# EM Fields Reference

## Definitions

$$\epsilon_0 \doteq 8.854 \times 10^{-12}$$
$$k = \frac{1}{4\pi\epsilon_0}$$
$$\doteq 8.988 \times 10^9$$
$$\Psi = \text{electric flux}$$
$$\mathbf{D} = \text{flux density}$$

## Common Integrals

$$\int x\left(y+x^2\right)^a \, \mathrm{d}x = \frac{\left(y+x^2\right)^{a+1}}{2a+2}$$

## Vector Operations

### Dot Product

$$\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}|\,|\mathbf{B}|\cos\theta_{AB}$$
$$= A_xB_x + A_yB_y + A_zB_z$$
$$\theta = \cos^{-1}\frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}|\,|\mathbf{B}|}$$

### Cross Product

$$\mathbf{A}\times\mathbf{B} = \mathbf{a}_N\,|\mathbf{A}|\,|\mathbf{B}|\sin\theta_{AB}$$
$$= \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$
$$= \left(A_yB_z - A_zB_y\right)\mathbf{a}_x + \left(A_zB_x - A_xB_z\right)\mathbf{a}_y + \left(A_xB_y - A_yB_x\right)\mathbf{a}_z$$

## Coordinate Systems

### Cartesian

$$A = A\left(x,y,z\right)$$
$$\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$$
$$|\mathbf{A}| = \sqrt{A_x^2+A_y^2+A_z^2}$$
$$\mathbf{a}_A = \frac{\mathbf{A}}{|\mathbf{A}|}$$
$$= \frac{\mathbf{A}}{\sqrt{A_x^2+A_y^2+A_z^2}}$$

### Cylindrical

$$A = A\left(\rho,\phi,z\right)$$
$$\mathbf{A} = A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$$
$$|\mathbf{A}| = \sqrt{\rho^2+z^2}$$

### Cylindrical to Cartesian

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

### Cylindrical to Spherical

$$r = \sqrt{\rho^2+z^2}$$
$$\phi = \phi$$
$$\theta = \tan^{-1}\frac{\rho}{z}$$

### Area and Volume

$$\mathrm{d}S = \rho\,\mathrm{d}\phi\,\mathrm{d}z$$
$$S = \iint_s \mathrm{d}S$$
$$= \int_0^{2\pi}\int_0^z \rho\,\mathrm{d}\phi\,\mathrm{d}z$$
$$= 2\pi\rho z$$
$$\mathrm{d}V = \rho\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}z$$
$$V = \iiint_v \mathrm{d}V$$
$$= \int_0^\rho\int_0^{2\pi}\int_0^z \rho\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}z$$
$$= \pi\rho^2 z$$

### Spherical

$$A = A\left(r,\phi,\theta\right)$$
$$\mathbf{A} = A_r\mathbf{a}_r + A_\phi\mathbf{a}_\phi + A_\theta\mathbf{a}_\theta$$
$$|\mathbf{A}| = A_r$$

### Spherical to Cartesian

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\cos\phi$$
$$z = r\cos\theta$$

### Spherical to Cylindrical

$$\rho = r\sin\theta$$
$$\phi = \phi$$
$$z = r\cos\theta$$

### Area and Volume

$$\mathrm{d}S = r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$
$$S = \iint_s \mathrm{d}S$$
$$= \int_0^\pi\int_0^{2\pi} r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$
$$= 2\pi r^2\int_0^\pi \sin\theta\,\mathrm{d}\theta$$
$$= 2\pi r^2\left[-\cos\pi + \cos 0\right]$$
$$= 4\pi r^2$$
$$\mathrm{d}V = r^2\,\mathrm{d}r\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$
$$V = \iiint_v \mathrm{d}V$$
$$= \int_0^r\int_0^\pi\int_0^{2\pi} r^2\,\mathrm{d}r\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$
$$= 2\pi\int_0^r\int_0^\pi r^2\,\mathrm{d}r\sin\theta\,\mathrm{d}\theta$$
$$= \frac{2}{3}\pi r^3\int_0^\pi \sin\theta\,\mathrm{d}\theta$$
$$= \frac{4}{3}\pi r^3$$

# Electric Field and Flux

## Coulombs Law

$$F = \frac{Q_1Q_2}{4\pi\epsilon_0R^2}\ \mathrm{N}$$
$$\mathbf{F} = \frac{Q_1Q_2}{4\pi\epsilon_0R_{12}^2}\mathbf{a}_{21}\ \mathrm{N}$$

## Field Intensity

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0R^2}\mathbf{a}_R\ \frac{\mathrm{V}}{\mathrm{m}}$$

## Charge Distributions

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\mathbf{a}_\rho \qquad \text{line charge}$$
$$\mathbf{E} = \frac{\rho_s}{2\epsilon_0}\mathbf{a}_N \qquad \text{sheet charge}$$

## Flux Density

$$\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r \qquad \text{point charge}$$
$$= \frac{\rho_L}{2\pi\rho}\mathbf{a}_\rho \qquad \text{line charge}$$
$$= \frac{\rho_S}{2}\mathbf{a}_N \qquad \text{sheet charge}$$
$$= \epsilon_0\mathbf{E} \qquad \text{in free space}$$
$$\Psi = \oint_s \mathbf{D}_s\cdot d\mathbf{S} = Q \qquad \text{Gauss's law}$$

## Divergence

$$\mathrm{div}\mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \qquad \text{rect}$$
$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho D_\rho\right) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z} \qquad \text{cyl}$$
$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2D_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\ D_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi} \qquad \text{sph}$$
$$\nabla = \frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z$$
$$\nabla\cdot\mathbf{D} = \left(\frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z\right)\cdot\left(D_x\mathbf{a}_x + D_y\mathbf{a}_y + D_z\mathbf{a}_z\right)$$
$$= \mathrm{div}\mathbf{D}$$
$$\oint_S \mathbf{D}\cdot d\mathbf{S} = \int_{\mathrm{vol}} \nabla\cdot\mathbf{D}\,\mathrm{d}v$$