/sequences/?q=3+1+4+5+9+14+23+37+60+97+157+254+411&language=english&go=Search>) Accordingly, I'm calling this

sequence the Wythoff sequence. Here's the Wythoff Triangle (or a small bit of it). The Wythoff Triangle is generated by the seed 3, 1. This makes the triangle asymmetrical. Therefore, additive sequences derived from left-leaning and right-leaning diagonals will be different, as will the scales derived from them.

```

      1/3
      1 3
      1 4 3
      1 5 7 3
      1 6 12 10 3
      1 7 18 22 13 3
      1 8 25 40 35 16 3
      1 9 33 65 75 51 19 3
      1 10 42 98 140 126 70 22 3
```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

#### Generator (3-1) Wythoff Scales - Right Wing Versions

##### Wythoff (3-1) Scale A Right - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 3, 1

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 3 1 4 5 9 14 23 37 60 97 157 254 411...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	37/32	251.344 37th harmonic
3:	157/128	353.545
4:	5/4	386.314 major third
5:	23/16	628.274 23rd harmonic
6:	3/2	701.955 perfect fifth
7:	97/64	719.895
8:	411/256	819.594
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	127/64	1186.422
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 9/8 7/4 23/16 37/32 15/8 97/64 157/128 127/64 411/256

Subsets:

MOS 7 = 0 1 2 4 5 6 9 (C C# D E F F# A)

MOS 10 = all except 11 and 8 (no G# or B)

**Wythoff (3-1) Scale B Right - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 3, 1, 1

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 3 1 1 4 5 6 10 15 21 31 46 67 98 144 211 309...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	9/8	203.910 major whole tone
3:	309/256	325.756
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	49/32	737.652
9:	211/128	865.319
10:	15/8	1088.269 classic major seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 15/8 21/16 31/16 23/16 67/64 49/32 9/8 211/128 309/256

Subsets:

MOS 5: 0 4 5 7 10 (C E F G Bb)

MOS 7: 0 4 5 6 7 10 11 (C E F F# G Bb B)

MOS 9: 0 1 4 5 6 7 8 10 11 (C C# E F F# G Ab Bb B)

MOS 11: all except 3 (no D#)

**Wythoff (3-1) Scale C Right - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 3, 0, 1

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 3 0 1 3 1 4 4 5 8 9 13 17 22 30 39 52 69 91 121...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	69/64	130.229
3:	9/8	203.910 major whole tone
4:	39/32	342.483 39th harmonic
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	91/64	609.354
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	15/8	1088.269 classic major seventh
11:	121/64	1102.636
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 9/8 13/8 17/16 11/8 15/8 39/32 69/64 91/64 121/64

Subsets:

MOS 5: 0 3 5 8 9 (C Eb F Ab A)

MOS 7: 0 1 3 5 6 8 9 (C Db Eb F Gb Ab A)

MOS 12: all tones

**Wythoff (3-1) Scale D Right - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 3, 1, 1, 1

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 3 1 1 1 4 5 6 7 11 16 22 29 40 56 78 107 147 203 281 388...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	281/256	161.312
2:	147/128	239.607
3:	39/32	342.483 39th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	97/64	719.895
8:	203/128	798.403
9:	107/64	889.760
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 11/8 29/16 39/32 107/64 147/128 203/128 281/256 97/64

Subsets:

MOS 5: 0 4 5 6 10 (C E F Gb Bb)

MOS 7: 0 3 4 5 6 10 11 (C D# E F Gb Bb B)

MOS 9: 0 2 3 4 5 6 9 10 11 (C D Eb E F F# A Bb B)

MOS 11: all except 7 (no G)

**Wythoff (3-1) Scale E Right - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 3, 0, 0, 1

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 3 0 0 1 3 0 1 4 3 1 5 7 4 6 12 11 10 18 23 21 28 41 44 49 69 85...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	69/64	130.229
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	41/32	429.062
5:	21/16	470.781 narrow fourth
6:	85/64	491.269
7:	11/8	551.318 undecimal semi-augmented fourth
8:	23/16	628.274 23rd harmonic
9:	3/2	701.955 perfect fifth
10:	49/32	737.652
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 11/8 9/8 23/16 21/16 41/32 49/32 69/64 85/64

Subsets:

MOS 7: 0 2 3 7 8 9 11 (C D Eb G Ab A B)

MOS 10: all except 1 and 6 (no C# or F#)

**Wythoff (3-1) Scale F Right - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 3, 1, 1, 1, 1

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 3 1 1 1 1 4 5 6 7 8 12 17 23 30 38 50 67 90 120 158...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	17/16	104.955 17th harmonic
3:	19/16	297.513 19th harmonic
4:	79/64	364.537
5:	5/4	386.314 major third
6:	45/32	590.224 diatonic tritone
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	25/16	772.627 classic augmented fifth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 17/16 23/16 15/8 19/16 25/16 67/64 45/32 79/64

Subsets:

MOS 5: 0 2 5 8 10 (C D F Ab Bb)

MOS 7: 0 2 5 7 8 10 11 (C D F G Ab Bb B)

MOS 12: all tones

**Wythoff (3-1) Scale G Right - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 3, 0, 1, 0, 1

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 3 0 1 0 1 3 1 4 1 5 4 6 8 7 13 11 19 19 26 32 37 51 56 77 88 114 139...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	139/128	142.729
2:	37/32	251.344 37th harmonic
3:	19/16	297.513 19th harmonic
4:	77/64	320.144
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	51/32	806.910
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	57/32	999.468
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 13/8 11/8 19/16 37/32 51/32 77/64 57/32 139/128

Subsets:

MOS 7: 0 2 3 5 6 7 9 (C D Eb F F# G A)

MOS 10: all except 1 and 11 (no C# or B)

**Wythoff (3-1) Scale H Right - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 3, 0, 0, 1, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 3 0 0 1 0 3 1 0 4 1 3 5 1 7 6 4 12 7 11  
18 11 23 25 22 41 36 45 66 58...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	41/32	429.062
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	25/16	772.627 classic augmented fifth
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 11/8 9/8 23/16 25/16 3 41/32 45/32 33/32 29/16

Subsets:

MOS 7: 0 2 3 5 7 8 10 ( C D Eb F G Ab Bb)

MOS 11: all except 11 (no B)

**Wythoff (3-1) Scale I Right - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 3, 0, 0, 0, 1

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 3 0 0 0 1 3 0 0 1 4 3 0 1 5 7 3 1 6 12  
10 4 7 18 22 14 11 25 40 36 25 36 65 76 61 61 101 141...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	141/128	167.462
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	101/64	789.854
10:	7/4	968.826 harmonic seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 11/8 25/16 65/64 19/16 61/32 101/64 141/128

Subsets:

MOS 5: 0 3 5 7 10 ( C Eb F G Bb)

MOS 9: 0 1 3 4 5 6 7 8 10 ( C Db Eb E F F# G Ab Bb)

**Wythoff (3-1) Scale J Right - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 3, 1, 1, 1, 1, 1

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 3 1 1 1 1 1 4 5 6 7 8 9 13 18 24 31 39  
48 61 79 103 134...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	9/8	203.910 major whole tone
3:	39/32	342.483 39th harmonic
4:	79/64	364.537
5:	5/4	386.314 major third
6:	3/2	701.955 perfect fifth
7:	103/64	823.801
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	61/32	1116.885
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 13/8 31/16 39/32 61/32 79/64 103/64 67/64

Subsets:

MOS 5 = 0 2 5 6 9 (C D F F# A)

MOS 8 = 0 2 3 5 6 8 9 11 (C D Eb F F# G# A B)

MOS 11 = all except 1 (no C#)

**Wythoff (3-1) Scale K Right - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 3, 0, 0, 0, 0, 1

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 3 0 0 0 0 1 3 0 0 0 1 4 3 0 0 1 5 7 3 0  
1 6 12 10 3 1 7 18 22 13 4 8 25 40 35 17 12 33 65...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
3:	17/16	104.955 17th harmonic
4:	35/32	155.140 septimal neutral second
5:	9/8	203.910 major whole tone
6:	5/4	386.314 major third
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	25/16	772.627 classic augmented fifth
10:	13/8	840.528 tridecimal neutral sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 11/8 13/8 25/16 35/32 17/16 33/32 65/64

Subsets:

MOS 5 = 0 5 6 8 11 (C F F# G# B)

MOS 6 = 0 5 6 7 8 11 (C F F# G Ab B)

MOS 11 = all except 1 (no C#)



## Generator (3-1) Wythoff Scales - Left Wing Versions

### Wythoff (3-1) Scale A Left - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 1, 3

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 1 3 4 7 11 18..

**This is the Lucas Series - the scale here is the same as the Lucas (2-1) Scale A Right.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	521/512	30.167
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	161/128	397.100
5:	11/8	551.318 undecimal semi-augmented fourth
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	199/128	763.950
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	123/64	1131.017
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 11/8 9/8 29/16 47/32 19/16 123/64 199/128 161/128 521/512

Subsets:

MOS 7 = 0 2 5 6 7 9 10 (C D F F# G A Bb)

MOS 10 = all except 1 and 4 (no C# or E)

### Wythoff (3-1) Scale B Left - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 1, 3, 3

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 1 3 3 4 7 10 14 21 31 45 66 97 142 208 305...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	71/64	179.697
3:	305/256	303.199
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	45/32	590.224 diatonic tritone
7:	3/2	701.955 perfect fifth
8:	97/64	719.895
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 21/16 31/16 45/32 33/32 97/64 71/64 13/8 305/256

Subsets:

MOS 5: 0 4 5 7 10 (C E F G Bb)

MOS 7: 0 4 5 6 7 10 11 (C E F F# G Bb B)

MOS 9: 0 1 4 5 6 7 8 10 11 (C C# E F F# G Ab Bb B)

MOS 11: all except 3 (no D#)

**Wythoff (3-1) Scale C Left - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 1, 0, 3

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 1 0 3 1 3 4 4 7 8 11 15 19 26 34 45 60  
79 105 139...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	139/128	142.729
3:	19/16	297.513 19th harmonic
4:	79/64	364.537
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	105/64	857.095 septimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 11/8 15/8 19/16 13/8 17/16 45/32 79/64 105/64 139/128

Subsets:

MOS 5: 0 5 7 10 11 (C F G Bb B)

MOS 7: 0 3 5 7 8 10 11 (C Eb F G Ab Bb B)

MOS 12: all tones

**Wythoff (3-1) Scale D Left - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 1, 3, 3, 3

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 1 3 3 3 4 7 10 13 17 24 34 47 64 88 122  
169 233 321 ...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	5/4	386.314 major third
3:	321/256	391.715
4:	169/128	481.055
5:	11/8	551.318 undecimal semi-augmented fourth
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	233/128	1037.023
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 17/16 47/32 11/8 61/32 169/128 233/128 321/256

Subsets:

MOS 5: 0 2 7 8 9 (C D G Ab A)

MOS 7: 0 1 2 6 7 8 9 (C C# D F# G Ab A)

MOS 9: 0 1 2 5 6 7 8 9 11 (C C# D F F# G Ab A B)

MOS 11: all except 3 (no Eb)

**Wythoff (3-1) Scale E Left - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 1, 0, 0, 3

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 1 0 0 3 1 0 3 4 1 3 7 5 4 10 12 9 14 22  
21 23 36 43 44 59 79 87...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	79/64	364.537
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	43/32	511.518
6:	87/64	531.532
7:	11/8	551.318 undecimal semi-augmented fourth
8:	23/16	628.274 23rd harmonic
9:	3/2	701.955 perfect fifth
10:	7/4	968.826 harmonic seventh
11:	59/32	1059.172
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 9/8 11/8 21/16 23/16 43/32 59/32 79/64 87/64

Subsets:

MOS 7: 0 1 3 4 7 9 10 (C C# Eb E G A Bb)

MOS 10: all except 2 and 6 (no D or F#)

**Wythoff (3-1) Scale F Left - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 1, 3, 3, 3, 3

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 1 3 3 3 3 4 7 10 13 16 20 27 37 50 66 86  
113 150...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	37/32	251.344 37th harmonic
3:	75/64	274.582 classic augmented second
4:	5/4	386.314 major third
5:	43/32	511.518
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	113/64	984.215
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 27/16 37/32 25/16 33/32 43/32 113/64 75/64

Subsets:

MOS 5: 0 4 6 8 10 (C E Gb Ab Bb)

MOS 7: 0 2 4 6 8 9 10 (C D E Gb Ab A Bb)

MOS 12: all tones

**Wythoff (3-1) Scale G Left - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 1, 0, 3, 0, 3

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 1 0 3 0 3 1 3 4 3 7 4 10 8 13 15 17 25  
25 38 40 55 65 80 103...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	17/16	104.955 17th harmonic
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	3/2	701.955 perfect fifth
6:	25/16	772.627 classic augmented fifth
7:	103/64	823.801
8:	13/8	840.528 tridecimal neutral sixth
9:	55/32	937.632
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 15/8 17/16 25/16 19/16 55/32 65/64 103/64

Subsets:

MOS 7: 0 2 4 5 8 10 11 (C D E F Ab Bb B)

MOS 10: all except 1 and 7 (no C# or G)

**Wythoff (3-1) Scale H Left - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 1, 0, 0, 3, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 1 0 0 3 0 1 3 0 4 3 1 7 3 5 10 4 12 13 9  
22 17 21 35 26 43 52 47...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	43/32	511.518
7:	11/8	551.318 undecimal semi-augmented fourth
8:	47/32	665.507
9:	3/2	701.955 perfect fifth
10:	13/8	840.528 tridecimal neutral sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 9/8 11/8 17/16 21/16 35/32 43/32 47/52

Subsets:

MOS 7: 0 3 4 7 9 10 11 (C Eb E G A Bb B)

MOS 11: all except 8 (no Ab)

**Wythoff (3-1) Scale I Left - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 1, 0, 0, 0, 3

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 1 0 0 0 3 1 0 0 3 4 1 0 3 7 5 1 3 10 12  
6 4 13 22 18 10 17 35 40 28 27 52 75 68 55...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	75/64	274.582 classic augmented second
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	55/32	937.632
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 11/8 9/8 17/16 35/32 27/16 75/64 55/32

Subsets:

MOS 5: 0 5 7 8 11 (C F G Ab B)

MOS 9: 0 1 2 3 5 6 7 8 11 (C C# D Eb F F# G Ab B)

**Wythoff (3-1) Scale J Left - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 1, 3, 3, 3, 3, 3

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 1 3 3 3 3 3 4 7 10 13 16 19 23 30 40 53  
69 88 111...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	69/64	130.229
2:	19/16	297.513 19th harmonic
3:	5/4	386.314 major third
4:	11/8	551.318 undecimal semi-augmented fourth
5:	23/16	628.274 23rd harmonic
6:	3/2	701.955 perfect fifth
7:	13/8	840.528 tridecimal neutral sixth
8:	53/32	873.505
9:	111/64	953.299
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 19/16 23/16 15/8 53/32 69/64 11/8 111/64

Subsets:

MOS 5 = 0 3 6 7 10 (C Eb F# G Bb)

MOS 8 = 0 2 3 5 6 7 10 11 (C D Eb F F# G Bb B)

MOS 11 = all except 9 (no A)

**Wythoff (3-1) Scale K Left - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 1, 0, 0, 0, 0, 3

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 1 0 0 0 0 3 1 0 0 0 3 4 1 0 0 3 7 5 1 0  
3 10 12 6 1 3 13 22 18 7 4 16 35 40 25 11 20 51 75...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	75/64	274.582 classic augmented second
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	51/32	806.910
10:	13/8	840.528 tridecimal neutral sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 11/8 9/8 35/32 25/16 51/32 75/64 65/64

Subsets:

MOS 5 = 0 5 7 10 11 (C F G Bb B)

MOS 6 = 0 5 6 7 10 11 (C F F# G Bb B)

MOS 11 = all except 1 (no C#)

#### Part 4: The YASSER SCALES (generator 3, 2)

If the seed 3, 2 is used with the Fibonacci rule, the resulting sequence is 3 2 5 7 12 19 31 50... This is the same sequence that Joseph Yasser used for scale generation and partitioning in his "A Theory of Evolving Tonality" (Yasser, American Library of Musicology, New York, 1932). Accordingly, I'm calling the scales derived from the 3, 2 series and triangle, the Yasser scales. Here is a part of the Yasser (3, 2) triangle. Having 2 different generators makes the triangle asymmetrical. Therefore, additive sequences derived from left-leaning and right-leaning diagonals will be different, as will the scales derived from them.

```

                2/3
                2 3
                2 5 3
                2 7 8 3
                2 9 15 11 3
                2 11 24 26 14 3
                2 13 35 50 40 17 3
```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

#### Generator (3-2) Yasser Scales - Right Wing Versions

##### Yasser (3-2) Scale A Right - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 3, 2

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 3 2 5 7 12 19 31 50 81 131 212 343 555...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	131/128	40.108
2:	555/512	139.613
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	81/64	407.820 Pythagorean major third
6:	343/256	506.478
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	53/32	873.505
10:	7/4	968.826 harmonic seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 19/16 31/16 25/16 81/64 131/128 53/32 343/256 555/512

Subsets:

MOS 7 = 0 3 4 7 8 10 11 (C D# E G Ab Bb B)

MOS 10 = all except 6 and 2 (no D or F#)

**Yasser (3-2) Scale B Right - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 3, 2, 2

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 3 2 2 5 7 9 14 21 30 44 65 95 139 204...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	139/128	142.729
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	95/64	683.827
8:	3/2	701.955 perfect fifth
9:	51/32	806.910
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 21/16 15/8 11/8 65/64 95/64 139/128 51/32

Subsets:

MOS 5: 0 3 4 8 10 (C Eb E Ab Bb)

MOS 7: 0 3 4 5 8 10 11 (C Eb E F Ab Bb B)

MOS 9: 0 1 3 4 5 6 8 10 11 (C Db Eb E F Gb Ab Bb B)

MOS 11: all except 9 (no A)

**Yasser (3-2) Scale C Right - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 3, 0, 2

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 3 0 2 3 2 5 5 7 10 12 17 22 29 39 51 68 90 119 158...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	39/32	342.483 39th harmonic
3:	79/64	364.537
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	3/2	701.955 perfect fifth
8:	51/32	806.910
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	119/64	1073.781
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 17/16 11/8 29/16 39/32 51/32 45/32 119/64 79/64

Subsets:

MOS 5: 0 1 4 7 9 (C C# E G A)

MOS 7: 0 1 4 5 7 9 10 (C C# E F G A Bb)

MOS 12: all tones



**Yasser (3-2) Scale D Right - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 3, 2, 2, 2

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 3 2 2 2 5 7 9 11 16 23 32 43 59 82 114 157...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	157/128	353.545
3:	5/4	386.314 major third
4:	41/32	429.062
5:	43/32	511.518
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	7/4	968.826 harmonic seventh
10:	57/32	999.468
11:	59/32	1059.172
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 11/8 23/16 43/32 59/32 41/32 57/32 157/128

Subsets:

MOS 5: 0 1 3 8 10 (C Db Eb Ab Bb)

MOS 7: 0 1 3 6 7 8 10 (C Db Eb F# G Ab Bb)

MOS 9: 0 1 3 5 6 7 8 10 11 (C Db Eb F F# G Ab Bb B)

MOS 11: all except 2 (no D)

**Yasser (3-2) Scale E Right - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 3, 0, 0, 2

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 3 0 0 2 3 0 2 5 3 2 7 8 5 9 15 13 14 24 28 27 38 52 55 65 90...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	45/32	590.224 diatonic tritone
6:	3/2	701.955 perfect fifth
7:	13/8	840.528 tridecimal neutral sixth
8:	27/16	905.865 Pythagorean major sixth
9:	55/32	937.632
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 15/8 13/8 27/16 19/16 55/32 65/64 45/32

Subsets:

MOS 7: 0 2 4 6 7 10 11 (C D E F# G Bb B)

MOS 10: all except 1 and 5 (no C# or F)

**Yasser (3-2) Scale F Right - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 3, 2, 2, 2, 2

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 3 2 2 2 2 5 7 9 11 13 18 25 34 45 58 76...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 11/8 13/8 25/16 17/16 45/32 29/16 19/16

Subsets:

MOS 5: 0 2 4 7 10 (C D E G Bb)

MOS 7: 0 2 4 5 7 9 10 (C D E F G A Bb)

MOS 12: all tones

**Yasser (3-2) Scale G Right - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 3, 0, 2, 0, 2

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 3 0 2 0 2 3 2 5 2 7 5 9 10 11 17 16 26 26 37 43 53 69...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	69/64	130.229
3:	9/8	203.910 major whole tone
4:	37/32	251.344 37th harmonic
5:	5/4	386.314 major third
6:	43/32	511.518
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	53/32	873.505
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 11/8 17/16 13/8 37/32 43/32 53/32 69/64

Subsets:

MOS 7: 0 1 3 5 7 8 11 (C Db Eb F G Ab B)

MOS 10: all except 2 and 10 (no D or Bb)

**Yasser (3-2) Scale H Right - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 3, 0, 0, 2, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 3 0 0 2 0 3 2 0 5 2 3 7 2 8 9 5 15 11 13  
24 16 28 35 29 52 51 57...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	11/8	551.318 undecimal semi-augmented fourth
5:	3/2	701.955 perfect fifth
6:	51/32	806.910
7:	13/8	840.528 tridecimal neutral sixth
8:	7/4	968.826 harmonic seventh
9:	57/32	999.468
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 15/8 11/8 13/8 35/32 29/16 51/32 57/32

Subsets:

MOS 7: 0 2 3 4 5 8 11 (C D Eb E F Ab B)

MOS 11: all except 9 (no A)

**Yasser (3-2) Scale I Right - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 3, 0, 0, 0, 2

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 3 0 0 0 2 3 0 0 2 5 3 0 2 7 8 3 2 9 15  
11 5 11 24 26 16 16 35 50 42 32 51...

...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	51/32	806.910
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 15/8 11/8 13/8 35/32 25/16 21/16 51/32

Subsets:

MOS 5: 0 2 3 6 10 (C D Eb Gb Bb)

MOS 9: 0 1 2 3 5 6 9 10 11 (C C# D Eb F F# A Bb B)

**Yasser (3-2) Scale J Right - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 3, 2, 2, 2, 2, 2

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 3 2 2 2 2 2 5 7 9 11 13 15 20 27 36 47  
60 75 95...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	75/64	274.582 classic augmented second
3:	5/4	386.314 major third
4:	11/8	551.318 undecimal semi-augmented fourth
5:	47/32	665.507
6:	95/64	683.827
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 11/8 13/8 15/8 27/16 47/32 75/64 95/64

Subsets:

MOS 5 = 0 1 3 7 10 (C Db Eb G Bb)

MOS 8 = 0 1 3 4 7 8 10 11 (C Db Eb E G Ab Bb B)

MOS 11 = all except 6 (no F#)

**Yasser (3-2) Scale K Right - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 3, 0, 0, 0, 0, 2

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 3 0 0 0 0 2 3 0 0 0 2 5 3 0 0 2 7 8 3 0  
2 9 15 11 3 2 11 24 26 14 5 13 35 50 40 19 18 48 85...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	85/64	491.269
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 9/8 15/8 11/8 13/8 35/32 25/16 19/16 85/64

Subsets:

MOS 5 = 0 2 4 7 10 (C D E G Bb)

MOS 6 = 0 2 4 7 10 11 (C D E G Bb B)

MOS 11 = all except 5 (no F)

## Generator (3-2) Yasser Scales - Left Wing Versions

### Yasser (3-2) Scale A Left - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 2, 3

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 2 3 5 8 13 21...

**This is the Fibonacci sequence - it forms the same scale as Meru(1-1) Scale A. Whether left wing or right wing is immaterial - the Meru(1-1) triangle is symmetrical.**

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	305/256	303.199
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	89/64	570.880
7:	377/256	670.105
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	55/32	937.632
11:	233/128	1037.023
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 13/8 21/16 17/16 55/32 89/64 9/8 233/128 377/256 305/256

Subsets:

MOS 7 = 0 1 4 5 8 9 10 (C C# E F G# A Bb)

MOS 10 = all except 3 and 7 (no Eb or G)

### Yasser (3-2) Scale B Left - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 2, 3, 3

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 2 3 3 5 8 11 16 24 35 51 75 110 161 236 346 507...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	75/64	274.582 classic augmented second
3:	5/4	386.314 major third
4:	161/128	397.100
5:	173/128	521.554
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	51/32	806.910
9:	55/32	937.632
10:	59/32	1059.172
11:	507/256	1183.010
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 11/8 35/32 51/32 75/64 55/32 161/128 59/32 173/128 507/256

Subsets:

MOS 5: 0 1 3 6 7 (C C# D# F# G)

MOS 7: 0 1 2 3 6 7 8 (C C# D D# F# G Ab)

MOS 9: 0 1 2 3 4 6 7 8 9 (C C# D D# E F# G Ab A)

MOS 11: all except 11 (no B)

**Yasser (3-2) Scale C Left - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 2, 0, 3

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 2 0 3 2 3 5 5 8 10 13 18 23 31 41 54 72  
95 126 167 ...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	5/4	386.314 major third
3:	41/32	429.062
4:	167/128	460.445
5:	23/16	628.274 23rd harmonic
6:	95/64	683.827
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	31/16	1145.036 31st harmonic
11:	63/32	1172.736 octave - septimal comma
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 13/8 9/8 23/16 31/16 41/32 27/16 95/64 63/32 167/128

Subsets:

MOS 5: 0 1 2 7 8 (C C# D G Ab)

MOS 7: 0 1 2 5 7 8 10 (C C# D F G Ab Bb)

MOS 12: all tones

**Yasser (3-2) Scale D Left - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 2, 3, 3, 3

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 2 3 3 3 5 8 11 14 19 27 38 52 71 98 136  
188...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	71/64	179.697
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	49/32	737.652
9:	13/8	840.528 tridecimal neutral sixth
10:	27/16	905.865 Pythagorean major sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 11/8 7/4 19/16 27/16 13/8 71/64 49/32 17/16 47/32

Subsets:

MOS 5: 0 4 5 7 11 (C E F G B)

MOS 7: 0 3 4 5 7 10 11 (C D# E F G Bb B)

MOS 9: 0 2 3 4 5 7 9 10 11 (C D D# E F G A Bb B)

MOS 11: all except 6 (no F#)

**Yasser (3-2) Scale E Left - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 2, 0, 0, 3

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 2 0 0 3 2 0 3 5 2 3 8 7 5 11 15 12 16 26  
27 28 42 53 55 70...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	5/4	386.314 major third
3:	21/16	470.781 narrow fourth
4:	11/8	551.318 undecimal semi-augmented fourth
5:	3/2	701.955 perfect fifth
6:	13/8	840.528 tridecimal neutral sixth
7:	53/32	873.505
8:	27/16	905.865 Pythagorean major sixth
9:	55/32	937.632
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 11/8 15/8 13/8 27/16 21/16 53/32 55/32 35/32

Subsets:

MOS 7: 0 2 4 5 6 10 11 (C D E F F# Bb B)

MOS 10: all except 1 and 9 (no C# or A)

**Yasser (3-2) Scale F Left - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 2, 3, 3, 3, 3

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 2 3 3 3 3 5 8 11 14 17 22 30 41 55 72 94  
124...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	41/32	429.062
5:	11/8	551.318 undecimal semi-augmented fourth
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	55/32	937.632
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 11/8 7/4 17/16 15/8 41/32 55/32 9/8 47/32 31/16

Subsets:

MOS 5: 0 3 5 7 9 (C Eb F G A)

MOS 7: 0 1 3 5 7 9 10 (C C# Eb F G A Bb)

MOS 12: all tones

**Yasser (3-2) Scale G Left - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 2, 0, 3, 0, 3

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 2 0 3 0 3 2 3 5 3 8 5 11 10 14 18 19 29  
29 43 47 62 76 91...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	19/16	297.513 19th harmonic
3:	5/4	386.314 major third
4:	43/32	511.518
5:	11/8	551.318 undecimal semi-augmented fourth
6:	91/64	609.354
7:	47/32	665.507
8:	3/2	701.955 perfect fifth
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 11/8 7/4 9/8 19/16 29/16 43/32 47/32 31/16 91/64

Subsets:

MOS 7: 0 1 2 3 5 8 9 (C C# D Eb F Ab A)

MOS 10: all except 6 and 11 (no F# or B)

**Yasser (3-2) Scale H Left - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 2, 0, 0, 3, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 2 0 0 3 0 2 3 0 5 3 2 8 3 7 11 5 15 14  
12 26 19 27 40 31 53 59...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	19/16	297.513 19th harmonic
2:	5/4	386.314 major third
3:	11/8	551.318 undecimal semi-augmented fourth
4:	3/2	701.955 perfect fifth
5:	13/8	840.528 tridecimal neutral sixth
6:	53/32	873.505
7:	27/16	905.865 Pythagorean major sixth
8:	7/4	968.826 harmonic seventh
9:	59/32	1059.172
10:	15/8	1088.269 classic major seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 11/8 15/8 13/8 19/16 27/16 31/16 53/32 59/32

Subsets:

MOS 7: 0 2 3 4 5 8 10 (C D Eb E F Ab Bb)

MOS 11: all except 9 (no A)



**Yasser (3-2) Scale I Left - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 2, 0, 0, 0, 3

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 2 0 0 0 3 2 0 0 3 5 2 0 3 8 7 2 3 11 15  
9 5 14 26 24 14 19 40 50 38 33 59...

...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	59/32	1059.172
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 11/8 15/8 9/8 13/8 19/16 25/16 33/32 59/32

Subsets:

MOS 5: 0 4 5 6 9 (C E F F# A)

MOS 9: 0 2 3 4 5 6 8 9 11 (C D Eb E F F# Ab A B)

**Yasser (3-2) Scale J Left - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 2, 3, 3, 3, 3, 3

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 2 3 3 3 3 3 5 8 11 14 17 20 25 33 44 58  
75 95 120...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	75/64	274.582 classic augmented second
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	95/64	683.827
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 11/8 7/4 17/16 25/16 33/32 29/16 75/64 95/64 15/8

Subsets:

MOS 5 = 0 4 5 7 9 (C E F G A)

MOS 8 = 0 1 2 4 5 7 8 9 (C C# D E F G Ab A)

MOS 11 = all except 11 (no B)

**Yasser (3-2) Scale K Left - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 2, 0, 0, 0, 0, 3

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 2 0 0 0 0 3 2 0 0 0 3 5 2 0 0 3 8 7 2 0  
3 11 15 9 2 3 14 26 24 11 5 17 40 50 35 16 22 57 ...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	57/32	999.468
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 5/4 7/4 11/8 15/8 9/8 13/8 17/16 25/16 35/32 57/32

Subsets:

MOS 5 = 0 4 5 6 9 (C E F F# A)

MOS 6 = 0 4 5 6 9 11 (C E F F# A B)

MOS 11 = all except 10 (no Bb)

## Part 5: The 4-1 SCALES (generator 4, 1)

Sequences following the Fibonacci rule with generators beyond 4,1 do not have names. So from this point on, the scales will only be named by their seeds. If the seed 4,1 is used with the Fibonacci rule, the resulting sequence is 4 1 5 6 11 17 28 45... Here is a part of the (4, 1) triangle. Having 2 different generators makes the triangle asymmetrical. Therefore, additive sequences derived from left-leaning and right-leaning diagonals will be different, as will the scales derived from them.

```

      1/4
      1 4
     1 5 4
    1 6 9 4
   1 7 15 13 4
  1 8 22 28 17 4
 1 9 30 50 45 21 4
```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

### Generator (4-1) Scales - Right Wing Versions

#### (4-1) Scale A Right - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 4, 1

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 4 1 5 6 11 17 28 45 73 118 191 309 500...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	73/64	227.789
3:	309/256	325.756
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	191/128	692.915
8:	3/2	701.955 perfect fifth
9:	7/4	968.826 harmonic seventh
10:	59/32	1059.172
11:	125/64	1158.941 classic augmented seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 17/16 7/4 45/32 73/64 59/32 191/128 309/256 125/64

Subsets:

MOS 7 = 0 1 4 5 6 8 9 (C C# E F F# G# A)

MOS 10 = all except 3 and 11 (no Eb or B)

**(4-1) Scale B Right - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 4, 1, 1

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 4 1 1 5 6 7 12 18 25 37 55 80 117 172 252 369...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	37/32	251.344 37th harmonic
3:	5/4	386.314 major third
4:	43/32	511.518
5:	369/256	632.972
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	55/32	937.632
9:	7/4	968.826 harmonic seventh
10:	117/64	1044.438
11:	63/32	1172.736 octave - septimal comma
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 7/4 9/8 25/16 37/32 55/32 117/64 43/32 63/32 369/256

Subsets:

MOS 5: 0 1 3 6 9 (C Db Eb F# A)

MOS 7: 0 1 2 3 6 7 9 (C Db D Eb F# G A)

MOS 9: 0 1 2 3 6 7 8 9 10 (C Db D Eb F# G Ab A Bb)

MOS 11: all except 5 (no F)

**(4-1) Scale C Right - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 4, 0, 1

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 4 0 1 4 1 5 5 6 10 11 16 21 27 37 48 64 85 112 149 197 261...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	261/256	33.487 vicesimononal comma
2:	37/32	251.344 37th harmonic
3:	149/128	263.002
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	85/64	491.269
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	197/128	746.462
10:	27/16	905.865 Pythagorean major sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 21/16 27/16 37/32 85/64 7/4 149/128 197/128 261/256

Subsets:

MOS 5: 0 4 5 7 8 (C E F G Ab)

MOS 7: 0 2 4 5 7 8 10 (C D E F G Ab Bb)

MOS 12: all tones

**(4-1) Scale D Right** - 12 tones

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 4, 1, 1, 1

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 4 1 1 1 5 6 7 8 13 19 26 34 47 66 92 126 173...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	173/128	521.554
6:	23/16	628.274 23rd harmonic
7:	47/32	665.507
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	63/32	1172.736 octave - septimal comma
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 7/4 13/8 19/16 17/16 47/32 33/32 23/16 63/32 173/128

Subsets:

MOS 5: 0 4 8 9 10 (C E Ab A Bb)

MOS 7: 0 2 3 4 8 9 10 (C D Eb E Ab A Bb)

MOS 9: 0 1 2 3 4 7 8 9 10 (C C# D Eb E G Ab A Bb)

MOS 11: all except 5 (no F)

**(4-1) Scale E Right** - 12 tones

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 4, 0, 0, 1

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 4 0 0 1 4 0 1 5 4 1 6 9 5 7 15 14 12 22 29 26 34 51 55...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	11/8	551.318 undecimal semi-augmented fourth
5:	3/2	701.955 perfect fifth
6:	51/32	806.910
7:	13/8	840.528 tridecimal neutral sixth
8:	55/32	937.632
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 9/8 7/4 15/8 11/8 29/16 13/8 17/16 51/32 55/32

Subsets:

MOS 7: 0 2 3 4 5 9 11 (C D Eb E F A B)

MOS 10: all except 6 and 8 (no F# or Ab)

**(4-1) Scale F Right - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 4, 1, 1, 1, 1

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 4 1 1 1 1 5 6 7 8 9 14 20 27 35 44 58 78 105 140 184...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	9/8	203.910 major whole tone
3:	39/32	342.483 39th harmonic, Zalzal wosta of Ibn

Sina

4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	105/64	857.095 septimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 7/4 9/8 27/16 35/32 11/8 29/16 39/32 105/64 23/16

Subsets:

MOS 5: 0 2 4 7 10 (C D E G Bb)

MOS 7: 0 1 2 4 7 9 10 (C C# D E G A Bb)

MOS 12: all tones

**(4-1) Scale G Right - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 4, 0, 1, 0, 1

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 4 0 1 0 1 4 1 5 1 6 5 7 10 8 16 13 23 23 31 39 44 62 67 93 106...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	39/32	342.483 39th harmonic
3:	5/4	386.314 major third
4:	11/8	551.318 undecimal semi-augmented fourth
5:	23/16	628.274 23rd harmonic
6:	93/64	646.991
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	53/32	873.505
10:	7/4	968.826 harmonic seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 7/4 13/8 23/16 31/16 39/32 11/8 67/64 93/64 53/32

Subsets:

MOS 7: 0 3 5 7 8 10 11 (C Eb F G Ab Bb B)

MOS 10: all except 6 and 9 (no F# or A)

**(4-1) Scale H Right - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 4, 0, 0, 1, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 4 0 0 1 0 4 1 0 5 1 4 6 1 9 7 5 15 8 14  
22 13 29 30 27 51 43...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	5/4	386.314 major third
3:	43/32	511.518
4:	11/8	551.318 undecimal semi-augmented fourth
5:	3/2	701.955 perfect fifth
6:	51/32	806.910
7:	13/8	840.528 tridecimal neutral sixth
8:	27/16	905.865 Pythagorean major sixth
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 9/8 7/4 15/8 11/8 13/8 29/16 27/16 51/32 43/32

Subsets:

MOS 7: 0 1 2 5 7 9 11 (C C# D F G A B)

MOS 11: all except 3 (no Eb)

**(4-1) Scale I Right - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 4, 0, 0, 0, 1

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 4 0 0 0 1 4 0 0 1 5 4 0 1 6 9 4 1 7 15  
13 5 8 22 28 18 13 30 50 46 31 43...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	5/4	386.314 major third
3:	43/32	511.518
4:	11/8	551.318 undecimal semi-augmented fourth
5:	23/16	628.274 23rd harmonic
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 9/8 7/4 15/8 13/8 11/8 25/16 23/16 31/16 43/32

Subsets:

MOS 5: 0 1 2 6 9 (C C# D F# A)

MOS 9: 0 1 2 4 6 7 8 9 10 (C C# D E F# G Ab A Bb)

**(4-1) Scale J Right - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 4, 1, 1, 1, 1, 1

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 4 1 1 1 1 1 5 6 7 8 9 10 15 21 28 36 45  
55 70 91 119...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	45/32	590.224 diatonic tritone
6:	91/64	609.354
7:	3/2	701.955 perfect fifth
8:	55/32	937.632
9:	7/4	968.826 harmonic seventh
10:	119/64	1073.781
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 7/4 9/8 15/8 21/16 45/32 55/32 35/32 91/64 119/64

Subsets:

MOS 5 = 0 2 3 7 9 (C D Eb G A)

MOS 8 = 0 2 3 4 5 7 9 11 (C D Eb E F G A B)

MOS 11 = all except 10 (no Bb)

**(4-1) Scale K Right - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 4, 0, 0, 0, 0, 1

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 4 0 0 0 0 1 4 0 0 0 1 5 4 0 0 1 6 9 4 0  
1 7 15 13 4 1 8 22 28 17 5 9 30 50 45 22 14 39...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	39/32	342.483 39th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 9/8 7/4 15/8 13/8 11/8 17/16 25/16 45/32 39/32

Subsets:

MOS 5 = 0 2 4 7 10 (C D E G Bb)

MOS 6 = 0 2 4 7 10 11 (C D E G Bb B)

MOS 11 = all except 3 (no Eb)



## Generator (4-1) Scales - Left Wing Versions

### (4-1) Scale A Left - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 1, 4

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 1 4 5 9 14 23 37 60 97 157 254 411 665...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	37/32	251.344 37th harmonic
3:	157/128	353.545
4:	5/4	386.314 major third
5:	665/512	452.653
6:	23/16	628.274 23rd harmonic
7:	97/64	719.895
8:	411/256	819.594
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	127/64	1186.422
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 23/16 37/32 15/8 97/64 157/128 127/64 411/256 665/512

Subsets:

MOS 7 = 0 1 2 4 6 9 10 (C C# D E F# A Bb)

MOS 10 = all except 5 and 8 (no F or Ab)

### (4-1) Scale B Left - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 1, 4, 4

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 1 4 4 5 9 13 18 27 40 58 85 125 183 268 393 576 844...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	85/64	491.269
5:	183/128	618.840
6:	393/256	742.063
7:	13/8	840.528 tridecimal neutral sixth
8:	211/128	865.319
9:	27/16	905.865 Pythagorean major sixth
10:	29/16	1029.577 29th harmonic
11:	125/64	1158.941 classic augmented seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 13/8 27/16 29/16 85/64 125/64 183/128 67/64 393/256 211/128

Subsets:

MOS 5: 0 2 3 7 9 (C D Eb G A)

MOS 7: 0 2 3 4 7 9 10 (C D Eb E G A Bb)

MOS 9: 0 2 3 4 5 7 9 10 11 (C D Eb E F G A Bb B)

MOS 11: all except 8 (no Ab)

**(4-1) Scale C Left - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 1, 0, 4

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 1 0 4 1 4 5 5 9 10 14 19 24 33 43 57 76 100 133 176...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	133/128	66.339
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	43/32	511.518
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	25/16	772.627 classic augmented fifth
10:	7/4	968.826 harmonic seventh
11:	57/32	999.468
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 19/16 3/2 33/32 43/32 57/32 25/16 133/128 11/8

Subsets:

MOS 5: 0 3 4 5 10 (C Eb E F Bb)

MOS 7: 0 1 3 4 5 8 10 (C C# Eb E F Ab Bb)

MOS 12: all tones

**(4-1) Scale D Left - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 1, 4, 4, 4

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 1 4 4 4 5 9 13 17 22 31 44 61 83 144 158 219 302...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	151/128	286.086
4:	79/64	364.537
5:	5/4	386.314 major third
6:	83/64	450.047
7:	11/8	551.318 undecimal semi-augmented fourth
8:	13/8	840.528 tridecimal neutral sixth
9:	219/128	929.744
10:	61/32	1116.885
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 13/8 17/16 11/8 31/16 61/32 83/64 79/64 219/128 151/128

Subsets:

MOS 5: 0 1 2 5 8 (C C# D F Ab)

MOS 7: 0 1 2 5 7 8 11 (C C# D F G Ab B)

MOS 9: 0 1 2 5 6 7 8 10 11 (C C# D F F# G Ab Bb B)

MOS 11: all except 3 (no Eb)

**(4-1) Scale E Left - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 1, 0, 0, 4

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 1 0 0 4 1 0 4 5 1 4 9 6 5 13 15 11 18 28 26 29 46 54 55...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	5/4	386.314 major third
3:	11/8	551.318 undecimal semi-augmented fourth
4:	23/16	628.274 23rd harmonic
5:	3/2	701.955 perfect fifth
6:	13/8	840.528 tridecimal neutral sixth
7:	27/16	905.865 Pythagorean major sixth
8:	55/32	937.632
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 3/2 13/8 15/8 11/8 7/4 29/16 23/16 27/16 55/32

Subsets:

MOS 7: 0 1 2 3 5 6 11 (C C# D Eb F F# B)

MOS 10: all except 7 and 8 (no G or Ab)

**(4-1) Scale F Left - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 1, 4, 4, 4, 4

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 1 4 4 4 4 5 9 13 17 21 26 35 48 65 86 112 147...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	17/16	104.955 17th harmonic
3:	35/32	155.140 septimal neutral second
4:	9/8	203.910 major whole tone
5:	147/128	239.607
6:	5/4	386.314 major third
7:	21/16	470.781 narrow fourth
8:	43/32	511.518
9:	3/2	701.955 perfect fifth
10:	13/8	840.528 tridecimal neutral sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 13/8 17/16 21/16 35/32 3/2 65/64 43/32 7/4 147/128

Subsets:

MOS 5: 0 2 4 6 10 (C D E F# Bb)

MOS 7: 0 2 3 4 6 7 10 (C D Eb E F# G Bb)

MOS 12: all tones

**(4-1) Scale G Left - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 1, 0, 4, 0, 4

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 1 0 4 0 4 1 4 5 4 9 5 13 10 17 19 22 32 32 49 51 71 83 103...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	71/64	179.697
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	83/64	450.047
7:	11/8	551.318 undecimal semi-augmented fourth
8:	49/32	737.652
9:	51/32	806.910
10:	103/64	823.801
11:	13/8	840.528 tridecimal neutral sixth
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 13/8 17/16 19/16 11/8 49/32 51/32 71/64 83/64 103/64

Subsets:

MOS 7: 0 1 3 4 5 7 11 (C C# Eb E F G B)

MOS 10: all except 6 and 10 (no F# or Bb)

**(4-1) Scale H Left - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 1, 0, 0, 4, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 1 0 0 4 0 1 4 0 5 4 1 9 4 6 13 5 15 17 11 28 22 26 45 33 54...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 3/2 13/8 15/8 17/16 11/8 7/4 45/32 33/32 27/16

Subsets:

MOS 7: 0 2 3 4 7 8 11 (C D Eb E G Ab B)

MOS 11: all except 9 (no A)

**(4-1) Scale I Left - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 1, 0, 0, 0, 4

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 1 0 0 0 4 1 0 0 4 5 1 0 4 9 6 1 4 13 15  
7 5 17 28 22 12 22 45 50 34 34 67...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	17/16	104.955 17th harmonic
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 3/2 13/8 15/8 7/4 17/16 11/8 45/32 25/16 67/64

Subsets:

MOS 5: 0 3 4 7 9 (C Eb E G A)

MOS 9: 0 2 3 4 5 7 9 10 11 (C D Eb E F G A Bb B)

**(4-1) Scale J Left - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 1, 4, 4, 4, 4, 4

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 1 4 4 4 4 4 5 9 13 17 21 25 30 39 52 69  
90 115...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	69/64	130.229
3:	9/8	203.910 major whole tone
4:	39/32	342.483 39th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	45/32	590.224 diatonic tritone
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	115/64	1014.588
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 13/8 17/16 21/16 25/16 15/8 39/32 69/64 45/32 115/64

Subsets:

MOS 5 = 0 1 3 5 9 (C C# Eb F A)

MOS 8 = 0 1 3 5 6 8 9 11 (C C# Eb F F# Ab A B)

MOS 11 = all except 10 (no Bb)

**(4-1) Scale K Left - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 1, 0, 0, 0, 0, 4

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 1 0 0 0 0 4 1 0 0 0 4 5 1 0 0 4 9 6 1 0  
4 13 15 7 1 4 17 28 22 8 5 21 45 50...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 3/2 13/8 15/8 7/4 17/16 11/8 21/16 45/32 25/16

Subsets:

MOS 5 = 0 2 3 7 9 (C D Eb G A)

MOS 6 = 0 2 3 7 9 11 (C D Eb G A B)

MOS 11 = all except 8 (no Ab)

## Part 6: The 4-3 SCALES (generator 4, 3)

If the seed 4, 3 is used with the Fibonacci rule, the resulting sequence is 4 3 7 10 17 27 44 71 115... Here is a part of the (4, 3) triangle. Having 2 different generators makes the triangle asymmetrical. Therefore, additive sequences derived from left-leaning and right-leaning diagonals will be different, as will the scales derived from them.

```

      3/4
     3 4
    3 7 4
   3 10 11 4
  3 13 21 15 4
 3 16 34 36 19 4
3 19 50 70 55 23 4
```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

### Generator (4-3) Scales - Right Wing Versions

#### (4-3) Scale A Right - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 4, 3

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 4 3 7 10 17 27 44 71 115 186 301 487...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	71/64	179.697
3:	301/256	280.344
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	93/64	646.991
7:	3/2	701.955 perfect fifth
8:	27/16	905.865 Pythagorean major sixth
9:	7/4	968.826 harmonic seventh
10:	115/64	1014.588
11:	487/256	1113.334
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 17/16 27/16 11/8 71/64 115/64 93/64 301/256 487/256

Subsets:

MOS 7 = 0 1 4 5 7 8 9 (C C# E F G Ab A)

MOS 10 = all except 3 and 11 (no Eb or B)

**(4-3) Scale B Right - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 4, 3, 3

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 4 3 3 7 10 13 20 30 43 63 93 136 199 292...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	73/64	227.789
3:	5/4	386.314 major third
4:	43/32	511.518
5:	93/64	646.991
6:	3/2	701.955 perfect fifth
7:	199/128	763.950
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	63/32	1172.736 octave - septimal comma
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 15/8 43/32 63/32 93/64 17/16 199/128 73/64

Subsets:

MOS 5: 0 3 6 8 9 (C Eb Gb Ab A)

MOS 7: 0 3 4 6 8 9 10 (C Eb E Gb Ab A Bb)

MOS 9: 0 3 4 5 6 8 9 10 11 (C Eb E F Gb A Bb B)

MOS 11: all except 2 (no D)

**(4-3) Scale C Right - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 4, 0, 3

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 4 0 3 4 3 7 7 10 14 17 24 31 41 55 72 96 127 168 223...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	41/32	429.062
5:	21/16	470.781 narrow fourth
6:	3/2	701.955 perfect fifth
7:	55/32	937.632
8:	223/128	961.080
9:	7/4	968.826 harmonic seventh
10:	31/16	1145.036 31st harmonic
11:	127/64	1186.422
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 17/16 31/16 41/32 55/32 9/8 127/64 21/16 223/128

Subsets:

MOS 5: 0 1 3 6 9 (C Db Eb Gb A)

MOS 7: 0 1 3 4 6 9 10 (C Db Eb E Gb A Bb)

MOS 12: all tones



**(4-3) Scale D Right** - 12 tones

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 4, 3, 3, 3

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 4 3 3 3 7 10 13 16 23 33 46 62 85 118 164 226...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	5/4	386.314 major third
3:	41/32	429.062
4:	85/64	491.269
5:	23/16	628.274 23rd harmonic
6:	3/2	701.955 perfect fifth
7:	13/8	840.528 tridecimal neutral sixth
8:	7/4	968.826 harmonic seventh
9:	113/64	984.215
10:	59/32	1059.172
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 23/16 33/32 31/16 85/64 59/32 41/32 113/64

Subsets:

MOS 5: 0 2 6 7 8 (C D F# G Ab)

MOS 7: 0 1 2 5 6 7 8 (C C# D F F# G Ab)

MOS 9: 0 1 2 4 5 6 7 8 11 (C C# D E F F# G Ab B)

MOS 11: all except 9 (no A)

**(4-3) Scale E Right** - 12 tones

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 4, 0, 0, 3

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 4 0 0 3 4 0 3 7 4 3 10 11 7 13 21 18 20 34 39 38 54...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	39/32	342.483 39th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	27/16	905.865 Pythagorean major sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 11/8 13/8 21/16 9/8 17/16 39/32 19/16 27/16

Subsets:

MOS 7: 0 5 6 7 8 9 11 (C F F# G Ab A B)

MOS 10: all except 3 and 10 (no Eb or Bb)

**(4-3) Scale F Right - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 4, 3, 3, 3, 3

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 4 3 3 3 3 7 10 13 16 19 26 36 49 65 84 110 146...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	9/8	203.910 major whole tone
3:	73/64	227.789
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	3/2	701.955 perfect fifth
8:	49/32	737.652
9:	13/8	840.528 tridecimal neutral sixth
10:	55/32	937.632
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 19/16 9/8 49/32 65/64 21/16 55/32 73/64

Subsets:

MOS 5: 0 5 7 9 11 (C F G A B)

MOS 7: 0 2 4 5 7 9 11 (C D E F G A B)

MOS 12: all tones

**(4-3) Scale G Right - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 4, 0, 3, 0, 3

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 4 0 3 0 3 4 3 7 3 10 7 13 14 16 24 23 37 37 53 61 76 98 113...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	37/32	251.344 37th harmonic
2:	19/16	297.513 19th harmonic
3:	5/4	386.314 major third
4:	23/16	628.274 23rd harmonic
5:	3/2	701.955 perfect fifth
6:	49/32	737.652
7:	13/8	840.528 tridecimal neutral sixth
8:	53/32	873.505
9:	7/4	968.826 harmonic seventh
10:	113/64	984.215
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 23/16 37/32 53/32 61/32 19/16 49/32 113/64

Subsets:

MOS 7: 0 1 3 4 5 7 9 (C Db Eb E F G A)

MOS 10: all except 6 and 10 (no F# or Bb)

**(4-3) Scale H Right - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 4, 0, 0, 3, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 4 0 0 3 0 4 3 0 7 3 4 10 3 11 13 7 21 16  
18 34 23 39 50...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	39/32	342.483 39th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	25/16	772.627 classic augmented fifth
10:	13/8	840.528 tridecimal neutral sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 11/8 13/8 21/16 9/8 17/16 23/16 39/32 25/16

Subsets:

MOS 7: 0 4 5 6 8 10 11 (C E F Gb Ab Bb B)

MOS 11: all except 9 (no A)

**(4-3) Scale I Right - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 4, 0, 0, 0, 3

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 4 0 0 0 3 4 0 0 3 7 4 0 3 10 11 4 3 13  
21 15 7 16 34 36 22 23 50...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 11/8 13/8 21/16 15/8 17/16 9/8 23/16 25/16

Subsets:

MOS 5: 0 3 5 7 10 (C Eb F G Bb)

MOS 9: 0 1 3 4 5 7 9 10 11 (C Db Eb E F G A Bb B)

**(4-3) Scale J Right - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 4, 3, 3, 3, 3, 3

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 4 3 3 3 3 3 7 10 13 16 19 22 29 39 52 68 87 109...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	19/16	297.513 19th harmonic
3:	39/32	342.483 39th harmonic
4:	5/4	386.314 major third
5:	87/64	531.532
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	109/64	921.821
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 13/8 19/16 11/8 29/16 39/32 17/16 87/64 109/64

Subsets:

MOS 5 = 0 4 7 8 10 (C E G Ab Bb)

MOS 8 = 0 2 4 6 7 8 10 11 (C D E F# G Ab Bb B)

MOS 11 = all except 9 (no A)

**(4-3) Scale K Right - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 4, 0, 0, 0, 0, 3

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 4 0 0 0 3 4 0 0 0 3 7 4 0 0 3 10 11 4 0 3 13 21 15 4 3 16 34 36 19 7 19 50...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 5/4 11/8 13/8 21/16 15/8 17/16 9/8 19/16 25/16

Subsets:

MOS 5 = 0 4 6 7 10 (C E F# G Bb)

MOS 6 = 0 4 6 7 9 10 (C E F# G A Bb)

MOS 11 = all except 8 (no Ab)

## Generator (4-3) Scales - Left Wing Versions

### (4-3) Scale A Left - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 3, 4

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 3 4 7 11 18 29 47 76 123 199 322 521...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	521/512	30.167
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	161/128	397.100
5:	11/8	551.318 undecimal semi-augmented fourth
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	199/128	763.950
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	123/64	1131.017
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 11/8 9/8 29/16 47/32 19/16 123/64 199/128 161/128 521/512

Subsets:

MOS 7 = 0 2 5 6 7 9 10 (C D F F# G A Bb)

MOS 10 = all except 1 and 4 (no C# or E)

### (4-3) Scale B Left - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 3, 4, 4

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 3 4 4 7 11 15 22 33 48 70 103 151 221 324 475...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	35/32	155.140 septimal neutral second
3:	151/128	286.086
4:	81/64	407.820 Pythagorean major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	103/64	823.801
8:	221/128	945.483
9:	7/4	968.826 harmonic seventh
10:	475/256	1070.140
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 11/8 15/8 33/32 35/32 103/64 151/128 221/128 81/64 475/256

Subsets:

MOS 5: 0 5 6 9 11 (C F F# A B)

MOS 7: 0 1 2 5 6 9 11 (C C# D F F# A B)

MOS 9: 0 1 2 3 5 6 7 9 11 (C C# D Eb F F# G A B)

MOS 11: all except 10 (no Bb)

**(4-3) Scale C Left - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 3, 0, 4

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 3 0 4 3 4 7 7 11 14 18 25 32 43 57 75  
100 132 175 232...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	9/8	203.910 major whole tone
3:	75/64	274.582 classic augmented second
4:	43/32	511.518
5:	175/128	541.453
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	7/4	968.826 harmonic seventh
10:	57/32	999.468
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 11/8 9/8 25/16 43/32 57/32 75/64 33/32 175/128 29/16

Subsets:

MOS 5: 0 2 6 7 9 (C D F# G A)

MOS 7: 0 2 4 6 7 8 9 (C D E F# G Ab A)

MOS 12: all tones

**(4-3) Scale D Left - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 3, 4, 4, 4

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 3 4 4 4 7 11 15 19 26 37 52 71 97 134  
186...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	71/64	179.697
3:	37/32	251.344 37th harmonic
4:	19/16	297.513 19th harmonic
5:	11/8	551.318 undecimal semi-augmented fourth
6:	93/64	646.991
7:	3/2	701.955 perfect fifth
8:	97/64	719.895
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 11/8 15/8 19/16 13/8 37/32 71/64 97/64 67/64 93/64

Subsets:

MOS 5: 0 5 7 10 11 (C F G Bb B)

MOS 7: 0 4 5 7 9 10 11 (C E F G A Bb B)

MOS 9: 0 2 3 4 5 7 9 10 11 (C D Eb E F G A Bb B)

MOS 11: all except 6 (no F#)

**(4-3) Scale E Left - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 3, 0, 0, 4

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 3 0 0 4 3 0 4 7 3 4 11 10 7 15 21 17 22  
36 38 39 58...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	39/32	342.483 39th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 11/8 5/4 15/8 21/16 17/16 9/8 19/16 39/32 29/16

Subsets:

MOS 7: 0 5 6 7 8 9 11 (C F F# G Ab A B)

MOS 10: all except 4 and 10 (no E or Bb)

**(4-3) Scale F Left - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 3, 4, 4, 4, 4

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 3 4 4 4 4 7 11 15 19 23 30 41 56 75 98  
128 169 225...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	75/64	274.582 classic augmented second
2:	19/16	297.513 19th harmonic
3:	41/32	429.062
4:	169/128	481.055
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	49/32	737.652
9:	7/4	968.826 harmonic seventh
10:	225/128	976.537 augmented sixth
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 11/8 15/8 19/16 23/16 41/32 75/64 49/32 169/128 225/128

Subsets:

MOS 5: 0 5 7 9 11 (C F G A B)

MOS 7: 0 2 5 6 7 9 11 (C D F F# G A B)

MOS 12: all tones

**(4-3) Scale G Left - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 3, 0, 4, 0, 4

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 3 0 4 0 4 3 4 7 4 11 7 15 14 19 25 26 40 40 59 65 85...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	19/16	297.513 19th harmonic
3:	5/4	386.314 major third
4:	85/64	491.269
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	59/32	1059.172
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 11/8 15/8 19/16 25/16 13/8 5/4 59/32 65/64 85/64

Subsets:

MOS 7: 0 2 5 6 7 9 11 (C D F F# G A B)

MOS 10: all except 1 and 4 (no C# or E)

**(4-3) Scale H Left - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 3, 0, 0, 4, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 3 0 0 4 0 3 4 0 7 4 3 11 4 10 15 7 21 19 17 36 26 38 55...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	55/32	937.632
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 11/8 5/4 15/8 21/16 19/16 17/16 9/8 13/8 55/32

Subsets:

MOS 7: 0 4 5 6 7 10 11 (C E F F# G Bb B)

MOS 11: all except 9 (no A)



**(4-3) Scale I Left - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 3, 0, 0, 0, 4

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 3 0 0 0 4 3 0 0 4 7 3 0 4 11 10 3 4 15  
21 13 7 19 36 34 20 26 55...

**Amazingly, this 12 note scale is IDENTICAL with the scale above ((4-3) Scale H Left). The order of the tones generated is different, though, as are the MOS subsets.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	55/32	937.632
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 11/8 5/4 15/8 21/16 13/8 19/16 9/8 17/16 55/32

Subsets:

MOS 5: 0 4 6 7 10 (C E F# G Bb)

MOS 9: 0 3 4 5 6 7 8 10 11 (C Eb E F F# G Ab Bb B)

**(4-3) Scale J Left - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 3, 4, 4, 4, 4, 4

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 3 4 4 4 4 4 7 11 15 19 23 27 34 45 60 79  
102...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	19/16	297.513 19th harmonic
3:	79/64	364.537
4:	11/8	551.318 undecimal semi-augmented fourth
5:	45/32	590.224 diatonic tritone
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	51/32	806.910
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 11/8 15/8 19/16 23/16 27/16 17/16 45/32 79/64 51/32

Subsets:

MOS 5 = 0 4 7 10 11 (C E G Bb B)

MOS 8 = 0 2 4 6 7 9 10 11 (C D E F# G A Bb B)

MOS 11 = all except 8 (no Ab)

**(4-3) Scale K Left - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 3, 0, 0, 0, 0, 4

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 3 0 0 0 0 4 3 0 0 0 4 7 3 0 0 4 11 10 3  
0 4 15 21 13 3 4 19 36 34 16 7 23...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 11/8 5/4 15/8 21/16 13/8 19/16 9/8 17/16 23/16

Subsets:

MOS 5 = 0 4 6 8 10 (C E F# Ab Bb)

MOS 6 = 0 4 6 8 10 11 (C E F# Ab Bb B)

MOS 11 = all except 7 (no G)

## Part 7: The 5-1 SCALES (generator 5, 1)

If the seed 5, 1 is used with the Fibonacci rule, the resulting sequence is 5 1 6 7 13 20 33 53 86 139 225... Here is a part of the (5, 1) triangle. Having 2 different generators makes the triangle asymmetrical. Therefore, additive sequences derived from left-leaning and right-leaning diagonals will be different, as will the scales derived from them.

```

      1/5
      1 5
     1 6 5
    1 7 11 5
   1 8 18 16 5
  1 9 26 34 21 5
 1 10 35 60 55 26 5
```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

### Generator (5-1) Scales - Right Wing Versions

#### (5-1) Scale A Right - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 5, 1

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 5 1 6 7 13 20 33 53 86 139 225 364 589...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	139/128	142.729
3:	589/512	242.549
4:	5/4	386.314 major third
5:	43/32	511.518
6:	91/64	609.354
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	53/32	873.505
10:	7/4	968.826 harmonic seventh
11:	225/128	976.537 augmented sixth
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 3/2 7/4 13/8 33/32 53/32 43/32 139/128 225/128 91/64 589/512

Subsets:

MOS 7 = 0 1 4 7 8 9 10 (C C# E G Ab A Bb)

MOS 10 = all except 3 and 6 (no Eb or F#)

**(5-1) Scale B Right - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 5, 1, 1

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 5 1 1 6 7 8 14 21 29 43 64 93 136 200 293 429...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	293/256	233.708
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	43/32	511.518
6:	93/64	646.991
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	429/256	893.801
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 3/2 7/4 21/16 29/16 43/32 93/64 17/16 25/16 293/256 429/256

Subsets:

MOS 5: 0 3 4 7 10 (C Eb E G Bb)

MOS 7: 0 3 4 5 7 10 11 (C Eb E F G Bb B)

MOS 9: 0 1 3 4 5 6 7 10 11 (C Db Eb E F F# G Bb B)

MOS 11: all except 9 (no A)

**(5-1) Scale C Right - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 5, 0, 1

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 5 0 1 5 1 6 6 7 12 13 19 25 32 44 57 76 101 133 177...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	133/128	66.339
2:	19/16	297.513 19th harmonic
3:	5/4	386.314 major third
4:	11/8	551.318 undecimal semi-augmented fourth
5:	177/128	561.127
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	101/64	789.854
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	57/32	999.468
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 3/2 7/4 13/8 19/16 25/16 11/8 57/32 101/64 133/128 177/128

Subsets:

MOS 5: 0 3 6 9 10 (C Eb Gb A Bb)

MOS 7: 0 2 3 6 7 9 10 (C D Eb Gb G A Bb)

MOS 12: all tones

**(5-1) Scale D Right** - 12 tones

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 5, 1, 1, 1

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 5 1 1 1 6 7 8 9 15 22 30 39 54 76 106 145...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	145/128	215.891
3:	19/16	297.513 19th harmonic
4:	39/32	342.483 39th harmonic
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	53/32	873.505
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 3/2 7/4 9/8 15/8 11/8 39/32 27/16 19/16 53/32 145/128

Subsets:

MOS 5: 0 1 5 7 10 (C C# F G Bb)

MOS 7: 0 1 5 6 7 10 11 (C C# F F# G Bb B)

MOS 9: 0 1 4 5 6 7 9 10 11 (C C# E F F# G A Bb B)

MOS 11: all except 2 (no D)

**(5-1) Scale E Right** - 12 tones

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 5, 0, 0, 1

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 5 0 0 1 5 0 1 6 5 1 7 11 6 8 18 17 14 26 35 31 40 61 66...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	35/32	155.140 septimal neutral second
4:	9/8	203.910 major whole tone
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	61/32	1116.885
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 3/2 7/4 11/8 9/8 17/16 13/8 35/32 31/16 61/32 33/32

Subsets:

MOS 7: 0 2 4 5 6 7 9 (C D E F F# G A)

MOS 10: all except 1 and 10 (no C# or Bb)

**(5-1) Scale F Right - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 5, 1, 1, 1, 1

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 5 1 1 1 1 6 7 8 9 10 16 23 31 40 50 66  
89 120 160 210...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	89/64	570.880
5:	23/16	628.274 23rd harmonic
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	105/64	857.095 septimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 3/2 7/4 9/8 23/16 31/16 25/16 33/32 89/64 15/8 105/64

Subsets:

MOS 5: 0 2 3 6 9 (C D Eb Gb A)

MOS 7: 0 2 3 5 6 9 11 (C D Eb F Gb A B)

MOS 12: all tones

**(5-1) Scale G Right - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 5, 0, 1, 0, 1

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 5 0 1 0 1 5 1 6 1 7 6 8 12 9 19 15 27 27  
36 46 51 73 78...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	73/64	227.789
3:	19/16	297.513 19th harmonic
4:	39/32	342.483 39th harmonic
5:	5/4	386.314 major third
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	51/32	806.910
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 3/2 7/4 9/8 19/16 15/8 27/16 23/16 51/32 73/64 39/32

Subsets:

MOS 7: 0 1 3 5 7 10 11 (C Db Eb F G Bb B)

MOS 10: all except 2 and 4 (no D or E)

**(5-1) Scale H Right - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 5, 0, 0, 1, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 5 0 0 1 0 5 1 0 6 1 5 7 1 11 8 6 18 9 17  
26 15 35 35 32 61 50...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 3/2 7/4 11/8 9/8 17/16 13/8 15/8 35/32 61/32 25/16

Subsets:

MOS 7: 0 1 3 4 5 6 9 (C C# D# E F F# A)

MOS 11: all except 7 (no G)

**(5-1) Scale I Right - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 5, 0, 0, 0, 1

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 5 0 0 0 1 5 0 0 1 6 5 0 1 7 11 5 1 8 18  
16 6 9 26 34 22 15 35 60 56 37 50...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	37/32	251.344 37th harmonic
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 3/2 7/4 11/8 9/8 13/8 17/16 15/8 35/32 37/32 25/16

Subsets:

MOS 5: 0 5 6 7 10 (C F F# G Bb)

MOS 9: 0 1 3 5 6 7 9 10 11 (C Db Eb F F# G A Bb B)

**(5-1) Scale J Right** - 12 tones

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 5, 1, 1, 1, 1, 1

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 5 1 1 1 1 1 6 7 8 9 10 11 17 24 32 41 51 62 79 103...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	79/64	364.537
4:	5/4	386.314 major third
5:	41/32	429.062
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	51/32	806.910
9:	103/64	823.801
10:	7/4	968.826 harmonic seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 3/2 7/4 9/8 11/8 17/16 41/32 51/32 31/16 79/64 103/64

Subsets:

MOS 5 = 0 2 4 7 10 (C D E G Bb)

MOS 8 = 0 1 2 4 5 6 7 10 (C C# D E F F# G Bb)

MOS 11 = all except 9 (no A)

**(5-1) Scale K Right** - 12 tones

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 5, 0, 0, 0, 0, 1

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 5 0 0 0 1 5 0 0 0 1 6 5 0 0 1 7 11 5 0 1 8 18 16 5 1 9 26 34 21 6 10 35 60 55...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	55/32	937.632
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 3/2 7/4 11/8 9/8 13/8 17/16 21/16 35/32 15/8 55/32

Subsets:

MOS 5 = 0 4 6 7 10 (C E F# G Bb)

MOS 6 = 0 3 4 6 7 10 (C Eb E F# G Bb)

MOS 11 = all except 9 (no A)



## Generator (5-1) Scales - Left Wing Versions

### (5-1) Scale A Left - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 1, 5

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 1 5 6 11 17 28 45 73 118 191 309 500...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	73/64	227.789
3:	309/256	325.756
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	191/128	692.915
8:	3/2	701.955 perfect fifth
9:	7/4	968.826 harmonic seventh
10:	59/32	1059.172
11:	125/64	1158.941 classic augmented seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 17/16 7/4 45/32 73/64 59/32 191/128 309/256 125/64

Subsets:

MOS 7 = 0 1 4 5 6 8 9 ( C C# E F F# Ab A)

MOS 10 = all except 3 and 11 (no Eb or B)

### (5-1) Scale B Left - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 1, 5, 5

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 1 5 5 6 11 16 22 33 49 71 104 153 224 328 481...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	71/64	179.697
3:	153/128	308.865
4:	5/4	386.314 major third
5:	41/32	429.062
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	49/32	737.652
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	481/256	1091.872
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 33/32 49/32 71/64 13/8 153/128 7/4 41/32 481/256

Subsets:

MOS 5: 0 1 4 6 7 (C C# E F# G)

MOS 7: 0 1 2 4 6 7 8 (C C# D E F# G Ab)

MOS 9: 0 1 2 3 4 6 7 8 9 (C C# D Eb E F# G Ab A)

MOS 11: all except 11 (no B)

**(5-1) Scale C Left - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 1, 0, 5

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 1 0 5 1 5 6 6 11 12 17 23 29 40 52 69 92 121 161 213...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	69/64	130.229
3:	5/4	386.314 major third
4:	161/128	397.100
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	213/128	881.652
10:	29/16	1029.577 29th harmonic
11:	121/64	1102.636
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 17/16 23/16 29/16 13/8 69/64 121/64 161/128 213/128

Subsets:

MOS 5: 0 1 3 5 7 (C C# Eb F G)

MOS 7: 0 1 3 5 6 7 10 (C C# Eb F F# G Bb)

MOS 12: all tones

**(5-1) Scale D Left - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 1, 5, 5, 5

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 1 5 5 5 6 11 16 21 27 38 54 75 102 140 194 269...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	269/256	85.755
2:	35/32	155.140 septimal neutral second
3:	75/64	274.582 classic augmented second
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	97/64	719.895
10:	51/32	806.910
11:	27/16	905.865 Pythagorean major sixth
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 21/16 27/16 19/16 75/64 51/32 35/32 97/64 269/256

Subsets:

MOS 5: 0 5 6 7 8 (C F F# G Ab)

MOS 7: 0 4 5 6 7 8 11 (C E F F# G Ab B)

MOS 9: 0 3 4 5 6 7 8 10 11 (C Eb E F F# G Ab Bb B)

MOS 11: all except 1 (no C#)

**(5-1) Scale E Left - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 1, 0, 0, 5

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 1 0 0 5 1 0 5 6 1 5 11 7 6 16 18 13 22  
34 31 35 56 65 66...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
3:	17/16	104.955 17th harmonic
4:	35/32	155.140 septimal neutral second
5:	9/8	203.910 major whole tone
6:	5/4	386.314 major third
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 7/4 9/8 13/8 17/16 31/16 35/32 65/64 33/32

Subsets:

MOS 7: 0 5 6 7 8 9 10 ( C F F# G Ab A Bb)

MOS 10: all except 1 and 2 (no C# or D)

**(5-1) Scale F Left - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 1, 5, 5, 5, 5

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 1 5 5 5 5 6 11 16 21 26 32 43 59 80 106  
138 181 240...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	69/64	130.229
2:	5/4	386.314 major third
3:	21/16	470.781 narrow fourth
4:	43/32	511.518
5:	11/8	551.318 undecimal semi-augmented fourth
6:	181/128	599.815
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	53/32	873.505
10:	59/32	1059.172
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 21/16 13/8 43/32 59/32 53/32 69/64 181/128 15/8

Subsets:

MOS 5: 0 2 3 5 7 (C D Eb F G)

MOS 7: 0 2 3 4 5 7 8 (C D Eb E F G Ab)

MOS 12: all tones

**(5-1) Scale G Left - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 1, 0, 5, 0, 5

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 1 0 5 0 5 1 5 6 5 11 6 16 12 21 23 27 39  
39 60 62 87 101...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	39/32	342.483 39th harmonic
2:	5/4	386.314 major third
3:	21/16	470.781 narrow fourth
4:	87/64	531.532
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	101/64	789.854
9:	27/16	905.865 Pythagorean major sixth
10:	15/8	1088.269 classic major seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 21/16 23/16 27/16 39/32 15/8 31/16 87/64 101/64

Subsets:

MOS 7: 0 2 3 5 6 7 9 (C D Eb F F# G A)

MOS 10: all except 4 and 8 (no E or Ab)

**(5-1) Scale H Left - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 1, 0, 0, 5, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 1 0 0 5 0 1 5 0 6 5 1 11 5 7 16 6 18 21  
13 34 27 31 55...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	13/8	840.528 tridecimal neutral sixth
8:	27/16	905.865 Pythagorean major sixth
9:	55/32	937.632
10:	7/4	968.826 harmonic seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 7/4 9/8 21/16 13/8 17/16 27/16 31/16 55/32

Subsets:

MOS 7: 0 2 3 4 5 6 10 (C D Eb E F F# Bb)

MOS 11: all except 9 (no A)

**(5-1) Scale I Left - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 1, 0, 0, 0, 5

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 1 0 0 0 5 1 0 0 5 6 1 0 5 11 7 1 5 16 18  
8 6 21 34 26 14 27 55 60...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	13/8	840.528 tridecimal neutral sixth
8:	27/16	905.865 Pythagorean major sixth
9:	55/32	937.632
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 7/4 9/8 21/16 17/16 13/8 27/16 55/32 15/8

Subsets:

MOS 5: 0 3 5 6 10 (C Eb F F# Bb)

MOS 9: 0 1 2 3 4 5 6 7 10 (C C# D Eb E F F# G Bb)

**(5-1) Scale J Left - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 1, 5, 5, 5, 5, 5

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 1 5 5 5 5 5 6 11 16 21 26 31 37 48 64 85  
111 142 179...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	71/64	179.697
2:	37/32	251.344 37th harmonic
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	85/64	491.269
6:	11/8	551.318 undecimal semi-augmented fourth
7:	179/128	580.579
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	111/64	953.299
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 21/16 13/8 31/16 37/32 85/64 111/64 71/64 179/128

Subsets:

MOS 5 = 0 3 4 6 8 (C Eb E F# Ab)

MOS 8 = 0 2 3 4 6 8 9 11 (C D Eb E F# Ab A B)

MOS 11 = all except 7 (no G)

**(5-1) Scale K Left - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 1, 0, 0, 0, 0, 5

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 1 0 0 0 0 5 1 0 0 0 5 6 1 0 0 5 11 7 1 0  
5 16 18 8 1 5 21 34 26 9 6 26 55 60 35...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	55/32	937.632
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 3/2 11/8 7/4 9/8 21/16 17/16 13/8 55/32 15/8 35/32

Subsets:

MOS 5 = 0 4 6 7 10 (C E F# G Bb)

MOS 6 = 0 3 4 6 7 10 (C Eb E F# G Bb)

MOS 11 = all except 2 (no D)

## Part 8: The 5-2 SCALES (generator 5, 2)

If the seed 5, 2 is used with the Fibonacci rule, the resulting sequence is 5 2 7 9 16 25 41 66 107 173... Here is a part of the (5, 2) triangle. Having 2 different generators makes the triangle asymmetrical. Therefore, additive sequences derived from left-leaning and right-leaning diagonals will be different, as will the scales derived from them.

```
      2/5
     2 5
    2 7 5
   2 9 12 5
  2 11 21 17 5
 2 13 32 38 22 5
```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

### Generator (5-2) Scales - Right Wing Versions

#### (5-2) Scale A Right - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 5, 2

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 5 2 7 9 16 25 41 66 107 173 280 453 733...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	41/32	429.062
6:	173/128	521.554
7:	733/512	621.203
8:	25/16	772.627 classic augmented fifth
9:	107/64	889.760
10:	7/4	968.826 harmonic seventh
11:	453/256	988.041
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 7/4 9/8 25/16 41/32 33/32 107/64 173/128 35/32 453/256 733/512

Subsets:

MOS 7 = 0 1 3 4 5 8 10 (C C# D# E F Ab Bb)

MOS 10 = all except 7 and 11 (no G or B)

**(5-2) Scale B Right - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 5, 2, 2

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 5 2 2 7 9 11 18 27 38 56 83 121 177 260 381...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	83/64	450.047
6:	11/8	551.318 undecimal semi-augmented fourth
7:	177/128	561.127
8:	381/256	688.377
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	121/64	1102.636
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 7/4 9/8 11/8 27/16 19/16 83/64 121/64 177/128 65/64 381/256

Subsets:

MOS 5: 0 2 4 6 10 (C D E F# Bb)

MOS 7: 0 2 3 4 6 9 10 (C D Eb E F# A Bb)

MOS 9: 0 2 3 4 5 6 9 10 11 (C D Eb E F F# A Bb B)

MOS 11: all except 8 (no Ab)

**(5-2) Scale C Right - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 5, 0, 2

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 5 0 2 5 2 7 7 9 14 16 23 30 39 53 69 92 122 161 214...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	69/64	130.229
2:	9/8	203.910 major whole tone
3:	39/32	342.483 39th harmonic
4:	5/4	386.314 major third
5:	161/128	397.100
6:	23/16	628.274 23rd harmonic
7:	53/32	873.505
8:	107/64	889.760
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 7/4 9/8 23/16 15/8 39/32 53/32 69/64 23/16 61/32 161/128 107/64

Subsets:

MOS 5: 0 2 4 6 9 (C D E F# A)

MOS 7: 0 2 3 4 6 9 10 (C D Eb E F# A Bb)

MOS 12: all tones



**(5-2) Scale D Right** - 12 tones

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 5, 2, 2, 2

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 5 2 2 2 7 9 11 13 20 29 40 53 73 102 142 195...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	71/64	179.697
2:	9/8	203.910 major whole tone
3:	73/64	227.789
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	195/128	728.796
7:	51/32	806.910
8:	13/8	840.528 tridecimal neutral sixth
9:	53/32	873.505
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 7/4 9/8 11/8 13/8 29/16 53/32 73/64 51/32 71/64 195/128

Subsets:

MOS 5: 0 2 4 5 10 (C D E F Bb)

MOS 7: 0 2 4 5 8 10 11 (C D E F Ab Bb B)

MOS 9: 0 2 3 4 5 8 9 10 11 (C D Eb E F Ab A Bb B)

MOS 11: all except 6 (no F#)

**(5-2) Scale E Right** - 12 tones

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 5, 0, 0, 2

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 5 0 0 2 5 0 2 7 5 2 9 12 7 11 21 19 18 32 40 37 50 72 77 87...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	37/32	251.344 37th harmonic
3:	19/16	297.513 19th harmonic
4:	77/64	320.144
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	87/64	531.532
8:	11/8	551.318 undecimal semi-augmented fourth
9:	3/2	701.955 perfect fifth
10:	25/16	772.627 classic augmented fifth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 7/4 9/8 3/2 11/8 21/16 19/16 37/32 25/16 77/64 87/64

Subsets:

MOS 7: 0 1 5 6 8 9 11 (C C# F F# Ab A B)

MOS 10: all except 4 and 7 (no E or G)

**(5-2) Scale F Right - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 5, 2, 2, 2, 2

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 5 2 2 2 2 7 9 11 13 15 22 31 42 55 70 92...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	13/8	840.528 tridecimal neutral sixth
8:	55/32	937.632
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 7/4 9/8 11/8 13/8 15/8 31/16 21/16 55/32 35/32 23/16

Subsets:

MOS 5: 0 2 3 5 9 (C D Eb F A)  
MOS 7: 0 2 3 5 7 9 10 (C D Eb F G A Bb)  
MOS 12: all tones

**(5-2) Scale G Right - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 5, 0, 2, 0, 2

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 5 0 2 0 2 5 2 7 2 9 7 11 14 13 23 20 34 34 47 57 67 91...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	17/16	104.955 17th harmonic
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	91/64	609.354
7:	23/16	628.274 23rd harmonic
8:	47/32	665.507
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	57/32	999.468
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 7/4 9/8 11/8 13/8 23/16 17/16 47/32 57/32 67/64 91/64

Subsets:

MOS 7: 0 3 4 5 7 9 10 (C Eb E F G A Bb)  
MOS 10: all except 1 and 6 (no C# or F#)

**(5-2) Scale H Right - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 5, 0, 0, 2, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 5 0 0 2 0 5 2 0 7 2 5 9 2 12 11 7 21 13  
19 32 20 40 45 39 72 65...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	39/32	342.483 39th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	11/8	551.318 undecimal semi-augmented fourth
8:	45/32	590.224 diatonic tritone
9:	3/2	701.955 perfect fifth
10:	13/8	840.528 tridecimal neutral sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 7/4 9/8 3/2 11/8 21/16 13/8 19/16 45/32 39/32 65/64

Subsets:

MOS 7: 0 2 5 6 7 9 11 (C D F F# G A B)

MOS 11: all except 1 (no C#)

**(5-2) Scale I Right - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 5, 0, 0, 0, 2

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 5 0 0 0 2 5 0 0 2 7 5 0 2 9 12 5 2 11 21  
17 7 13 32 38 24 20 45 70...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	11/8	551.318 undecimal semi-augmented fourth
8:	45/32	590.224 diatonic tritone
9:	3/2	701.955 perfect fifth
10:	13/8	840.528 tridecimal neutral sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 7/4 9/8 3/2 11/8 21/16 17/16 13/8 19/16 45/32 35/32

Subsets:

MOS 5: 0 3 5 9 11 (C Eb F A B)

MOS 9: 0 1 3 5 6 7 9 10 11 (C C# Eb F F# G A Bb B)

**(5-2) Scale J Right - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 5, 2, 2, 2, 2, 2

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 5 2 2 2 2 2 7 9 11 13 15 17 24 33 44 57 72 89...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	89/64	570.880
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	57/32	999.468
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 7/4 9/8 11/8 13/8 15/8 17/16 3/2 33/32 57/32 89/64

Subsets:

MOS 5 = 0 3 4 5 9 (C Eb E F A)

MOS 8 = 0 2 3 4 5 8 9 11 (C D Eb E F Ab A B)

MOS 11 = all except 6 (no F#)

**(5-2) Scale K Right - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 5, 0, 0, 0, 0, 2

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 5 0 0 0 0 2 5 0 0 0 2 7 5 0 0 2 9 12 5 0 2 11 21 17 5 2 13 32 38 22 7 15 45...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	45/32	590.224 diatonic tritone
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 7/4 9/8 3/2 11/8 21/16 17/16 13/8 19/16 15/8 45/32

Subsets:

MOS 5 = 0 2 4 8 10 (C D E Ab Bb)

MOS 6 = 0 2 4 6 8 10 (C D E F# Ab Bb)

MOS 11 = all except 7 (no G)

## Generator (5-2) Scales - Left Wing Versions

### (5-2) Scale A Left - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 2, 5

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 2 5 7 12 19 31 50 81 131 212 343 555...

### Identical to Yasser (3-2) Scale A Right.

Scale:

0:	1/1	0.000 unison, perfect prime
1:	131/128	40.108
2:	555/512	139.613
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	81/64	407.820 Pythagorean major third
6:	343/256	506.478
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	53/32	873.505
10:	7/4	968.826 harmonic seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 5/4 7/4 19/16 31/16 25/16 81/64 131/128 53/32 343/256 555/512

Subsets:

MOS 7 = 0 3 4 7 8 10 11 (C D# E G Ab Bb B)

MOS 10 = all except 6 and 2 (no D or F#)

### (5-2) Scale B Left - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 2, 5, 5

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 2 5 5 7 12 17 24 36 53 77 113 166 243 356...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	77/64	320.144
4:	5/4	386.314 major third
5:	83/64	450.047
6:	89/64	570.880
7:	3/2	701.955 perfect fifth
8:	53/32	873.505
9:	7/4	968.826 harmonic seventh
10:	113/64	984.215
11:	243/128	1109.775 Pythagorean major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 7/4 3/2 17/16 9/8 53/32 77/64 113/64 83/64 243/128 89/64

Subsets:

MOS 5: 0 1 4 7 9 (C C# E G A)

MOS 7: 0 1 2 4 7 8 9 (C C# D E G Ab A)

MOS 9: 0 1 2 3 4 7 8 9 10 (C C# D Eb E G Ab A Bb)

MOS 11: all except 6 (no F#)

**(5-2) Scale C Left - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 2, 0, 5

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 2 0 5 2 5 7 7 12 14 19 26 33 45 59 78 104 137 182...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	137/128	117.638
3:	19/16	297.513 19th harmonic
4:	39/32	342.483 39th harmonic
5:	5/4	386.314 major third
6:	45/32	590.224 diatonic tritone
7:	91/64	609.354
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	59/32	1059.172
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 7/4 3/2 19/16 13/8 33/32 45/32 59/32 39/32 137/128 91/64

Subsets:

MOS 5: 0 3 5 8 10 (C Eb F Ab Bb)

MOS 7: 0 1 3 5 8 9 10 (C C# Eb F Ab A Bb)

MOS 12: all tones

**(5-2) Scale D Left - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 2, 5, 5, 5

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 2 5 5 5 7 12 17 22 29 41 58 80 109 150 208 288...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	75/64	274.582 classic augmented second
4:	5/4	386.314 major third
5:	41/32	429.062
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	109/64	921.821
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 7/4 3/2 17/16 11/8 29/16 41/32 109/64 75/64 13/8 9/8

Subsets:

MOS 5: 0 1 4 7 10 (C C# E G Bb)

MOS 7: 0 1 4 6 7 10 11 (C C# E F# G Bb B)

MOS 9: 0 1 4 5 6 7 9 10 11 (C C# E F F# G A Bb B)

MOS 11: all except 2 (no D)

**(5-2) Scale E Left - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 2, 0, 0, 5

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 2 0 0 5 2 0 5 7 2 0 5 12 9 7 17 21 16 24  
38 37 40 62 75 77...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	37/32	251.344 37th harmonic
4:	75/64	274.582 classic augmented second
5:	19/16	297.513 19th harmonic
6:	77/64	320.144
7:	5/4	386.314 major third
8:	21/16	470.781 narrow fourth
9:	3/2	701.955 perfect fifth
10:	7/4	968.826 harmonic seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 7/4 3/2 9/8 17/16 21/16 19/16 37/32 31/16 75/64 77/64

Subsets:

MOS 7: 0 1 2 7 8 9 10 (C C# D G Ab A Bb)

MOS 10: all except 4 and 6 (no E or F#)

**(5-2) Scale F Left - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 2, 5, 5, 5, 5

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 2 5 5 5 5 7 12 17 22 27 34 46 63 85 112  
146 192 255...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	73/64	227.789
3:	5/4	386.314 major third
4:	85/64	491.269
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	27/16	905.865 Pythagorean major sixth
9:	7/4	968.826 harmonic seventh
10:	63/32	1172.736 octave - septimal comma
11:	255/128	1193.224
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 7/4 3/2 17/16 11/8 27/16 23/16 63/32 85/64 73/64 255/128

Subsets:

MOS 5: 0 1 3 7 9 (C C# Eb G A)

MOS 7: 0 1 3 5 7 8 9 (C C# Eb F G Ab A)

MOS 12: all tones

**(5-2) Scale G Left - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 2, 0, 5, 0, 5

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 2 0 5 0 5 2 5 7 5 12 7 17 14 22 26 29 43 43 65 69 94...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	17/16	104.955 17th harmonic
3:	69/64	130.229
4:	5/4	386.314 major third
5:	43/32	511.518
6:	11/8	551.318 undecimal semi-augmented fourth
7:	47/32	665.507
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 7/4 3/2 17/16 11/8 13/8 29/16 43/32 65/64 69/64 47/32

Subsets:

MOS 7: 0 2 4 6 8 9 10 (C D E F# Ab A Bb)

MOS 10: all except 3 and 7 (no Eb or G)

**(5-2) Scale H Left - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 2, 0, 0, 5, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 2 0 0 5 0 2 5 0 7 5 2 12 5 9 17 7 21 22 16 38 29 37 60...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	37/32	251.344 37th harmonic
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 7/4 3/2 9/8 17/16 21/16 11/8 19/16 29/16 37/32 15/8

Subsets:

MOS 7: 0 1 2 5 6 8 9 (C C# D F F# Ab A)

MOS 11: all except 11 (no B)



**(5-2) Scale I Left - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 2, 0, 0, 0, 5

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 2 0 0 0 5 2 0 0 5 7 2 0 5 12 9 2 5 17 21  
11 7 22 38 32 18 29 60 70...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 7/4 3/2 9/8 17/16 21/16 11/8 19/16 29/16 15/8 35/32

Subsets:

MOS 5: 0 3 5 8 9 (C Eb F Ab A)

MOS 9: 0 1 3 4 5 6 7 8 9 (C C# Eb E F F# G Ab A)

**(5-2) Scale J Left - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 2, 5, 5, 5, 5, 5

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 2 5 5 5 5 5 7 12 17 22 27 32 39 51 68 90  
117 149...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	149/128	263.002
3:	39/32	342.483 39th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	3/2	701.955 perfect fifth
8:	51/32	806.910
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	117/64	1044.438
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 7/4 3/2 17/16 11/8 27/16 39/32 51/32 45/32 117/64 149/128

Subsets:

MOS 5 = 0 1 4 7 10 (C C# E G Bb)

MOS 8 = 0 1 3 4 5 7 9 10 (C C# Eb E F G A Bb)

MOS 11 = all except 2 (no D)

**(5-2) Scale K Left - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 2, 0, 0, 0, 0, 5

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 2 0 0 0 0 5 2 0 0 0 5 7 2 0 0 5 12 9 2 0  
5 17 21 11 2 5 22 38 32 13 7 27 60...

**This is about as close to a "normal chromatic" scale as exists in this catalog.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 7/4 3/2 9/8 17/16 21/16 11/8 19/16 13/8 27/16 15/8

Subsets:

MOS 5 = 0 2 4 7 10 (C D E G Bb)

MOS 6 = 0 1 2 4 7 10 (C C# D E G Bb)

MOS 11 = all except 11 (no B)

## Part 9: The 5-3 SCALES (generator 5, 3)

If the seed 5, 3 is used with the Fibonacci rule, the resulting sequence is 5 3 8 11 19 30 49 79 128... Here is a part of the (5, 3) triangle. Having 2 different generators makes the triangle asymmetrical. Therefore, additive sequences derived from left-leaning and right-leaning diagonals will be different, as will the scales derived from them.

```
      3/5
     3 5
    3 8 5
   3 11 13 5
  3 14 24 18 5
 3 17 38 42 23 5
3 20 55 80 65 28 5
```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

### Generator (5-3) Scales - Right Wing Versions

#### (5-3) Scale A Right - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 5, 3

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 5 3 8 11 19 30 49 79 128 207 335 542 877...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	271/256	98.579
2:	19/16	297.513 19th harmonic
3:	79/64	364.537
4:	5/4	386.314 major third
5:	335/256	465.621
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	49/32	737.652
9:	207/128	832.184
10:	877/512	931.720
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 1/1 11/8 19/16 15/8 49/32 79/64 207/128 335/256 271/256 877/512

Subsets:

MOS 7 = 0 2 4 6 7 8 11 (C D E F# G Ab B)

MOS 10 = all except 1 and 10 (no C# or Bb)

**(5-3) Scale B Right - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 5, 3, 3

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 5 3 3 8 11 14 22 33 47 69 102 149 218 320 469...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	69/64	130.229
3:	149/128	263.002
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	51/32	806.910
9:	109/64	921.821
10:	7/4	968.826 harmonic seventh
11:	469/256	1048.133
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 1/1 11/8 7/4 33/32 47/32 69/64 51/32 149/128 109/64 469/256

Subsets:

MOS 5: 0 4 5 7 10 (C E F G Bb)

MOS 7: 0 1 4 5 6 7 10 (C C# E F F# G Bb)

MOS 9: 0 1 2 4 5 6 7 8 10 (C C# D E F F# G Ab Bb)

MOS 11: all except 11 (no B)

**(5-3) Scale C Right - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 5, 0, 3

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 5 0 3 5 3 8 8 11 16 19 27 35 46 62 81 108 143 189...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	143/128	191.846
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	81/64	407.820 Pythagorean major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	189/128	674.691
9:	3/2	701.955 perfect fifth
10:	27/16	905.865 Pythagorean major sixth
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 1/1 11/8 19/16 27/16 35/32 23/16 31/16 81/64 143/128 198/128

Subsets:

MOS 5: 0 3 4 6 9 (C Eb E F# A)

MOS 7: 0 1 3 4 6 9 10 (C C# Eb E F# A Bb)

MOS 12: all tones

**(5-3) Scale D Right - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 5, 3, 3, 3

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 5 3 3 3 8 11 14 17 25 36 50 67 92 128 178 245...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	17/16	104.955 17th harmonic
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	89/64	570.880
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	25/16	772.627 classic augmented fifth
10:	7/4	968.826 harmonic seventh
11:	245/128	1123.966
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 1/1 11/8 7/4 17/16 25/16 9/8 67/64 23/16 89/64 245/128

Subsets:

MOS 5: 0 4 5 8 10 (C E F Ab Bb)

MOS 7: 0 2 4 5 8 9 10 (C D E F Ab A Bb)

MOS 9: 0 1 2 3 4 5 8 9 10 (C C# D Eb E F Ab A Bb)

MOS 11: all except 11 (no B)

**(5-3) Scale E Right - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 5, 0, 0, 3

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 5 0 0 3 5 0 3 8 5 3 11 13 8 14 24 21 22 38 45 43 60 83...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	19/16	297.513 19th harmonic
2:	5/4	386.314 major third
3:	83/64	450.047
4:	21/16	470.781 narrow fourth
5:	43/32	511.518
6:	11/8	551.318 undecimal semi-augmented fourth
7:	45/32	590.224 diatonic tritone
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 1/1 11/8 13/8 7/4 21/16 19/16 45/32 43/32 15/8 83/64

Subsets:

MOS 7: 0 2 4 6 8 9 10 (C D E F# Ab A Bb)

MOS 10: all except 3 and 11 (no Eb or B)

**(5-3) Scale F Right - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 5, 3, 3, 3, 3

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 5 3 3 3 3 8 11 14 17 20 28 39 53 70 90 118 157...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	39/32	342.483 39th harmonic
4:	157/128	353.545
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	45/32	590.224 diatonic tritone
8:	3/2	701.955 perfect fifth
9:	53/32	873.505
10:	7/4	968.826 harmonic seventh
11:	59/32	1059.172
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 1/1 11/8 7/4 17/16 39/32 53/32 35/32 45/32 59/32 157/128

Subsets:

MOS 5: 0 5 6 8 10 (C F F# Ab Bb)

MOS 7: 0 1 3 5 6 8 10 (C C# D# F Gb Ab Bb)

MOS 12: all tones

**(5-3) Scale G Right - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 5, 0, 3, 0, 3

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 5 0 3 0 3 5 3 8 3 11 8 14 16 17 27 25 41 41 58 68 83 109...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	5/4	386.314 major third
3:	41/32	429.062
4:	83/64	450.047
5:	11/8	551.318 undecimal semi-augmented fourth
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	27/16	905.865 Pythagorean major sixth
9:	109/64	921.821
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 1/1 11/8 7/4 17/16 27/16 25/16 41/32 29/16 83/64 109/64

Subsets:

MOS 7: 0 1 2 5 6 8 10 (C C# D F Gb Ab Bb)

MOS 10: all except 4 and 9 (no E or A)

**(5-3) Scale H Right - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 5, 0, 0, 3, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 5 0 0 3 0 5 3 0 8 3 5 11 3 13 14 8 24 17  
21 38 25 45 55...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	19/16	297.513 19th harmonic
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	55/32	937.632
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 1/1 11/8 13/8 7/4 17/16 21/16 19/16 25/16 45/32 55/32

Subsets:

MOS 7: 0 1 3 5 7 9 11 (C Db Eb F G A B)

MOS 11: all except 10 (no Bb)

**(5-3) Scale I Right - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 5, 0, 0, 0, 3

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 5 0 0 0 3 5 0 0 3 8 5 0 3 11 13 5 3 14  
24 18 8 17 38 42 26 25 55...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	55/32	937.632
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 1/1 11/8 13/8 7/4 9/8 17/16 19/16 21/16 25/16 55/32

Subsets:

MOS 5: 0 4 6 7 9 (C E F# G A)

MOS 9: 0 1 2 3 4 6 7 9 11 (C C# D Eb E F# G A B)

**(5-3) Scale J Right - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 5, 3, 3, 3, 3, 3

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 5 3 3 3 3 3 8 11 14 17 20 23 31 42 56 73 93 116...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	73/64	227.789
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	93/64	646.991
8:	3/2	701.955 perfect fifth
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 1/1 11/8 7/4 17/16 23/16 31/16 21/16 73/64 93/64 29/16

Subsets:

MOS 5 = 0 3 5 8 9 (C Eb F Ab A)

MOS 8 = 0 1 3 5 6 8 9 11 (C Db Eb F Gb Ab A B)

MOS 11 = all except 10 (no Bb)

**(5-3) Scale K Right - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 5, 0, 0, 0, 0, 3

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 5 0 0 0 0 3 5 0 0 0 3 8 5 0 0 3 11 13 5 0 3 14 24 18 5 3 17 38 42 23 8 20 55...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	55/32	937.632
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 1/1 11/8 13/8 7/4 9/8 17/16 19/16 21/16 23/16 55/32

Subsets:

MOS 5 = 0 4 6 8 9 (C E F# G# A)

MOS 6 = 0 4 6 8 9 11 (C E F# G# A B)

MOS 11 = all except 10 (no Bb)



### Generator (5-3) Scales Left Wing Versions

#### 5-3) Scale A Left - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 3, 5

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 3 5 8 13 21 34 55 89 144 233 377 605...

#### Identical with Meru (1-1) Scale A Right

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	605/512	288.950
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	89/64	570.880
7:	377/256	670.105
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	55/32	937.632
11:	233/128	1037.023
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 1/1 13/8 21/16 17/16 55/32 89/64 9/8 233/128 377/256 305/256

Subsets:

MOS 7 = 0 1 4 5 8 9 10 (C C# E F G# A Bb)

MOS 10 = all except 3 and 7 (no Eb or G)

#### (5-3) Scale B Left - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 3, 5, 5

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 3 5 5 8 13 18 26 39 57 83 122 179 262 384 563...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	131/128	40.108
2:	563/512	164.389
3:	9/8	203.910 major whole tone
4:	39/32	342.483 39th harmonic
5:	5/4	386.314 major third
6:	83/64	450.047
7:	179/128	580.579
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	57/32	999.468
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 1/1 13/8 9/8 39/32 57/32 83/64 61/32 179/128 131/128 563/512

Subsets:

MOS 5: 0 3 5 8 9 (C Eb F Ab A)

MOS 7: 0 3 4 5 8 9 10 (C Eb E F Ab A Bb)

MOS 9: 0 3 4 5 6 8 9 10 11 (C Eb E F F# Ab A Bb B)

MOS 11: all except 2 (no D)

**(5-3) Scale C Left - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 3, 0, 5

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 3 0 5 3 5 8 8 13 16 21 29 37 50 66 87 116 153 203...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	37/32	251.344 37th harmonic
3:	153/128	308.865
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	87/64	531.532
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	203/128	798.403
10:	13/8	840.528 tridecimal neutral sixth
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 1/1 13/8 21/16 29/16 37/32 25/16 33/32 87/64 153/128 203/128

Subsets:

MOS 5: 0 4 5 7 10 (C E F G Bb)

MOS 7: 0 2 4 5 7 10 11 (C D E F G Bb B)

MOS 12: all tones

**(5-3) Scale D Left - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 3, 5, 5, 5

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 3 5 5 5 8 13 18 23 31 44 62 85 116 160 222 307...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	307/256	314.514
3:	5/4	386.314 major third
4:	85/64	491.269
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	111/64	953.299
10:	29/16	1029.577 29th harmonic
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 1/1 13/8 9/8 23/16 31/16 11/8 85/64 29/16 111/64 307/256

Subsets:

MOS 5: 0 1 3 7 8 (C C# Eb G Ab)

MOS 7: 0 1 3 6 7 8 11 (C C# Eb F# G Ab B)

MOS 9: 0 1 3 4 5 6 7 8 11 (C C# Eb E F F# G Ab B)

MOS 11: all except 2 (no D)

**(5-3) Scale E Left - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 3, 0, 0, 5

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 3 0 0 5 3 0 5 8 3 5 13 11 8 18 24 19 26  
42 43 45 68 85...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	85/64	491.269
7:	43/32	511.518
8:	11/8	551.318 undecimal semi-augmented fourth
9:	45/32	590.224 diatonic tritone
10:	3/2	701.955 perfect fifth
11:	13/8	840.528 tridecimal neutral sixth
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 1/1 13/8 11/8 9/8 19/16 21/16 43/32 45/32 17/16 85/64

Subsets:

MOS 7: 0 2 3 4 8 10 11 (C D Eb E Ab Bb B)

MOS 10: all except 1 and 6 (no C# or F#)

**(5-3) Scale F Left - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 3, 5, 5, 5, 5

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 3 5 5 5 5 8 13 18 23 28 36 49 67 90 118  
154...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	9/8	203.910 major whole tone
3:	77/64	320.144
4:	5/4	386.314 major third
5:	45/32	590.224 diatonic tritone
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	49/32	737.652
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	59/32	1059.172
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 1/1 13/8 9/8 23/16 7/4 49/32 67/64 45/32 59/32 77/64

Subsets:

MOS 5: 0 2 4 7 9 (C D E G A)

MOS 7: 0 2 4 6 7 9 10 (C D E F# G A Bb)

MOS 12: all tones

**(5-3) Scale G Left - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 3, 0, 5, 0, 5

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 3 0 5 0 5 3 5 8 5 13 8 18 16 23 29 31 47  
47 70 76 101...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	23/16	628.274 23rd harmonic
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	101/64	789.854
9:	13/8	840.528 tridecimal neutral sixth
10:	29/16	1029.577 29th harmonic
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 1/1 13/8 9/8 23/16 29/16 31/16 47/32 35/32 19/16 101/64

Subsets:

MOS 7: 0 2 4 5 7 9 10 (C D E F G A Bb)

MOS 10: all except 3 and 8 (no Eb or Ab)

**(5-3) Scale H Left - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 3, 0, 0, 5, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 3 0 0 5 0 3 5 0 8 5 3 13 5 11 18 8 24 23  
19 42 31 43 65...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	43/32	511.518
7:	11/8	551.318 undecimal semi-augmented fourth
8:	23/16	628.274 23rd harmonic
9:	3/2	701.955 perfect fifth
10:	13/8	840.528 tridecimal neutral sixth
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 1/1 13/8 11/8 9/8 23/16 19/16 21/16 31/16 43/32 65/64

Subsets:

MOS 7: 0 2 4 7 8 9 10 (C D E G Ab A Bb)

MOS 11: all except 1 (no C#)

**(5-3) Scale I Left - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 3, 0, 0, 0, 5

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 3 0 0 0 5 3 0 0 5 8 3 0 5 13 11 3 5 18  
24 14 8 23 42 38 22 31 65...

**Only 1 tone different from the previous scale.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 1/1 13/8 11/8 9/8 7/4 23/16 21/16 19/16 31/16 65/64

Subsets:

MOS 5: 0 4 6 8 9 (C E F# Ab A)

MOS 9: 0 2 4 5 6 7 8 9 10 (C D E F F# G Ab A Bb)

**(5-3) Scale J Left - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 3, 5, 5, 5, 5, 5

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 3 5 5 5 5 5 8 13 18 23 28 33 41 54 72 95  
123...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	41/32	429.062
5:	23/16	628.274 23rd harmonic
6:	95/64	683.827
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	123/64	1131.017
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 1/1 13/8 9/8 23/16 7/4 33/32 41/32 27/16 95/64 123/64

Subsets:

MOS 5 = 0 2 3 7 8 (C D Eb G Ab)

MOS 8 = 0 1 2 3 5 7 8 10 (C C# D Eb F G Ab Bb)

MOS 11 = all except 11 (no B)

**(5-3) Scale K Left - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 3, 0, 0, 0, 0, 5

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 3 0 0 0 0 5 3 0 0 0 5 8 3 0 0 5 13 11 3  
0 5 18 24 14 3 5 23 42 38 17 8 28 65...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	17/16	104.955 17th harmonic
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	11/8	551.318 undecimal semi-augmented fourth
8:	23/16	628.274 23rd harmonic
9:	3/2	701.955 perfect fifth
10:	13/8	840.528 tridecimal neutral sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 1/1 13/8 11/8 9/8 7/4 23/16 21/16 19/16 17/16 65/64

Subsets:

MOS 5 = 0 5 7 9 10 (C F G A Bb)

MOS 6 = 0 3 5 7 9 10 (C Eb F G A Bb)

MOS 11 = all except 1 (no C#)

## Part 10: The 5-4 SCALES (generator 5, 4)

If the seed 5, 4 is used with the Fibonacci rule, the resulting sequence is 5 4 9 13 22 35 57 92 149 241... Here is a part of the (5, 4) triangle. Having 2 different generators makes the triangle asymmetrical. Therefore, additive sequences derived from left-leaning and right-leaning diagonals will be different, as will the scales derived from them.

```

      4/5
     4 5
    4 9 5
   4 13 14 5
  4 17 27 19 5
 4 21 44 46 24 5
4 25 65 90 70 29 5
```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

### Generator (5-4) Scales - Right Wing Versions

#### (5-4) Scale A Right - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 5, 4

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 5 4 9 13 22 35 57 92 149 241 390 631...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	9/8	203.910 major whole tone
3:	149/128	263.002
4:	631/512	361.795
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	195/128	728.796
9:	13/8	840.528 tridecimal neutral sixth
10:	57/32	999.468
11:	241/128	1095.467
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 9/8 13/8 11/8 35/32 57/32 23/16 149/128 241/128 195/128 631/512

Subsets:

MOS 7 = 0 1 2 5 6 9 10 (C C# D F F# A Bb)

MOS 10 = all except 4 and 8 (no E or Ab)

**(5-4) Scale B Right** - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 5, 4, 4

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 5 4 4 9 13 17 26 39 56 82 121 177 259 380...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	259/256	20.170
2:	17/16	104.955 17th harmonic
3:	9/8	203.910 major whole tone
4:	39/32	342.483 39th harmonic, Zalzal wosta of Ibn Sina
5:	5/4	386.314 major third
6:	41/32	429.062
7:	177/128	561.127
8:	95/64	683.827
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	121/64	1102.636
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 9/8 13/8 17/16 39/32 7/4 41/32 121/64 177/128 259/256 95/64

Subsets:

MOS 5: 0 2 3 5 9 (C D Eb F A)  
MOS 7: 0 2 3 4 5 9 10 (C D Eb E F A Bb)  
MOS 9: 0 2 3 4 5 6 9 10 11 (C D Eb E F F# A Bb B)  
MOS 11: all except 8 (no Ab)

**(5-4) Scale C Right** - 12 tones

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 5, 0, 4

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 5 0 4 5 4 9 9 13 18 22 31 40 53 71 93 124 164 217 288 381...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	71/64	179.697
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	41/32	429.062
5:	11/8	551.318 undecimal semi-augmented fourth
6:	93/64	646.991
7:	381/256	688.377
8:	13/8	840.528 tridecimal neutral sixth
9:	53/32	873.505
10:	217/128	913.861
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 9/8 13/8 11/8 31/16 53/32 71/64 93/64 41/32 217/128 381/256

Subsets:

MOS 5: 0 2 3 5 8 (C D Eb F Ab)  
MOS 7: 0 2 3 5 8 9 11 (C D Eb F Ab A B)  
MOS 12: all tones



**(5-4) Scale D Right** - 12 tones

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 5, 4, 4, 4

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 5 4 4 4 9 13 17 21 30 43 60 81 111 154 214...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	77/64	320.144
4:	5/4	386.314 major third
5:	81/64	407.820 Pythagorean major third
6:	21/16	470.781 narrow fourth
7:	43/32	511.518
8:	13/8	840.528 tridecimal neutral sixth
9:	107/64	889.760
10:	111/64	953.299
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 9/8 13/8 17/16 21/16 15/8 43/32 81/64 111/64 77/64 107/64

Subsets:

MOS 5: 0 1 2 4 8 (C C# D E Ab)

MOS 7: 0 1 2 4 6 8 11 (C C# D E Gb Ab B)

MOS 9: 0 1 2 4 5 6 7 8 11 (C C# D E F F# G Ab B)

MOS 11: all except 9 (no A)

**(5-4) Scale E Right** - 12 tones

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 5, 0, 0, 4

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 5 0 0 4 5 0 4 9 5 4 13 14 9 17 27 23 26 44 50 49 70 ...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	49/32	737.652
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	27/16	905.865 Pythagorean major sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 9/8 13/8 7/4 17/16 27/16 23/16 11/8 25/16 49/32 35/32

Subsets:

MOS 7: 0 1 3 4 9 10 11 (C C# D# E A Bb B)

MOS 10: all except 2 and 7 (no D or G)

**(5-4) Scale F Right - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 5, 4, 4, 4, 4

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 5 4 4 4 4 9 13 17 21 25 34 47 64 85 110  
144 191 255...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	85/64	491.269
6:	47/32	665.507
7:	191/128	692.915
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	55/32	937.632
11:	255/128	1193.224
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 9/8 13/8 17/16 21/16 25/16 47/32 85/64 55/32 191/128 255/128

Subsets:

MOS 5: 0 1 2 3 9 (C C# D Eb A)

MOS 7: 0 1 2 3 4 8 9 (C C# D Eb E Ab A)

MOS 12: all tones

**(5-4) Scale G Right - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 5, 0, 4, 0, 4

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 5 0 4 0 4 5 4 9 4 13 9 17 18 21 31 30 48  
48 69 79 99...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	69/64	130.229
3:	9/8	203.910 major whole tone
4:	79/64	364.537
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	3/2	701.955 perfect fifth
8:	99/64	755.228
9:	13/8	840.528 tridecimal neutral sixth
10:	15/8	1088.269 classic major seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 9/8 13/8 17/16 21/16 31/16 15/8 3/2 69/64 79/64 99/64

Subsets:

MOS 7: 0 1 3 5 6 9 11 (C Db Eb F Gb A B)

MOS 10: all except 4 and 8 (no E or Ab)

**(5-4) Scale H Right - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 5, 0, 0, 4, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 5 0 0 4 0 5 4 0 9 4 5 13 4 14 17 9 27 21 23 44 30 50...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	25/16	772.627 classic augmented fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 9/8 13/8 7/4 17/16 27/16 21/16 23/16 11/8 15/8 25/16

Subsets:

MOS 7: 0 1 2 3 8 9 10 (C C# D Eb Ab A Bb)

MOS 11: all except 7 (no G)

**(5-4) Scale I Right - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 5, 0, 0, 0, 4

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 5 0 0 0 4 5 0 0 4 9 5 0 4 13 14 5 4 17 27 19 9 21 44 46 28 30...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 9/8 13/8 7/4 17/16 27/16 19/16 21/16 11/8 23/16 15/8

Subsets:

MOS 5: 0 2 4 8 10 (C D E Ab Bb)

MOS 9: 0 1 2 3 4 5 8 9 10 (C C# D Eb E F Ab A Bb)

**(5-4) Scale J Right** - 12 tones

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 5, 4, 4, 4, 4, 4

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 5 4 4 4 4 4 9 13 17 21 25 29 38 51 68 89 114...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	89/64	570.880
7:	25/16	772.627 classic augmented fifth
8:	51/32	806.910
9:	13/8	840.528 tridecimal neutral sixth
10:	57/32	999.468
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 9/8 13/8 17/16 21/16 25/16 29/16 19/16 51/32 89/64 57/32

Subsets:

MOS 5 = 0 1 2 4 9 (C C# D E A)

MOS 8 = 0 1 2 4 5 7 9 11 (C C# D E F G A B)

MOS 11 = all except 10 (no Bb)

**(5-4) Scale K Right** - 12 tones

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 5, 0, 0, 0, 0, 4

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 5 0 0 0 0 4 5 0 0 0 4 9 5 0 0 4 13 14 5 0 4 17 27 19 5 4 21 44 46 24...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	27/16	905.865 Pythagorean major sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 1/1 9/8 13/8 7/4 17/16 27/16 19/16 21/16 11/8 23/16 3/2

Subsets:

MOS 5 = 0 2 4 9 11 (C D E A B)

MOS 6 = 0 1 2 4 9 11 (C C# D E A B)

MOS 11 = all except 8 (no Ab)

## Generator (5-4) Scales - Left Wing Versions

### (5-4) Scale A Left - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 4, 5

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 4 5 9 14 23 37 60 97 157 254 411 665...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	37/32	251.344 37th harmonic
3:	157/128	353.545
4:	5/4	386.314 major third
5:	665/512	452.653
6:	23/16	628.274 23rd harmonic
7:	97/64	719.895
8:	411/256	819.594
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	127/64	1186.422
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 23/16 37/32 15/8 97/64 157/128 127/64 411/256 665/512

Subsets:

MOS 7 = 0 1 2 4 6 9 10 (C C# D E F# A Bb)

MOS 10 = all except 5 and 8 (no F or Ab)

### (5-4) Scale B Left - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 4, 5, 5

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 4 5 5 9 14 19 28 42 61 89 131 192 281 412...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	131/128	40.108
2:	281/256	161.312
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	89/64	570.880
8:	3/2	701.955 perfect fifth
9:	103/64	823.801
10:	7/4	968.826 harmonic seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 19/16 21/16 61/32 89/64 131/128 3/2 281/256 103/64

Subsets:

MOS 5: 0 3 4 5 10 (C Eb E F Bb)

MOS 7: 0 3 4 5 6 10 11 (C Eb E F F# Bb B)

MOS 9: 0 1 3 4 5 6 7 10 11 (C C# Eb E F F# G Bb B)

MOS 11: all except 9 (no A)

**(5-4) Scale C Left - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 4, 0, 5

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 4 0 5 4 5 9 9 14 18 23 32 41 55 73 96  
128 169 224 297 393...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	73/64	227.789
3:	297/256	257.183
4:	5/4	386.314 major third
5:	41/32	429.062
6:	169/128	481.055
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	393/256	742.063
10:	55/32	937.632
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 23/16 41/32 55/32 73/64 3/2 169/128 297/256 393/256

Subsets:

MOS 5: 0 1 4 7 11 (C C# E G B)

MOS 7: 0 1 4 5 7 10 11 (C C# E F G Bb B)

MOS 12: all tones

**(5-4) Scale D Left - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 4, 5, 5, 5

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 4 5 5 5 9 14 19 24 33 47 66 90 123 170  
236...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	85/64	491.269
6:	45/32	590.224 diatonic tritone
7:	47/32	665.507
8:	3/2	701.955 perfect fifth
9:	7/4	968.826 harmonic seventh
10:	59/32	1059.172
11:	123/64	1131.017
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 19/16 3/2 33/32 47/32 45/32 123/64 85/64 59/32

Subsets:

MOS 5: 0 2 3 4 9 (C D Eb E A)

MOS 7: 0 1 2 3 4 8 9 (C C# D Eb E Ab A)

MOS 9: 0 1 2 3 4 6 7 8 9 (C C# D Eb E F F# Ab A)

MOS 11: all except 10 (no Bb)

**(5-4) Scale E Left - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 4, 0, 0, 5

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 4 0 0 5 4 0 5 9 4 5 14 13 9 19 27 22 28 46 49 50 74...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	9/8	203.910 major whole tone
2:	37/32	251.344 37th harmonic
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	49/32	737.652
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	27/16	905.865 Pythagorean major sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 13/8 19/16 27/16 11/8 23/16 49/32 25/16 37/32

Subsets:

MOS 7: 0 1 3 4 9 10 11 (C C# Eb E A Bb B)

MOS 10: all except 2 and 8 (no D or Ab)

**(5-4) Scale F Left - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 4, 5, 5, 5, 5

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 4 5 5 5 5 9 14 19 24 29 38 52 71 95 124 162...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	71/64	179.697
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	81/64	407.820 Pythagorean major third
6:	95/64	683.827
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 19/16 3/2 29/16 13/8 71/64 95/64 31/16 81/64

Subsets:

MOS 5: 0 2 3 4 9 (C D Eb E A)

MOS 7: 0 2 3 4 7 9 10 (C D Eb E G A Bb)

MOS 12: all tones

**(5-4) Scale G Left - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 4, 0, 5, 0, 5

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 4 0 5 0 5 4 5 9 5 14 9 19 18 24 32 33 51  
51 75 83 108 134...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	67/64	79.307
3:	9/8	203.910 major whole tone
4:	75/64	274.582 classic augmented second
5:	19/16	297.513 19th harmonic
6:	5/4	386.314 major third
7:	83/64	450.047
8:	3/2	701.955 perfect fifth
9:	51/32	806.910
10:	27/16	905.865 Pythagorean major sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 19/16 3/2 33/32 51/32 75/64 83/64 27/16 67/64

Subsets:

MOS 7: 0 1 3 5 6 8 11 (C C# Eb F F# Ab B)

MOS 10: all except 2 and 10 (no D or Bb)

**(5-4) Scale H Left - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 4, 0, 0, 5, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 4 0 0 5 0 4 5 0 9 5 4 14 5 13 19 9 27 24  
22 46 33 49...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	49/32	737.652
9:	13/8	840.528 tridecimal neutral sixth
10:	27/16	905.865 Pythagorean major sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 13/8 19/16 27/16 3/2 11/8 23/16 33/32 49/32

Subsets:

MOS 7: 0 2 3 4 9 10 11 (C D Eb E A Bb B)

MOS 11: all except 8 (no Ab)



**(5-4) Scale I Left - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 4, 0, 0, 0, 5

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 4 0 0 0 5 4 0 0 5 9 4 0 5 14 13 4 5 19  
27 17 9 24 46 44 26 33...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	27/16	905.865 Pythagorean major sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 13/8 19/16 27/16 17/16 3/2 23/16 11/8 33/32

Subsets:

MOS 5: 0 3 5 9 11 (C Eb F A B)

MOS 9: 0 2 3 4 5 8 9 10 11 (C D Eb E F Ab A Bb B)

**(5-4) Scale J Left - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 4, 5, 5, 5, 5, 5

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 4 5 5 5 5 5 9 14 19 24 29 34 43 57 76  
100 129...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	129/128	13.473
2:	17/16	104.955 17th harmonic
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	43/32	511.518
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	7/4	968.826 harmonic seventh
10:	57/32	999.468
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 19/16 3/2 29/16 17/16 43/32 57/32 25/16 129/128

Subsets:

MOS 5 = 0 3 4 5 9 (C Eb E F A)

MOS 8 = 0 2 3 4 5 7 9 11 (C D Eb E F G A B)

MOS 11 = all except 1 (no C#)

**(5-4) Scale K Left - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 4, 0, 0, 0, 0, 5

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 4 0 0 0 0 5 4 0 0 0 5 9 4 0 0 5 14 13 4  
0 5 19 27 17 4 5 24 46 44 21...

**This scale is identical with (5-4) Scale K Right, although MOS 6 is different by 1 degree.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	27/16	905.865 Pythagorean major sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 5/4 9/8 7/4 13/8 19/16 27/16 17/16 3/2 23/16 11/8 21/16

Subsets:

MOS 5 = 0 2 4 9 11 (C D E A B)

MOS 6 = 0 2 3 4 9 11 (C D Eb E A B)

MOS 11 = all except 5 (no E)

## Part 11: The 6-1 SCALES (generator 6, 1)

If the seed 6, 1 is used with the Fibonacci rule, the resulting sequence is 6 1 7 8 15 23 38 61 99 160... Here is a part of the (6, 1) triangle. Having 2 different generators makes the triangle asymmetrical. Therefore, additive sequences derived from left-leaning and right-leaning diagonals will be different, as will the scales derived from them.

```

      1/6
      1 6
     1 7 6
    1 8 13 6
   1 9 21 19 6
  1 10 30 40 25 6
 1 11 40 70 65 31 6
```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

### Generator (6-1) Scales - Right Wing Versions

#### (6-1) Scale A Right - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 6,1

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 6 1 7 8 15 23 38 61 99 160 259 419 678...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	259/256	20.170
2:	19/16	297.513 19th harmonic
3:	5/4	386.314 major third
4:	339/256	486.170
5:	23/16	628.274 23rd harmonic
6:	3/2	701.955 perfect fifth
7:	99/64	755.228
8:	419/256	852.968
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 15/8 23/16 19/16 61/32 99/64 5/4 259/256 419/256 339/256

Subsets:

MOS 7 = 0 2 5 6 9 10 11 (C D F F# A Bb B)

MOS 10 = all except 4 and 8 (no E or Ab)

**(6-1) Scale B Right** - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 6, 1, 1

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 6 1 1 7 8 9 16 24 33 49 73 106 155 228 334 489...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	9/8	203.910 major whole tone
3:	73/64	227.789
4:	155/128	331.349
5:	167/128	460.445
6:	3/2	701.955 perfect fifth
7:	49/32	737.652
8:	53/32	873.505
9:	7/4	968.826 harmonic seventh
10:	57/32	999.468
11:	489/256	1120.429
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 9/8 33/32 49/32 73/64 53/32 155/128 57/32 167/128 489/256

Subsets:

MOS 5: 0 1 2 6 9 (C C# D F# A)

MOS 7: 0 1 2 3 6 7 9 (C C# D D# F# G A)

MOS 9: 0 1 2 3 4 6 7 8 9 (C C# D D# E F# G Ab A)

MOS 11: all except 11 (no B)

**(6-1) Scale C Right** - 12 tones

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 6, 0, 1

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 6 0 1 6 1 7 7 8 14 15 22 29 37 51 66 88 117 154 205...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	37/32	251.344 37th harmonic
3:	77/64	320.144
4:	11/8	551.318 undecimal semi-augmented fourth
5:	3/2	701.955 perfect fifth
6:	51/32	806.910
7:	205/128	815.376
8:	7/4	968.826 harmonic seventh
9:	29/16	1029.577 29th harmonic
10:	117/64	1044.438
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 15/8 11/8 29/16 37/32 51/32 33/32 117/64 77/64 205/128

Subsets:

MOS 5: 0 4 5 8 11 (C E F Ab B)

MOS 7: 0 2 4 5 8 9 11 (C D E F Ab A B)

MOS 12: all tones

**(6-1) Scale D Right - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 6, 1, 1, 1

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 6 1 1 1 7 8 9 10 17 25 34 44 61 86 120 164...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	41/32	429.062
5:	43/32	511.518
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 9/8 5/4 17/16 25/16 11/8 61/32 43/32 15/8 41/32

Subsets:

MOS 5: 0 2 3 7 9 (C D Eb G A)

MOS 7: 0 1 2 3 7 8 9 (C C# D Eb G Ab A)

MOS 9: 0 1 2 3 6 7 8 9 11 (C C# D Eb Gb G Ab A B)

MOS 11: all except 4 (no E)

**(6-1) Scale E Right - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 6, 0, 0, 1

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 6 0 0 1 6 0 1 7 6 1 8 13 7 9 21 20 16 30 41 36 46 71 77...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	71/64	179.697
2:	9/8	203.910 major whole tone
3:	77/64	320.144
4:	5/4	386.314 major third
5:	41/32	429.062
6:	21/16	470.781 narrow fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 13/8 9/8 21/16 5/4 15/8 41/32 23/16 71/64 77/64

Subsets:

MOS 7: 0 2 4 6 8 9 10 (C D E Gb Ab A Bb)

MOS 10: all except 1 and 3 (no C# or Eb)

**(6-1) Scale F Right - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 6, 1, 1, 1, 1

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 6 1 1 1 1 7 8 9 10 11 18 26 35 45 56 74 100 135...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	135/128	92.179 major chroma, major limma
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	37/32	251.344 37th harmonic
5:	5/4	386.314 major third
6:	11/8	551.318 undecimal semi-augmented fourth
7:	45/32	590.224 diatonic tritone
8:	3/2	701.955 perfect fifth
9:	25/16	772.627 classic augmented fifth
10:	13/8	840.528 tridecimal neutral sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 9/8 5/4 11/8 13/8 35/32 45/32 37/32 25/16 135/128

Subsets:

MOS 5: 0 3 5 8 11 (C Eb Gb Ab B)

MOS 7: 0 3 5 6 8 10 11 (C Eb F Gb Ab Bb B)

MOS 12: all tones

**(6-1) Scale G Right - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 6, 0, 1, 0, 1

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 6 0 1 0 1 6 1 7 1 8 7 9 14 10 22 17 31 31 41 53 58 84...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	41/32	429.062
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	53/32	873.505
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 9/8 5/4 11/8 17/16 31/16 41/32 53/32 29/16 21/16

Subsets:

MOS 7: 0 1 2 3 6 7 9 (C C# D Eb Gb G A)

MOS 10: all except 5 and 10 (no F or Bb)

**(6-1) Scale H Right - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 6, 0, 0, 1, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 6 0 0 1 0 6 1 0 7 1 6 8 1 13 9 7 21 10  
20 30 17 41 40 37 71...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	71/64	179.697
3:	9/8	203.910 major whole tone
4:	37/32	251.344 37th harmonic
5:	5/4	386.314 major third
6:	41/32	429.062
7:	21/16	470.781 narrow fourth
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 13/8 9/8 21/16 5/4 15/8 17/16 41/32 37/32 71/64

Subsets:

MOS 7: 0 3 5 7 8 9 10 (C Eb F G Ab A Bb)

MOS 11: all except 2 (no D)

**(6-1) Scale I Right - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 6, 0, 0, 0, 1

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 6 0 0 0 1 6 0 0 1 7 6 0 1 8 13 6 1 9 21  
19 7 10 30 40 26 17 40 70 66...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	35/32	155.140 septimal neutral second
4:	9/8	203.910 major whole tone
5:	19/16	297.513 19th harmonic
6:	5/4	386.314 major third
7:	21/16	470.781 narrow fourth
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 13/8 9/8 21/16 19/16 5/4 15/8 17/16 35/32 33/32

Subsets:

MOS 5: 0 4 8 9 10 (C E Ab A Bb)

MOS 9: 0 4 5 6 7 8 9 10 11 (C E F F# G Ab A Bb B)

**(6-1) Scale J Right** - 12 tones

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 6, 1, 1, 1, 1, 1

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 6 1 1 1 1 1 7 8 9 10 11 12 19 27 36 46  
57 69 88 115...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	69/64	130.229
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	27/16	905.865 Pythagorean major sixth
9:	7/4	968.826 harmonic seventh
10:	57/32	999.468
11:	115/64	1014.588
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 9/8 5/4 11/8 19/16 27/16 23/16 57/32 69/64 115/64

Subsets:

MOS 5 = 0 2 4 7 9 (C D E G A)

MOS 8 = 0 2 3 4 5 7 8 9 (C D Eb E F G Ab A)

MOS 11 = all except 11 (no B)

**(6-1) Scale K Right** - 12 tones

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 6, 0, 0, 0, 0, 1

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 6 0 0 0 1 6 0 0 0 1 7 6 0 0 1 8 13 6 0  
1 9 21 19 6 1 10 30 40 25 7 11 40 70...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 1/1 7/4 13/8 9/8 21/16 19/16 5/4 15/8 25/16 11/8 35/32

Subsets:

MOS 5 = 0 2 7 9 10 (C D G A Bb)

MOS 6 = 0 2 5 7 9 10 (C D F G A Bb)

MOS 11 = all except 1 (no C#)



## Generator (6-1) Scales - Left Wing Versions

### (6-1) Scale A Left - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 1, 6

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 1 6 7 13 20 33 53 86 139 225 364 589...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	139/128	142.729
3:	589/512	242.549
4:	5/4	386.314 major third
5:	43/32	511.518
6:	91/64	609.354
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	53/32	873.505
10:	7/4	968.826 harmonic seventh
11:	225/128	976.537 augmented sixth
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 13/8 5/4 33/32 53/32 43/32 139/128 225/128 91/64 589/512

Subsets:

MOS 7 = 0 1 4 7 8 9 10 (C C# E G Ab A Bb)

MOS 10 = all except 3 and 6 (no Eb or F#)

### (6-1) Scale B Left - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 1, 6, 6

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 1 6 6 7 13 19 26 39 58 84 123 181 265 388...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	265/256	59.818
2:	19/16	297.513 19th harmonic
3:	39/32	342.483 39th harmonic
4:	21/16	470.781 narrow fourth
5:	181/128	599.815
6:	3/2	701.955 perfect fifth
7:	97/64	719.895
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	123/64	1131.017
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 13/8 19/16 39/32 29/16 21/16 123/64 181/128 265/256 97/64

Subsets:

MOS 5: 0 2 6 8 9 (C D F# Ab A)

MOS 7: 0 2 3 6 8 9 10 (C D Eb F# Ab A Bb)

MOS 9: 0 2 3 4 6 8 9 10 11 (C D Eb E F# Ab A Bb B)

MOS 11: all except 7 (no G)

**(6-1) Scale C Left - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 1, 0, 6

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 1 0 6 1 6 7 7 13 14 20 27 34 47 61 81 108 142 189...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	71/64	179.697
3:	5/4	386.314 major third
4:	81/64	407.820 Pythagorean major third
5:	47/32	665.507
6:	189/128	674.691
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 13/8 5/4 27/16 17/16 47/32 61/32 81/64 71/64 189/128

Subsets:

MOS 5: 0 3 7 8 10 (C Eb G Ab Bb)

MOS 7: 0 1 3 7 8 9 10 (C C# Eb G Ab A Bb)

MOS 12: all tones

**(6-1) Scale D Left - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 1, 6, 6, 6

Resulting sequence:  $D_n = D_{n-4} + D_{n-1}$ : 1 6 6 6 7 13 19 25 32 45 64 89 121 166 230 319...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	19/16	297.513 19th harmonic
2:	319/256	380.895
3:	83/64	450.047
4:	89/64	570.880
5:	45/32	590.224 diatonic tritone
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	115/64	1014.588
11:	121/64	1102.636
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 13/8 19/16 25/16 45/32 89/64 121/64 83/64 115/64 319/256

Subsets:

MOS 5: 0 1 6 8 9 (C C# F# Ab A)

MOS 7: 0 1 5 6 7 8 9 (C C# F F# G Ab A)

MOS 9: 0 1 4 5 6 7 8 9 11 (C C# E F F# G Ab A B)

MOS 11: all except 2 (no D)

**(6-1) Scale E Left - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 1, 0, 0, 6

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 1 0 0 6 1 0 6 7 1 6 13 8 7 19 21 15 26  
40 36 41 66 76  
77...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	77/64	320.144
5:	5/4	386.314 major third
6:	41/32	429.062
7:	21/16	470.781 narrow fourth
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 13/8 19/16 21/16 15/8 5/4 9/8 41/32 33/32 77/64

Subsets:

MOS 7: 0 3 7 8 9 10 11 (C Eb G Ab A Bb B)

MOS 10: all except 1 and 4 (no C# or E)

**(6-1) Scale F Left - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 1, 6, 6, 6, 6

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 1 6 6 6 6 7 13 19 25 31 38 51 70 95 126  
164...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	19/16	297.513 19th harmonic
3:	41/32	429.062
4:	95/64	683.827
5:	3/2	701.955 perfect fifth
6:	25/16	772.627 classic augmented fifth
7:	51/32	806.910
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	31/16	1145.036 31st harmonic
11:	63/32	1172.736 octave - septimal comma
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 13/8 19/16 25/16 31/16 51/32 35/32 95/64 63/32 41/32

Subsets:

MOS 5: 0 2 5 8 9 (C D F Ab A)

MOS 7: 0 2 5 6 8 9 10 (C D F F# Ab A Bb)

MOS 12: all tones

**(6-1) Scale G Left - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 1, 0, 6, 0, 6

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 1 0 6 0 6 1 6 7 6 13 7 19 14 25 27 32 46  
46 71 73 103 119...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	71/64	179.697
2:	73/64	227.789
3:	19/16	297.513 19th harmonic
4:	23/16	628.274 23rd harmonic
5:	3/2	701.955 perfect fifth
6:	25/16	772.627 classic augmented fifth
7:	103/64	823.801
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	119/64	1073.781
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 13/8 19/16 25/16 27/16 23/16 71/64 73/64 103/64 119/64

Subsets:

MOS 7: 0 3 5 6 8 9 10 (C Eb F F# Ab A Bb)

MOS 10: all except 7 and 11 (no G or B)

**(6-1) Scale H Left - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 1, 0, 0, 6, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 1 0 0 6 0 1 6 0 7 6 1 13 6 8 19 7 21 25  
15 40 32 36 65 47...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 13/8 19/16 21/16 25/16 15/8 5/4 9/8 65/64 47/32

Subsets:

MOS 7: 0 3 5 7 8 9 10 (C Eb F G Ab A Bb)

MOS 11: all except 6 (no F#)

**(6-1) Scale I Left - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 1, 0, 0, 0, 6

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 1 0 0 0 6 1 0 0 6 7 1 0 6 13 8 1 6 19 21  
9 7 25 40 30 16 32 65 70...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	35/32	155.140 septimal neutral second
3:	9/8	203.910 major whole tone
4:	19/16	297.513 19th harmonic
5:	5/4	386.314 major third
6:	21/16	470.781 narrow fourth
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	7/4	968.826 harmonic seventh
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 13/8 19/16 21/16 9/8 25/16 5/4 15/8 65/64 35/32

Subsets:

MOS 5: 0 4 7 9 10 (C E G A Bb)

MOS 9: 0 3 4 5 6 7 8 9 10 (C Eb E F F# G Ab A Bb)

**(6-1) Scale J Left - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 1, 6, 6, 6, 6, 6

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 1 6 6 6 6 6 7 13 19 25 31 37 44 57 76  
101 132 ...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	37/32	251.344 37th harmonic
3:	19/16	297.513 19th harmonic
4:	11/8	551.318 undecimal semi-augmented fourth
5:	3/2	701.955 perfect fifth
6:	25/16	772.627 classic augmented fifth
7:	101/64	789.854
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	57/32	999.468
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 13/8 19/16 25/16 31/16 37/32 11/8 57/32 101/64 33/32

Subsets:

MOS 5 = 0 3 5 8 9 (C Eb F Ab A)

MOS 8 = 0 2 3 5 6 8 9 11 (C D Eb F F# Ab A B)

MOS 11 = all except 1 (no C#)

**(6-1) Scale K Left - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 1, 0, 0, 0, 0, 6

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 1 0 0 0 0 6 1 0 0 0 6 7 1 0 0 6 13 8 1 0  
6 19 21 9 1 6 25 40 30 10 7 31 65...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	9/8	203.910 major whole tone
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	3/2	701.955 perfect fifth
7:	25/16	772.627 classic augmented fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

1/1 3/2 7/4 13/8 19/16 21/16 9/8 25/16 5/4 15/8 31/16 65/64

Subsets:

MOS 5 = 0 3 6 8 9 (C Eb F# Ab A)

MOS 6 = 0 3 5 6 8 9 (C Eb F F# Ab A)

MOS 11 = all except 1 (no C#)

## Part 12: The 6-5 SCALES (generator 6, 5)

If the seed 6, 5 is used with the Fibonacci rule, the resulting sequence is 6 5 11 16 27 43 70 113 183 296... Here is a part of the (6, 5) triangle. Having 2 different generators makes the triangle asymmetrical. Therefore, additive sequences derived from left-leaning and right-leaning diagonals will be different, as will the scales derived from them. Note: with the left-leaning diagonals, sequences begin to be generated that, with at least the first 12 unique elements, do not have a 1 or 2 or their multiples in them. This means that scales without fundamentals begin to be generated here. This implies arriving at a different harmonic territory than that occupied by most of the other scales in this collection.

```
      5/6
     5 6
    5 11 6
   5 16 17 6
  5 21 33 23 6
 5 26 54 56 29 6
5 31 80 110 85 35 6
```

Here are the scales that generated by treating each non-zero element of each sequence as a harmonic, and then normalizing the resulting scale. The rule for the sequence, the seed string derived from the triangle, the resulting number sequence, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

### Generator (6-5) Scales - Right Wing Versions

#### (6-5) Scale A Right - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 6,5

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 6 5 11 16 27 43 70 113 183 296 479 775...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	35/32	155.140 septimal neutral second
2:	37/32	251.344 37th harmonic
3:	5/4	386.314 major third
4:	43/32	511.518
5:	11/8	551.318 undecimal semi-augmented fourth
6:	183/128	618.840
7:	3/2	701.955 perfect fifth
8:	775/512	717.663
9:	27/16	905.865 Pythagorean major sixth
10:	113/64	984.215
11:	479/256	1084.658
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 11/8 1/1 27/16 43/32 35/32 113/64 183/128 37/32 479/256 775/512

Subsets:

MOS 7 = 0 1 3 4 5 7 9 (C C# D# E F G A)

MOS 10 = all except 8 and 11 (no Ab or B)

**(6-5) Scale B Right** - 12 tones

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 6, 5, 5

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 6 5 5 11 16 21 32 48 69 101 149 218 319 468 686...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	69/64	130.229
2:	149/128	263.002
3:	319/256	380.895
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	343/256	506.478
7:	11/8	551.318 undecimal semi-augmented fourth
8:	3/2	701.955 perfect fifth
9:	101/64	789.854
10:	109/64	921.821
11:	117/64	1044.438
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 11/8 1/1 21/16 69/64 101/64 149/128 109/64 319/256 117/64 343/256

Subsets:

- MOS 5: 0 4 5 7 8 (C E F G Ab)
- MOS 7: 0 1 4 5 7 8 9 (C C# E F G Ab A)
- MOS 9: 0 1 2 4 5 7 8 9 10 (C C# D E F G Ab A Bb)
- MOS 11: all except 6 (no F#)

**(6-5) Scale C Right** - 12 tones

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 6, 0, 5

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 6 0 5 6 5 11 11 16 22 27 38 49 65 87 114 152 201 266...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	65/64	26.841 13th-partial chroma
2:	133/128	66.339
3:	19/16	297.513 19th harmonic
4:	5/4	386.314 major third
5:	87/64	531.532
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	49/32	737.652
9:	201/128	781.262
10:	27/16	905.865 Pythagorean major sixth
11:	57/32	999.468
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 11/8 1/1 27/16 19/16 49/32 65/64 87/64 57/32 201/128 133/128

Subsets:

- MOS 5: 0 4 6 7 10 (C E F# G Bb)
- MOS 7: 0 3 4 6 7 8 10 (C Eb E F# G Ab Bb)
- MOS 12: all tones



**(6-5) Scale D Right - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 6, 5, 5, 5

Resulting sequence: D  $D_n = D_{n-4} + D_{n-1}$ : 6 5 5 5 11 16 21 26 37 53 74 100 137 190 264...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	137/128	117.638
3:	37/32	251.344 37th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	95/64	683.827
8:	3/2	701.955 perfect fifth
9:	25/16	772.627 classic augmented fifth
10:	13/8	840.528 tridecimal neutral sixth
11:	53/32	873.505
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 11/8 1/1 21/16 13/8 37/32 53/32 25/16 137/128 95/64 33/32

Subsets:

MOS 5: 0 4 5 6 8 (C E F Gb Ab)

MOS 7: 0 3 4 5 6 8 10 (C Eb E F Gb Ab Bb)

MOS 9: 0 3 4 5 6 8 9 10 11 (C Eb E F Gb Ab A Bb B)

MOS 11: all except 1 (no C#)

**(6-5) Scale E Right - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 6, 0, 0, 5

Resulting sequence: E  $E_n = E_{n-4} + E_{n-3}$ : 6 0 0 5 6 0 5 11 6 5 16 17 11 21 33 28 32 54 61 60 86...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	43/32	511.518
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	27/16	905.865 Pythagorean major sixth
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 11/8 1/1 17/16 21/16 33/32 7/4 27/16 61/32 15/8 43/32

Subsets:

MOS 7: 0 1 2 3 4 6 7 (C C# D Eb E F# G)

MOS 10: all except 5 and 10 (no F or Bb)

**(6-5) Scale F Right - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 6, 5, 5, 5, 5

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 6 5 5 5 5 11 16 21 26 31 42 58 79 105 136 178...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	79/64	364.537
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	89/64	570.880
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	105/64	857.095 septimal neutral sixth
10:	29/16	1029.577 29th harmonic
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 11/8 1/1 21/16 13/8 31/16 21/16 29/16 79/64 105/64 17/16 89/64

Subsets:

MOS 5: 0 3 4 5 7 (C Eb E F G)  
MOS 7: 0 3 4 5 7 8 11 (C Eb E F G Ab B)  
MOS 12: all tones

**(6-5) Scale G Right - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 6, 0, 5, 0, 5

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 6 0 5 0 5 6 5 11 5 16 11 21 22 26 38 37 59 59 85 97 122...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	37/32	251.344 37th harmonic
2:	19/16	297.513 19th harmonic
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	85/64	491.269
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	97/64	719.895
9:	13/8	840.528 tridecimal neutral sixth
10:	59/32	1059.172
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 11/8 1/1 21/16 13/8 19/16 37/32 59/32 85/64 97/64 61/32

Subsets:

MOS 7: 0 2 3 4 6 7 9 (C D Eb E F# G A)  
MOS 10: all except 8 and 11 (no Ab or B)

**(6-5) Scale H Right - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 6, 0, 0, 5, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 6 0 0 5 0 6 5 0 11 5 6 16 5 17 21 11 33  
26 28 54 37 61...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	37/32	251.344 37th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 11/8 1/1 17/16 21/16 33/32 13/8 7/4 27/16 37/32 61/32

Subsets:

MOS 7: 0 1 2 4 5 6 7 (C C# D E F F# G)

MOS 11: all except 11 (no B)

**(6-5) Scale I Right - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 6, 0, 0, 0, 5

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 6 0 0 0 5 6 0 0 5 11 6 0 5 16 17 6 5 21  
33 23 11 26 54 56 34 37...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	37/32	251.344 37th harmonic
4:	5/4	386.314 major third
5:	21/16	470.781 narrow fourth
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	27/16	905.865 Pythagorean major sixth
11:	7/4	968.826 harmonic seventh
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 11/8 1/1 17/16 21/16 33/32 23/16 13/8 27/16 7/4 37/32

Subsets:

MOS 5: 0 2 4 6 8 (C D E Gb Ab)

MOS 9: 0 1 2 4 5 6 7 8 9 (C C# D E F F# G Ab A)

**(6-5) Scale J Right - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 6, 5, 5, 5, 5, 5

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 6 5 5 5 5 5 11 16 21 26 31 36 47 63 84 110 141...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	141/128	167.462
2:	9/8	203.910 major whole tone
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	47/32	665.507
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	55/32	937.632
10:	31/16	1145.036 31st harmonic
11:	63/32	1172.736 octave - septimal comma
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 11/8 1/1 21/16 13/8 31/16 9/8 47/32 63/32 55/32 141/128

Subsets:

MOS 5 = 0 3 4 5 7 (C Eb E F G)

MOS 8 = 0 2 3 4 5 7 8 10 (C D Eb E F G Ab Bb)

MOS 11 = all except 1 (no C#)

**(6-5) Scale K Right - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 6, 0, 0, 0, 0, 5

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 6 0 0 0 0 5 6 0 0 0 5 11 6 0 0 5 16 17 6 0 5 21 33 23 6 5 26 54 56 29...

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

3/2 5/4 11/8 1/1 17/16 21/16 33/32 23/16 13/8 27/16 7/4 29/16

Subsets:

MOS 5 = 0 2 3 5 7 (C D Eb F G)

MOS 6 = 0 2 3 4 5 7 (C D Eb E F G)

MOS 11 = all except 11 (no B)

## Generator (6-5) Scales - Left Wing Versions

In this set of scales, many of the scales actually don't have a 1/1. We retain the fundamental here for consistency with all other scales in the catalog, but the scales without a fundamental are noted below.

In this series the following scales DO NOT have a 1/1: A B C D F G J.

The following scales DO have a 1/1: E H I K.

Further investigation is required to see if this phenomenon happens in triangles based on 7, 8, 9 etc.

### (6-5) Scale A Left - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Seed string from triangle: 5, 6

Resulting sequence:  $A_n = A_{n-2} + A_{n-1}$ : 5 6 11 17 28 45 73 118 191 309 500 (809)...

This scale actually DOESN'T contain a 1/1, but in the interests of consistency with the other scales, we'll include it here, omitting the last relevant scale degree, which would be 809.

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	73/64	227.789
3:	309/256	325.756
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	45/32	590.224 diatonic tritone
7:	191/128	692.915
8:	3/2	701.955 perfect fifth
9:	7/4	968.826 harmonic seventh
10:	59/32	1059.172
11:	125/64	1158.941 classic augmented seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 11/8 17/16 7/4 45/32 73/64 59/32 191/128 309/256 125/64 (809/512)  
(NO 1/1)

Subsets:

MOS 7 = 1 2 4 5 6 8 9 (C# D E F F# Ab A)

MOS 10 = all except 0 and 11 (no C or B)

**(6-5) Scale B Left - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Seed string from triangle: 5, 6, 6

Resulting sequence:  $B_n = B_{n-3} + B_{n-1}$ : 5 6 6 11 17 23 34 51 74 108 159 233 341 (500)...

**Again, this scale DOESN'T have a 1/1, but we'll keep the 1/1 for consistency, and omit the last relevant tone, 500.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	37/32	251.344 37th harmonic
3:	159/128	375.460
4:	5/4	386.314 major third
5:	341/256	496.354
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	51/32	806.910
10:	27/16	905.865 Pythagorean major sixth
11:	233/128	1037.023
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 11/8 17/16 23/16 51/32 37/32 27/16 159/128 233/128 341/256 (500/256) (NO 1/1)

Subsets:

MOS 5: 1 4 6 7 8 (C# E F# G Ab)

MOS 7: 1 2 4 6 7 8 9 (C# D E F# G Ab A)

MOS 9: 1 2 3 4 6 7 8 9 10 (C# D Eb E F# G Ab A Bb)

MOS 11: all except 0 (no C)

**(6-5) Scale C Left - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Seed string from triangle: 5, 0, 6

Resulting sequence:  $C_n = C_{n-3} + C_{n-2}$ : 5 0 6 5 6 11 11 17 22 28 39 50 67 89 117  
156 206 (273)...

**Another scale without a 1/1! We may be on to something here - that is, we may have finally reached some other sort of harmonic territory.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	67/64	79.307
2:	17/16	104.955 17th harmonic
3:	39/32	342.483 39th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	89/64	570.880
7:	3/2	701.955 perfect fifth
8:	25/16	772.627 classic augmented fifth
9:	103/64	823.801
10:	7/4	968.826 harmonic seventh
11:	117/64	1044.438
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 11/8 17/16 7/4 39/32 25/16 67/64 89/64 117/64 103/64 (273/256) (NO 1/1)

Subsets:

MOS 5: 2 4 5 7 10 (D E F G Bb)

MOS 7: 2 3 4 5 7 8 10 (D Eb E F G Ab Bb)

MOS 12: all tones (except since there is no 273/256, probably there is no MOS 12 here)

**(6-5) Scale D Left - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Seed string from triangle: 5, 6, 6, 6

Resulting sequence: D  $D_n = D_{n-4} + D_{n-1}$ : 5 6 6 6 11 17 23 29 40 57 80 109 149  
206 286 (395).

**Yet again, no 1/1. Some of the 6-5 scales DO have a 1/1, but its much further out in the series than the 12<sup>th</sup> element. As a general characteristic, these scales lack a fundamental, which suggests that we've gotten into some new harmonic world.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	143/128	191.846
3:	149/128	263.002
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	103/64	823.801
9:	109/64	921.821
10:	57/32	999.468
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 11/8 17/16 23/16 29/16 57/32 109/64 149/128 103/64 143/128  
(395/256)

Subsets:

MOS 5: 1 4 5 6 7 (C# E F F# G)

MOS 7: 1 4 5 6 7 10 11 (C# E F F# G Bb B)

MOS 9: 1 3 4 5 6 7 9 10 11 (C# Eb E F F# G A Bb B)

MOS 11: all except 0 (no C)



**(6-5) Scale E Left - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Seed string from triangle: 5, 0, 0, 6

Resulting sequence:  $E_n = E_{n-4} + E_{n-3}$ : 5 0 0 6 5 0 6 11 5 6 17 16 11 23 33 27  
34 56 60 61 90...

**This DOES have a 1/1 in it, and it even fits in the smallest MOS subset.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	5/4	386.314 major third
4:	11/8	551.318 undecimal semi-augmented fourth
5:	45/32	590.224 diatonic tritone
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	27/16	905.865 Pythagorean major sixth
9:	7/4	968.826 harmonic seventh
10:	15/8	1088.269 classic major seventh
11:	61/32	1116.885
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 11/8 17/16 1/1 23/16 33/32 27/16 7/4 15/8 61/32 45/32

Subsets:

MOS 7: 0 1 2 3 4 6 7 (C C# D Eb E F# G)

MOS 10: all except 5 and 11 (no F or B)

**(6-5) Scale F Left - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Seed string from triangle: 5, 6, 6, 6, 6

Resulting Sequence:  $F_n = F_{n-5} + F_{n-1}$ : 5 6 6 6 6 11 17 23 29 35 46 63 86 115  
150 (196)...

**Another scale with no 1/1. Treated as before, leaving out 196, the last generated degree.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	35/32	155.140 septimal neutral second
3:	75/64	274.582 classic augmented second
4:	5/4	386.314 major third
5:	43/32	511.518
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	115/64	1014.588
10:	29/16	1029.577 29th harmonic
11:	63/32	1172.736 octave - septimal comma
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 11/8 17/16 23/16 29/16 35/32 63/32 43/32 115/64 75/64 (49/32) (NO 1/1)

Subsets:

MOS 5: 1 4 6 7 8 (C# E F# G Ab)

MOS 7: 1 2 4 6 7 8 10 (C# D E F# G Ab Bb)

MOS 12: all tones (but without a 49/32 we don't really have all 12 tones, so no MOS 12)

**(6-5) Scale G Left - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Seed string from triangle: 5, 0, 6, 0, 6

Resulting sequence:  $G_n = G_{n-5} + G_{n-2}$ : 5 0 6 0 6 5 6 11 6 17 11 23 22 29 39 40 62 62 91 101 131 (163)...

**Another scale with no 1/1. Treated as before, leaving out 163, the last generated scale degree.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	131/128	40.108
2:	17/16	104.955 17th harmonic
3:	39/32	342.483 39th harmonic
4:	5/4	386.314 major third
5:	11/8	551.318 undecimal semi-augmented fourth
6:	91/64	609.354
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	101/64	789.854
10:	29/16	1029.577 29th harmonic
11:	31/16	1145.036 31st harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 11/8 17/16 23/16 29/16 39/32 31/16 91/64 101/64 131/128 (163/128)  
(NO 1/1)

Subsets:

MOS 7: 2 3 4 5 7 8 10 (D Eb E F G Ab Bb)

MOS 10: all except 0 and 1 (no C or C#)

**(6-5) Scale H Left - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Seed string from triangle: 5, 0, 0, 6, 0

Resulting sequence:  $H_n = H_{n-5} + H_{n-3}$ : 5 0 0 6 0 5 6 0 11 6 5 17 6 16 23 11 33 29 27 56 40 60 85...

**This scale DOES have a 1/1, and it's even present in the smallest MOS subset.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	5/4	386.314 major third
4:	85/64	491.269
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	27/16	905.865 Pythagorean major sixth
9:	7/4	968.826 harmonic seventh
10:	29/16	1029.577 29th harmonic
11:	15/8	1088.269 classic major seventh
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 11/8 17/16 1/1 23/16 33/32 29/16 27/16 7/4 15/8 85/64

Subsets:

MOS 7: 0 1 2 3 5 6 7 (C C# D Eb F F# G)

MOS 11: all except 4 (no E)

**(6-5) Scale I Left - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Seed string from triangle: 5, 0, 0, 0, 6

Resulting sequence:  $I_n = I_{n-5} + I_{n-4}$ : 5 0 0 0 6 5 0 0 6 11 5 0 6 17 16 5 6 23  
33 21 11 29 56 54 32 40 85...

**This scale DOES have a 1/1, and it's even present in the smallest MOS subset.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	85/64	491.269
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 11/8 17/16 1/1 23/16 33/32 21/16 29/16 7/4 27/16 85/64

Subsets:

MOS 5: 0 2 3 6 8 (C D Eb F# Ab)

MOS 9: 0 1 2 3 4 6 7 8 11 (C C# D Eb E F# G Ab B)

**(6-5) Scale J Left - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Seed string from triangle: 5, 6, 6, 6, 6, 6

Resulting sequence:  $J_n = J_{n-6} + J_{n-1}$ : 5 6 6 6 6 6 11 17 23 29 35 41 52 69 92  
121 (156)...

**This scale DOES NOT have a 1/1. As before we leave out 156, the last generated scale degree.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17/16	104.955 17th harmonic
2:	69/64	130.229
3:	35/32	155.140 septimal neutral second
4:	5/4	386.314 major third
5:	41/32	429.062
6:	11/8	551.318 undecimal semi-augmented fourth
7:	23/16	628.274 23rd harmonic
8:	3/2	701.955 perfect fifth
9:	13/8	840.528 tridecimal neutral sixth
10:	29/16	1029.577 29th harmonic
11:	121/64	1102.636
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 11/8 17/16 23/16 29/16 35/32 41/32 13/8 69/64 121/64 (156/128 -  
39/32) (NO 1/1)

Subsets:

MOS 5 = 1 4 6 7 8 (C# E F# G Ab)

MOS 8 = 1 3 4 5 6 7 8 10 (C# Eb E F F# G Ab Bb)

MOS 11 = all except 0 (no C)

**(6-5) Scale K Left - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Seed string from triangle: 5, 0, 0, 0, 0, 6

Resulting sequence:  $K_n = K_{n-6} + K_{n-5}$ : 5 0 0 0 0 6 5 0 0 0 6 11 5 0 0 6 17 16 5  
0 6 23 33 21 5 6 29 56 54 26...

**This scale DOES have a 1/1, and it's even present in the smallest MOS subset.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	33/32	53.273 undecimal comma, al-Farabi's 1/4-tone
2:	17/16	104.955 17th harmonic
3:	5/4	386.314 major third
4:	21/16	470.781 narrow fourth
5:	11/8	551.318 undecimal semi-augmented fourth
6:	23/16	628.274 23rd harmonic
7:	3/2	701.955 perfect fifth
8:	13/8	840.528 tridecimal neutral sixth
9:	27/16	905.865 Pythagorean major sixth
10:	7/4	968.826 harmonic seventh
11:	29/16	1029.577 29th harmonic
12:	2/1	1200.000 octave

Order of generation:

5/4 3/2 11/8 17/16 1/1 23/16 33/32 21/16 29/16 7/4 27/16 13/8

Subsets:

MOS 5 = 0 2 3 5 7 (C D Eb F G)

MOS 6 = 0 2 3 5 6 7 (C D Eb F F# G)

MOS 11 = all except 8 (no Ab)

## APPENDIX - The Pythagorean Limit Scales

Each of these additive sequence rules tends towards a certain limit. That is, the ratio between any two successive elements of the sequence gets more and more similar the farther you move out in the sequence. This limit can be used as a generator for a Pythagorean scale, and there are several ways of doing this. The limit can be used logarithmically as a pitch. This means that the log of the limit can be taken, and by multiplying that by 1200, an interval in cents results. Some of the scales generated with the limit used as a logarithmic factor are MOS scales, and these MOS sizes are used with the just scales in this paper to derive subscales to play with. There are other ways of using the limit however. One might also use the limit as a linear factor. This involves multiplying the limit by 1200 and then subtracting 1200 to get an interval within an octave. If this interval is used as a generator for a Pythagorean scale, a set of MOS scales different from the ones derived from the "log factor generator interval" result. Some of these scales have already been explored in both *The Mossy Scales of Mt. Meru*, and in *Pythagoras' Babylonian Bathtub*, but we here include all the 12-note scales derived by these methods for completeness. In this appendix, first we list the scales derived from the limit used as a log factor, along with their MOS subsets, and then the scales generated by the limit used as a linear factor. Note that because each additive sequence tends towards a limit regardless of the generators used, the symmetry of the triangle that generated the sequences is irrelevant, and there is only one version of each scale.

### Part 13: The Limit Scales with the Limit Used as a Logarithmic Factor

Here are the scales that generated by using the limit ratio used as a logarithmic factor converted to an interval (as stated above, the log of the limit is taken, multiplied by 1200, and an interval in cents results), and that interval is used as a generator for a Pythagorean scale, and then the result is normalized. The rule for the sequence, limit ratio, the interval derived and used as the scale generator, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

#### Limit Log Scale A - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Limit ratio of rule: 1.61803398875...

Resulting interval (1200 / limit ratio): 833.090296 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	99.271 cents	99.271
2:	198.542 cents	198.542
3:	297.813 cents	297.813
4:	466.181 cents	466.181
5:	565.451 cents	565.451
6:	664.722 cents	664.722
7:	763.993 cents	763.993
8:	833.090 cents	833.090
9:	932.361 cents	932.361
10:	1031.632 cents	1031.632
11:	1130.903 cents	1130.903
12:	2/1	1200.000 octave

Order of generation:

0 8 4 1 9 5 2 10 6 3 11 7

Subsets:

MOS 7 = 0 1 2 4 5 8 9 (C C# D E F Ab A)

MOS 10 = all except 7 and 11 (no G or B)

**Limit Log Scale B - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Limit ratio of rule: 1.46557123188...

Resulting interval (1200 / limit ratio): 661.755708 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	79.313 cents	79.313
2:	123.511 cents	123.511
3:	247.023 cents	247.023
4:	370.534 cents	370.534
5:	494.046 cents	494.046
6:	617.557 cents	617.557
7:	661.756 cents	661.756
8:	785.267 cents	785.267
9:	908.779 cents	908.779
10:	1032.290 cents	1032.290
11:	1155.801 cents	1155.801
12:	2/1	1200.000 octave

Order of generation:

0 7 2 8 3 9 4 10 5 11 6 1

Subsets:

MOS 5 = 0 2 3 7 8 (C D Eb G Ab)

MOS 7 = 0 2 3 4 7 8 9 (C D Eb E G Ab A)

MOS 9 = 0 2 3 4 5 7 8 9 10 (C D Eb E F G Ab A Bb)

MOS 11 = all except 1 (no C#)

**Limit Log Scale C - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Limit ratio of rule: 1.32471795725...

Resulting interval (1200 / limit ratio): 486.822278 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	34.111 cents	34.111
2:	68.223 cents	68.223
3:	260.467 cents	260.467
4:	294.578 cents	294.578
5:	486.822 cents	486.822
6:	520.934 cents	520.934
7:	555.045 cents	555.045
8:	747.289 cents	747.289
9:	781.401 cents	781.401
10:	973.645 cents	973.645
11:	1007.756 cents	1007.756
12:	2/1	1200.000 octave

Order of generation:

0 5 10 3 8 1 6 11 4 9 2 7

Subsets:

MOS 5 = 0 3 5 8 10 (C Eb F Ab Bb)

MOS 7 = 0 1 3 5 6 8 10 (C Db Eb F Gb Ab Bb)

MOS 12 = all pitches



**Limit Log Scale D - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Limit ratio of rule: 1.38027756909...

Resulting interval (1200 / limit ratio): 557.950101 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	137.451 cents	137.451
2:	221.551 cents	221.551
3:	305.651 cents	305.651
4:	389.751 cents	389.751
5:	473.850 cents	473.850
6:	557.950 cents	557.950
7:	779.501 cents	779.501
8:	863.601 cents	863.601
9:	947.701 cents	947.701
10:	1031.800 cents	1031.800
11:	1115.900 cents	1115.900
12:	2/1	1200.000 octave

Order of generation:

0 6 11 5 10 4 9 3 8 2 7 1

Subsets:

MOS 5 = 0 5 6 10 11 (C F Gb Bb B)

MOS 7 = 0 4 5 6 9 10 11 (C E F Gb A Bb B)

MOS 9 = 0 3 4 5 6 8 9 10 11 (C Eb E F Gb Ab A Bb B)

MOS 11 = all except 1 (no C#)

**Limit Log Scale E - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Limit ratio of rule: 1.22074408461...

Resulting interval (1200 / limit ratio): 345.312945 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	17.191 cents	17.191
2:	181.252 cents	181.252
3:	198.442 cents	198.442
4:	345.313 cents	345.313
5:	362.504 cents	362.504
6:	526.565 cents	526.565
7:	690.626 cents	690.626
8:	707.817 cents	707.817
9:	871.878 cents	871.878
10:	1035.939 cents	1035.939
11:	1053.129 cents	1053.129
12:	2/1	1200.000 octave

Order of generation:

0 4 7 10 2 6 9 1 5 8 11 3

Subsets:

MOS 7 = 0 2 4 6 7 9 10 (C D E F# G A Bb)

MOS 10 = all except 3 and 11 (no Eb or B)

**Limit Log Scale F** - 12 tones - Rule:  $F_n = F_{n-5} + F_{n-1}$   
 Limit ratio of rule: 1.32471795724...  
 Resulting interval (1200 / limit ratio): 486.822278 cents

**Note: this is precisely the same as Scale C in this series.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	34.111 cents	34.111
2:	68.223 cents	68.223
3:	260.467 cents	260.467
4:	294.578 cents	294.578
5:	486.822 cents	486.822
6:	520.934 cents	520.934
7:	555.045 cents	555.045
8:	747.289 cents	747.289
9:	781.401 cents	781.401
10:	973.645 cents	973.645
11:	1007.756 cents	1007.756
12:	2/1	1200.000 octave

Order of generation:

0 5 10 3 8 1 6 11 4 9 2 7

Subsets:

MOS 5 = 0 3 5 8 10 (C Eb F Ab Bb)

MOS 7 = 0 1 3 5 6 8 10 (C Db Eb F Gb Ab Bb)

MOS 12 = all pitches

**Limit Log Scale G** - 12 tones

Rule:  $G_n = G_{n-5} + G_{n-2}$

Limit ratio of rule: 1.23650570339...

Resulting interval (1200 / limit ratio): 367.522673 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	75.227 cents	75.227
2:	172.659 cents	172.659
3:	270.091 cents	270.091
4:	367.523 cents	367.523
5:	442.749 cents	442.749
6:	540.181 cents	540.181
7:	637.613 cents	637.613
8:	735.045 cents	735.045
9:	907.704 cents	907.704
10:	1005.136 cents	1005.136
11:	1102.568 cents	1102.568
12:	2/1	1200.000 octave

Order of generation:

0 4 8 11 3 7 10 2 6 9 1 5

Subsets:

MOS 7 = 0 3 4 7 8 10 11 (C Eb E G Ab A Bb)

MOS 10 = all except 1 and 5 (no C# or F)

**Limit Log Scale H - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Limit ratio of rule: 1.19385911132...

Resulting interval (1200 / limit ratio): 306.759111 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	27.036 cents	27.036
2:	54.073 cents	54.073
3:	306.759 cents	306.759
4:	333.796 cents	333.796
5:	360.832 cents	360.832
6:	613.518 cents	613.518
7:	640.555 cents	640.555
8:	667.591 cents	667.591
9:	920.277 cents	920.277
10:	947.314 cents	947.314
11:	974.350 cents	974.350
12:	2/1	1200.000 octave

Order of generation:

0 3 6 9 1 4 7 10 2 5 8 11

Subsets:

MOS 7 = 0 1 3 4 6 7 9 (C Db Eb E F# G A)

MOS 11 = all except 11 (no B)

**Limit Log Scale I - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Limit ratio of rule: 1.16730397826...

Resulting interval (1200 / limit ratio): 267.816364 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	10.347 cents	10.347
2:	139.082 cents	139.082
3:	267.816 cents	267.816
4:	278.164 cents	278.164
5:	406.898 cents	406.898
6:	535.633 cents	535.633
7:	545.980 cents	545.980
8:	674.715 cents	674.715
9:	803.449 cents	803.449
10:	942.531 cents	942.531
11:	1071.265 cents	1071.265
12:	2/1	1200.000 octave

Order of generation:

0 3 6 9 11 2 5 8 10 1 4 7

Subsets:

MOS 5 = 0 3 6 9 11 (C Eb Gb A B)

MOS 9 = 0 2 3 5 6 8 9 10 11 (C D Eb F Gb Ab A Bb B)

**Limit Log Scale J - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Limit ratio of rule: 1.28519903326...

Resulting interval (1200 / limit ratio): 434.390161 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	103.170 cents	103.170
2:	206.341 cents	206.341
3:	309.511 cents	309.511
4:	434.390 cents	434.390
5:	537.561 cents	537.561
6:	640.731 cents	640.731
7:	743.902 cents	743.902
8:	868.780 cents	868.780
9:	971.951 cents	971.951
10:	1075.121 cents	1075.121
11:	1178.292 cents	1178.292
12:	2/1	1200.000 octave

Order of generation:

0 4 8 1 5 9 2 6 10 3 7 11

Subsets:

MOS 5 = 0 1 4 5 8 (C C# E F Ab)

MOS 8 = 0 1 2 4 5 6 8 9 (C C# D E F Gb Ab A)

MOS 11 = all except 1 (no C#)

**Limit Log Scale K - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Limit ratio of rule: 1.13472413840...

Resulting interval (1200 / limit ratio): 218.809930 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	6.909 cents	6.909
2:	112.860 cents	112.860
3:	218.810 cents	218.810
4:	331.670 cents	331.670
5:	437.620 cents	437.620
6:	550.479 cents	550.479
7:	656.430 cents	656.430
8:	769.289 cents	769.289
9:	875.240 cents	875.240
10:	988.099 cents	988.099
11:	1094.050 cents	1094.050
12:	2/1	1200.000 octave

Order of generation:

0 3 5 7 9 11 2 4 6 8 10 1

Subsets:

MOS 5 = 0 3 5 7 9 (C Eb F G A)

MOS 6 = 0 3 5 7 9 11 (C Eb F G A B)

MOS 11 = all except 1 (no C#)

#### Part 14: The Limit Scales with the Limit Used as a Linear Factor

If the limit ratio is used as a linear factor ( $[(1200 * \text{limit ratio}) - 1200]$  gives the interval in cents), a different set of MOS scales result. Each additive sequence rule has its own limit ratio and these, converted into an interval, can be used as the basis for a Pythagorean scale of any size.

This method of generation could be regarded as a clear mistake, as the MOS subsets are completely different than the ones used in the other scales in this catalog, which derive from the Limit Ratio used as a Logarithmic factor. However, since we developed a whole family of Limit Ratio Linear scales for *Pythagoras' Babylonian Bathub*, we felt we should continue the possible error, and list the limit ratio linear scales for these limits. A number of these scales are pretty funky, and lumpy. But they may be useful.

Here are the scales that generated by using the limit ratio used as a linear factor converted to an interval, and used as a generator for a Pythagorean scale, and then normalizing the result. The rule for the sequence, limit ratio, the interval derived and used as the scale generator, the 12-note scale, and the Pythagorean limit-derived MOS subsets are given below.

##### Limit Linear Scale A - 12 tones

Rule:  $A_n = A_{n-2} + A_{n-1}$

Limit ratio of rule: 1.61803398875...

Resulting interval  $((1200 * \text{limit ratio}) - 1200)$ : .741.6407 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	108.204 cents	108.204
2:	216.408 cents	216.408
3:	283.282 cents	283.282
4:	391.486 cents	391.486
5:	566.563 cents	566.563
6:	674.767 cents	674.767
7:	741.641 cents	741.641
8:	849.845 cents	849.845
9:	958.049 cents	958.049
10:	1024.922 cents	1024.922
11:	1133.126 cents	1133.126
12:	2/1	1200.000 octave

Order of generation:

0 7 3 10 5 1 8 4 11 6 2 9

Subsets:

MOS 5 = 0 3 5 7 10 (C Eb F G Bb)

MOS 8 = 0 1 3 4 5 7 8 10 (C Db Eb E F G Ab Bb)

**Limit Linear Scale B - 12 tones**

Rule:  $B_n = B_{n-3} + B_{n-1}$

Limit ratio of rule: 1.46557123188...

Resulting interval ((1200 \* limit ratio)-1200): 558.685478256 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	145.540 cents	145.540
2:	228.169 cents	228.169
3:	310.798 cents	310.798
4:	393.427 cents	393.427
5:	476.056 cents	476.056
6:	558.685 cents	558.685
7:	786.855 cents	786.855
8:	869.484 cents	869.484
9:	952.113 cents	952.113
10:	1034.742 cents	1034.742
11:	1117.371 cents	1117.371
12:	2/1	1200.000 octave

Order of generation:

0 6 11 5 10 4 9 3 8 2 7 1

Subsets:

MOS 5 = 0 5 6 10 11 (C F F# Bb B)

MOS 7 = 0 4 5 6 9 10 11 (C E F F# A Bb B)

MOS 9 = 0 3 4 5 6 8 9 10 11 (C D# E F Gb Ab A Bb B)

MOS 11 = all except 1 (no C#)

**Limit Linear Scale C - 12 tones**

Rule:  $C_n = C_{n-3} + C_{n-2}$

Limit ratio of rule: 1.32471795725...

Resulting interval ((1200 \* limit ratio)-1200): 389.6615487 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	296.615 cents	296.615
2:	327.631 cents	327.631
3:	358.646 cents	358.646
4:	389.662 cents	389.662
5:	686.277 cents	686.277
6:	717.292 cents	717.292
7:	748.308 cents	748.308
8:	779.323 cents	779.323
9:	1106.954 cents	1106.954
10:	1137.969 cents	1137.969
11:	1168.985 cents	1168.985
12:	2/1	1200.000 octave

Order of generation:

0 4 8 11 3 7 10 2 6 9 1 5

Subsets:

MOS 7 = 0 3 4 7 8 10 11 (C Eb E G Ab Bb B)

MOS 10 = all except 1 and 5 (no C# or F)

**Limit Linear Scale D - 12 tones**

Rule:  $D_n = D_{n-4} + D_{n-1}$

Limit ratio of rule: 1.38027756909...

Resulting interval ((1200 \* limit ratio)-1200): 456.333082908 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	50.665 cents	50.665
2:	168.999 cents	168.999
3:	219.664 cents	219.664
4:	337.998 cents	337.998
5:	456.333 cents	456.333
6:	506.998 cents	506.998
7:	625.332 cents	625.332
8:	794.332 cents	794.332
9:	912.666 cents	912.666
10:	963.331 cents	963.331
11:	1081.665 cents	1081.665
12:	2/1	1200.000 octave

Order of generation:

0 5 9 2 7 11 4 8 1 6 10 3

Subsets:

MOS 5 = 0 2 5 7 9 (C D F G A)

MOS 8 = 0 2 4 5 7 8 9 11 (C D E F G Ab A B)

**Limit Linear Scale E - 12 tones**

Rule:  $E_n = E_{n-4} + E_{n-3}$

Limit ratio of rule: 1.22074408461...

Resulting interval ((1200 \* limit ratio)-1200): 264.892901532 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	124.465 cents	124.465
2:	248.929 cents	248.929
3:	264.893 cents	264.893
4:	389.357 cents	389.357
5:	513.822 cents	513.822
6:	529.786 cents	529.786
7:	654.250 cents	654.250
8:	794.679 cents	794.679
9:	919.143 cents	919.143
10:	1059.572 cents	1059.572
11:	1184.036 cents	1184.036
12:	2/1	1200.000 octave

Order of generation:

0 3 6 8 10 1 4 7 9 11 2 5

Subsets:

MOS 5 = 0 3 6 8 10 (C Eb Gb Ab Bb)

MOS 9 = 0 1 3 4 6 7 8 9 10 (C Db Eb E Gb G Ab A Bb)

**Limit Linear Scale F - 12 tones**

Rule:  $F_n = F_{n-5} + F_{n-1}$

Limit ratio of rule: 1.32471795724...

Resulting interval ((1200 \* limit ratio)-1200): 389.661548688 cents

**Note: this is precisely the same as Scale C in this set of scales.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	296.615 cents	296.615
2:	327.631 cents	327.631
3:	358.646 cents	358.646
4:	389.662 cents	389.662
5:	686.277 cents	686.277
6:	717.292 cents	717.292
7:	748.308 cents	748.308
8:	779.323 cents	779.323
9:	1106.954 cents	1106.954
10:	1137.969 cents	1137.969
11:	1168.985 cents	1168.985
12:	2/1	1200.000 octave

Order of generation:

0 4 8 11 3 7 10 2 6 9 1 5

Subsets:

MOS 7 = 0 3 4 7 8 10 11 (C Eb E G Ab Bb B)

MOS 10 = all except 1 and 5 (no C# or F)

**Limit Linear Scale G - 12 tones**

Rule:  $G_n = G_{n-5} + G_{n-2}$

Limit ratio of rule: 1.23650570339...

Resulting interval ((1200 \* limit ratio)-1200): 283.806844068 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	154.262 cents	154.262
2:	219.034 cents	219.034
3:	283.807 cents	283.807
4:	438.068 cents	438.068
5:	502.841 cents	502.841
6:	567.614 cents	567.614
7:	721.875 cents	721.875
8:	786.648 cents	786.648
9:	851.421 cents	851.421
10:	1070.455 cents	1070.455
11:	1135.227 cents	1135.227
12:	2/1	1200.000 octave

Order of generation:

0 3 6 9 11 2 5 8 10 1 4 7

Subsets:

MOS 5 = 0 3 6 9 11 (C Eb Gb A B)

MOS 9 = 0 2 3 5 6 8 9 10 11 (C D Eb F Gb Ab A Bb B)



**Limit Linear Scale H - 12 tones**

Rule:  $H_n = H_{n-5} + H_{n-3}$

Limit ratio of rule: 1.19385911132...

Resulting interval ((1200 \* limit ratio)-1200): 232.630933584 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	158.940 cents	158.940
2:	195.786 cents	195.786
3:	232.631 cents	232.631
4:	428.417 cents	428.417
5:	465.262 cents	465.262
6:	661.047 cents	661.047
7:	697.893 cents	697.893
8:	893.678 cents	893.678
9:	930.524 cents	930.524
10:	1126.309 cents	1126.309
11:	1163.155 cents	1163.155
12:	2/1	1200.000 octave

Order of generation:

0 3 5 7 9 11 2 4 6 8 10 1

Subsets:

MOS 5 = 0 3 5 7 9 (C Eb F G A)

MOS 6 = 0 3 5 7 9 11 (C Eb F G A B)

MOS 11 = all except 1 (no C#)

**Limit Linear Scale I - 12 tones**

Rule:  $I_n = I_{n-5} + I_{n-4}$

Limit ratio of rule: 1.16730397826...

Resulting interval ((1200 \* limit ratio)-1200): 200.764773912 cents

**This scale is a whole tone scale with beatings!**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	4.589 cents	4.589
2:	200.765 cents	200.765
3:	205.353 cents	205.353
4:	401.530 cents	401.530
5:	406.118 cents	406.118
6:	602.294 cents	602.294
7:	606.883 cents	606.883
8:	803.059 cents	803.059
9:	807.648 cents	807.648
10:	1003.824 cents	1003.824
11:	1008.413 cents	1008.413
12:	2/1	1200.000 octave

Order of generation:

0 2 4 6 8 10 1 3 5 7 9 11

Subsets:

MOS 5 = 0 2 4 6 8 (C D E F# G#)

MOS 6 = 0 2 4 6 8 10 (C D E F# G# A#)

MOS 11 = all except 11 (no B)

**Limit Linear Scale J - 12 tones**

Rule:  $J_n = J_{n-6} + J_{n-1}$

Limit ratio of rule: 1.28519903326...

Resulting interval ((1200 \* limit ratio)-1200): 342.238839912 cents

**This is a 7 tone scale with 5 tones having partners that produce beating.**

Scale:

0:	1/1	0.000 unison, perfect prime
1:	164.627 cents	164.627
2:	168.955 cents	168.955
3:	337.911 cents	337.911
4:	342.239 cents	342.239
5:	511.194 cents	511.194
6:	680.150 cents	680.150
7:	684.478 cents	684.478
8:	853.433 cents	853.433
9:	1022.388 cents	1022.388
10:	1026.717 cents	1026.717
11:	1195.672 cents	1195.672
12:	2/1	1200.000 octave

Order of generation:

0 4 7 10 2 5 8 11 3 6 9 1

Subsets:

MOS 7 = 0 2 4 5 7 8 10 (C D E F G Ab Bb)

MOS 11 = all except 1 (no C#)

**Limit Linear Scale K - 12 tones**

Rule:  $K_n = K_{n-6} + K_{n-5}$

Limit ratio of rule: 1.13472413840...

Resulting interval ((1200 \* limit ratio)-1200): 161.66896608 cents

Scale:

0:	1/1	0.000 unison, perfect prime
1:	93.352 cents	93.352
2:	161.669 cents	161.669
3:	255.021 cents	255.021
4:	323.338 cents	323.338
5:	416.690 cents	416.690
6:	485.007 cents	485.007
7:	578.359 cents	578.359
8:	646.676 cents	646.676
9:	808.345 cents	808.345
10:	970.014 cents	970.014
11:	1131.683 cents	1131.683
12:	2/1	1200.000 octave

Order of generation:

0 2 4 6 8 9 10 11 1 3 5 7

Subsets:

MOS 5 = 0 2 4 6 8 (C D E F# G#)

MOS 6 = 0 2 4 6 8 9 (C D E F# G# A)

MOS 7 = 0 2 4 6 8 9 10 (C D E F# G# A Bb)