How to Rigorously Define Fine-Tuning

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Dear Readers:

For those who are not familiar with what a constant of physics is, I will briefly explain here: The fundamental constants of physics are the fundamental numbers (such as the gravitational constant G in Newton's law of gravity, $F = Gm_1m_2/r^2$) that help determine the basic structure of the universe. The "fine-tuning" of the cosmos refers to the claim that the fundamental constants of physics (and the initial conditions of the universe) are "balanced on a razor's edge" for life to occur. This is often taken to suggest that the universe was designed or that there are many universes in over which the constants vary at random and hence just by chance a universe exists in which the constants have the right values for life. So far, no one has adequately explicated what it means for a constant to be fine-tuned. This is what I attempt to do in this paper.

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I: Introduction:

In chapter two, we presented the evidence for the fine-tuning of the constants of physics for life. In that chapter, we claimed that a constant was fine-tuned if the width, W_r , of the range of values of the constant that permit, or are optimal for, the existence of intelligent life is small compared to the width, W_R , of some properly chosen comparison range R: that is, if Wr/WR << I, where << stands for much, much less than. (I) (Wr could also stand for the sum of the widths of the intelligent-life-permitting regions.) The range r of intelligent life-permitting values is determined via physical calculations and thus, apart from debates about what is meant by intelligent-life, is largely unproblematic from a philosophical perspective. In contrast, our choice of the comparison range R - of which r will always be taken to be a subrange - cannot be decided by physical considerations alone.

Insofar as the issue of how to choose the comparison range is addressed in the literature, it is typically merely asserted without argument that the comparison range is the range of conceivable values for the constant in question. This range is further assumed to go from minus infinity to infinity. For instance, in a recent article arguing against the fine-tuning argument, Timothy and Lydia McGrew and Eric Vestrup make this assumption without any justification being offered for their choice (2001). Besides lacking justification, such a choice for the comparison range already presupposes without argument that the laws and constants of physics can exist apart from a universe to instantiate them. As I argue in section IV below, for many values of the constants of physics - such as the gravitational constant - no universe could exist that could instantiate them. Hence such values are not even logically possible under metaphysical accounts of the laws of nature in which the laws must be instantiated, such as, for instance, the regularist view of the

laws of nature in which laws are merely merely descriptions of universal regularities in the universe. In any case, as I will argue in the appendix, even if the comparison range for some constant turns out to be infinite, that poses no problem for the fine-tuning argument, contrary to what McGrew and Vestrup claim.

An outstanding issue for developing the fine-tuning argument, therefore, is to find a plausible methodology for choosing the comparison range R. This is what we will do in this chapter. The proper way of choosing the comparison range hinges on our method of inference to design or many universes. We want to choose our comparison range in such a way that the method of inference to design or many universes that we use turns out to be sound assuming the validity of the fundamental principles underlying the method of inference in question. In chapter four, we present two methods of inference. One is a quasi-Bayesian method. As I show in chapter 4, defining the comparison range in such a way that the quasi-Bayesian method of inference turns out to be sound will result in the soundness of the other methods of inference, given that one grants the fundamental principles underlying the application of this method. Here we will only consider the inference to design since this is the primary issue we are concerned with but our method of choosing the comparison range will work as well for an inference to many universes. [confirmation instead of inference?]

Put simply, the method of choosing the comparison range that will result in the soundness of our quasi-Bayesian inference strategy is to choose the comparison range as the range of theoretically possible values for the constant in question - that is, the range of possible values allowed by our background theories, initial conditions, laws of physics, and other constants. Although the method is simple to state and seems intuitively plausible, providing a fundamental justification of this method (section III), spelling out how to choose the background theories and laws (section III), and then applying them to actual cases of fine-tuning (section IV) involves a fair amount of complexity. Finally, in the appendix, we will address the objection raised by McGrew and Vestrup (2001) against infinite comparison ranges, ranges which as mentioned above they assume without argument. Before presenting the details of our method in section III, we first must address some preliminaries, such as an outline of our quasi-Bayesian inference strategy and the notion of probability we will be employing.

II. Some Preliminaries

(i) Quasi-Bayesian Inference Strategy

As we stated in the last section, we want our method of choosing the comparison range to be such as to make the quasi-Bayesian method of inference underlying our argument in support of design a sound strategy. The quasi-Bayesian method of inference essentially involves four steps:

- 1. Argue that for certain constants of physics, the width of the intelligent-life-permitting range is much smaller than the width of the comparison range: that is, $Wr/WR \ll 1$.
- 2. Argue that if Wr/WR << 1 for some constant C, then it is epistemically very improbable for the constant to fall into the intelligent-life-permitting range under what I call the atheistic single-universe hypothesis: that is, P(Lc/ k' & As) << 1. [Here Lc denotes the claim that a constant C falls into the intelligent-life-permitting or intelligent-life-optimal range, k' denotes some appropriately chosen background information, and As denotes the atheistic single-universe hypothesis that is, the hypothesis that only one universe exists and that this universe exists as a brute fact without any reason for its existence. Finally, P(Lc/ k' & As) denotes the conditional epistemic probability of Lc on k' & As.] We will assume here that the probabilistic principle of indifference (see below) grounds the inference from Wr/WR << 1 to P(Lc/ k' & As) << 1, although using this principle is not absolutely necessary (see below).
- 3. Argue that it is not epistemically highly improbable for the constants of physics to have intelligent-life-permitting values under the theistic hypothesis. That is, argue that for any given constant C, P(Lc/k' & T) is not much, much less than 1, where T denotes the theistic hypothesis.

4. Using what I call the prime principle of confirmation (PPC) [see below], conclude from (2) and (3), that because the epistemic probability of certain constants having intelligent-life-permitting values is much greater under the theistic hypothesis than under the atheistic single-universe hypothesis, the fact that these constants fall into the intelligent-life-permitting region strongly confirms the theistic hypothesis over the atheistic single-universe hypothesis *relative* to background information k'. That is, for certain constants C, the fact that P(Lc/k' & T) > P(Lc/k' & As) implies that Lc confirms T over As.

The soundness of steps (1) and (2) crucially depend on how we define the comparison range R. This constitutes the major bulk of this chapter. That is, we want to define the comparison range in such a way that for at least some of the constants of physics, $Wr/WR \ll 1$ [step 1], while at the same time step (2), the inference from $Wr/WR \ll 1$ to $P(Lc/As \& k') \ll 1$, is legitimate assuming the validity of the principle of indifference. If we can do this, then the soundness of the fine-tuning argument will only depend on steps (3) and (4) since the truth of steps (1) and (2) will be guaranteed by our choice of the comparison range.

(ii) The Prime Principle of Confirmation

The prime principle of confirmation, which is also been called the *likelihood principle* or the *principle of relevance*, can be stated as follows. Let H1 and H2 be two competing non-ad-hoc hypotheses: that is, hypotheses that were not constructed merely to account for the data E in question and with no independent evidence in their favor. According to this principle, if a body of data E is more *epistemically* probable under hypothesis H1 than under hypothesis H2, then the data E provides evidence in favor of H1 over H2. Further, the strength of the evidence will be proportional to the ratio $P(E/H_1)/P(E/H_2)$, where P(/) represents conditional epistemic probability. (2)

(iii) Definition of Probability

As I argue for in more detail in chapter 4, the relevant notion of probability occurring in the fine-tuning argument is a widely recognized type of probability called *epistemic* probability. Roughly, the unconditional epistemic probability of a proposition can be thought of as the degree confidence or belief we rationally should have in the proposition. Further, the conditional epistemic probability of a proposition R on another proposition S--written as P(R/S)--can roughly be defined as the degree to which the proposition S *of itself* should rationally lead us to expect that R is true. Under the epistemic conception of probability, therefore, the claim that $P(Lc/As \& k') \ll 1$ is to be understood as making a statement about the degree to which As & k' should, of itself, rationally lead us to expect C to have an intelligent-life-permitting value. [For a more indepth account of epistemic probability along with an explication of the *of itself* clause, see chapter 7.]

The appropriate choice of background information k' is crucial here, since our total background information k includes the information that we are alive and hence by implication that the constants of physics have intelligent-life-permitting values. Accordingly P(Lc/As & k) = 1, and hence no confirmation argument can get off the ground if we use k as our background information. Thus we confront the much discussed problem of *old evidence* with Bayesian style inferences. In the section III below we will directly address this problem and propose a solution to it that will allow for a non-trivial conditional epistemic probability of a universe existing with intelligent life permitting values for its constants on the atheistic-single universe hypothesis.

It is important to stress here that, within the epistemic conception of probability, there is no statistical probability of a universe turning out fine-tuned in and of itself. One could only get a statistical probability if one models the universe as being generated by some universe generator, which churns out intelligent-

life-permitting universes a certain proportion of the time. The whole point of the atheistic single-universe hypothesis, however, is that there is no such universe generator. Rather, the universe exists as a brute fact, inexplicable fact. Thus, the probabilities in this case should not be confused with some sort of statistical probability. Rather, they are measures of the rational degree of belief of one proposition on another: for example, the rational degree of belief in the claim that the constant falls into the intelligent-life-permitting region on the atheistic single-universe hypothesis.

(iv) Inferring From Wr/WR << 1 to P(Lc/k' & As) << 1

The principle of inference used to infer from Wr/WR << 1 to P(Lc/ k' & As) << 1 is the epistemic principle of indifference. Roughly, this principle states that when we have no reason to prefer any value of a parameter over another, we should assign equal probabilities to equal ranges of the parameter, given that the parameter in question directly corresponds to some physical magnitude (or occurs in the simplest way of writing the fundamental theories in the relevant domain). A full statement and defense of this principle will be presented in chapter 6. Applied to the case at hand, since R will be chosen in such a way that the atheistic single-universe hypothesis conjoined with the appropriate background information k' is claimed to give us no reason to prefer any value of a constant over any other within the range R, it follows from the principle of indifference that we should put a uniform probability distribution over region R. This is sufficient to justify the inference from Wr/WR << 1 to P(Lc/As & k') << 1.

It should be noted, however, that in defining the comparison range this way, one need not be committed to the general validity of the principle of indifference. All we are doing here is defining the comparison range in such a way that if one were to use a quasi-Bayesian form of inference, and apply the principle of indifference, the crucial steps in the inference would turn out to be sound. A separate argument would need to be presented to show that this method of defining the comparison range works for other proposed methods of inference.

It should also be said, however, that any principled justification of the inference from Wr/WR << 1 to P(Lc/As & k') << 1 must put some constraints on our credence function over the comparison range R, however we choose that comparison range. (3) Without some constraint, one could always choose a credence function such that P(Lc/As & k') had any probability one likes. The principle of indifference simply provides a particularly strong constraint, requiring that one have a uniform credence function over R. One merit of a uniform credence function is that it seems to be the least arbitrary choice. It is possible, however, that one could still justify the inference from Wr/WR << 1 to P(Lc/As & k') << 1 by adopting some principle that yielded weaker constraints on the credence function.

(v): What it Means to Vary A Constant of Physics

Before presenting our procedure for determining the comparison range, we need to define more carefully what a constant of physics is and what it means to vary such constants. Intuitively there is a distinction between laws and constants, and physicists usually suppose such a distinction - that is why they talk about the fundamental constants. In current physics, most laws can be thought of as a mathematical description of the relation between certain physical quantities. These descriptions have a certain mathematical form, along with a set of numbers that are determined by experiment. So, for example, Newton's law of gravity, $F = GM_1M_2/r^2$ has a certain mathematical form, along with a number G determined by experiment. We can then think of a world in which the relation of force to mass and distance has the same mathematical form of being proportional to the product of the masses divided by the distance between them squared, but in which the number G is different. We could then say that such worlds have the same law of gravity, but a different

value for the gravitational constant G. A similar separation can be made between the mathematical form of an equation and the constants occurring in the equation, such as Maxwell's equation or the Schrödinger equation. So, when we conceive of worlds in which the constants of physics are different but in which the laws are the same, we are conceiving of worlds in which the mathematical form of the law remains the same, but in which the experimentally determined numbers are different.

In speaking of the laws in this way, we are already assuming a certain level of description of physical reality. The history of physics is one in which we find that a certain law is only a limiting case of some deeper law, such as Newton's law of gravity being a limiting case Einstein's equation of general relativity. This is especially relevant in the context of the current search for a grand unified theory. One hope is that such a grand unified theory will have no free parameters. The idea is that higher-level principles of physics - such as those that lie at the heart of quantum mechanics and general relativity - will uniquely determine the form of the grand unified theory along with the various fundamental constants of physics that are part of this theory. In this case, given the higher-level principles, it would be impossible for the constants to be any different than they are. Elsewhere (chapter 5), I address whether such a grand unified theory would undermine the fine-tuning argument, and conclude that it would not since it would still be very coincidental that the a universe is such that the higher-level principles it instantiates entail just those values for the constants of physics that are intelligent-life-permitting. Here, my point is simply that any discussion of fine-tuning must be explicated at a certain level of description of physical reality.

This issue also arises even in the context of our current understanding of physics. Consider, for instance, the strength of the strong force that holds neutrons and protons together in the nucleus. This force is actually not fundamental, according to current theory, but simply a product of a deeper force between the quark constituents of the protons and neutrons, much as the force of cohesion between molecules is not fundamental but rather a product of various electromagnetic (and exclusion principle) forces. This deeper force binding quarks together is given by quantum chromodynamics (QCD), which in current theory has its own set of free parameters. From the perspective of QCD, one cannot simply change the strength of the strong force while keeping everything else the same. Instead, one would have to change one or more of the parameters of QCD, which would in turn change not just the strength of the strong force, but other things such as the masses of the neutron and proton and the range of the strong force. Calculations of these effects are very difficult because of the complexity of the situation, and hence it is difficult to develop a rigorous argument for fine-tuning at the level of QCD. Thus, for practical purposes, at present we need to develop most of our arguments for fine-tuning at a higher-level of description, such as at the level of the phenomenological equation governing the strong force between nucleons.

I argue in more detail elsewhere (see chapter ____) that this is a legitimate procedure for calculating epistemic probabilities. The basic idea behind the argument is that epistemic probabilities are only useful in conditions of ignorance - they are attempts to generate degrees of expectation, or conditional degrees of expectation, when we do not know everything about a situation. For example, we have an unconditional epistemic probability that a coin will land on heads of 50% (instead of 100% or 0%) because we are ignorant of the physically determined side on which it will land. So, the only rule is that we should formulate our expectations using all the knowledge we have for which we are able to make calculations - for example, knowledge such as that the coin is not weighted in favor of either side. Accordingly, until we can make calculations relevant to fine-tuning at the level of QCD, we should perform our calculations at the level of a more phenomological description of the strong force.

The above procedure can be thought of as looking into "nearby" law structures in which the fundamental laws of physics are slightly different instead of merely the constants of physics being different. Suppose, for instance, that QCD provides the most fundamental set of laws governing the forces between nucleons. Then, varying the parameters of QCD would in effect be to vary the fundamental constants of physics. On the other hand, no set of variations in the fundamental parameters of QCD yield a variation in the strong force coupling constant g_s while at the same time keeping everything else constant. Thus, in effect, law structures in which g_s is different but everything else remains the same are laws structures in which the fundamental equations of physics are different. In this case, therefore, our fine-tuning is not so much the fine-tuning of a fundamental parameter of physics, but rather a fine-tuning with respect to a set of law

structures generated by variations in the parameter g_s . But I do not see where this presents a problem. There is nothing less fundamental about varying a law then a parameter. In the strict sense, when we vary the fundamental parameters, we also vary the laws since laws and parameters come as single packages in nature; it is only we humans that make the distinction. What matters is being able to calculate what happens for nearby law structures, and to provide a parameter that provides an epistemic measure over the reference class consisting of our law structure and these nearby possible law structures. (Finally, if one does still does not like this way of thinking about it, alternatively one can think of this fine-tuning as a fine-tuning with respect to possible universes, or variations of our universe, in which the phenomenological law governing the strong force remains the same but its strength as given by g_s remains the same.)

Of course, our choice of nearby laws structures must not be biased to yield the results we want, just as when we choose a sample of a population in testing the efficacy of a drug we should make sure that we choose our sample in as unbiased a way as possible, such as a random sampling technique. Certainly, however, the above method of choosing to vary the constants in the most fundamental theory for which we can make calculations is not biased in favor of fine-tuning. Finally, just as in one can almost always choose a sample out of a sufficiently large population that will yield any statistical result one wants, one could always select nearby law stuctures in such a way that no fine-tuning results - e.g., one could just choose for one's reference class the set of nearby universes that are intelligent-life-permitting. What is important for calculating epistemic probability, however, is that our method of selection be non-arbitrary.

That said, one must keep this whole enterprise in perspective. Our ultimate goal is not to provide exact numerical values for the degree to which we should be surprised that a certain constant falls into the intelligent-life-permitting range. Rather, our goal is to provide as much objective support for the claim that we should find the fact that some parameter falls into the intelligent-life-permitting range very surprising. That is, what we want to show is that the intuitive sense of surprise that we have is not based on some mistake in thinking or perception, and not just based in some merely subjective interpretation of the data, but rather can be grounded in a justified, non-arbitrary procedure. In doing this, we must remember that the exact values for the epistemic probabilities that we come up with will certainly turn out to be different if we are able to make these calculations from the perspective of a deeper theory, such as QCD. The fact that many constants of physics turn out to be fine-tuned at a lower level of description, however, gives us good reason to believe that there will be significant fine-tuning at a deeper level.

Finally, as I explicate in more detail in chapter 6 when we discuss the principle of indifference, part of the justification for our overall method of determining epistemic probabilities is based on the choice to eschew agnosticism and to seek the least arbitrary assignment of epistemic probabilities compatible with our knowledge and abilities to perform calculations. The method proposed for determining the comparison range, including our method of looking across possible law structures, is designed to implement this idea. (4)

III. Procedure for Determining Comparison Range

(i) The Method Explained

For a straightforward application of the principle of indifference, the comparison range can be defined as the range R of values for C such that: (i), we are epistemically indifferent with regard to the value of C under k' and the atheistic single-universe hypothesis; and (ii), k' implies that the value of constant C falls within R. If we define the comparison region in this way, then the principle of indifference implies that under k' and As, we should assign equal probabilities to equal ranges within R. Since k' implies that C falls within R, the total probability of C falling within R must be 1. This implies that $P(Lc/k' \& As) = W_r/W_R$, and hence that if Wr/Wr << 1 then P(Lc/k' & As) << 1. We will therefore adopt this as our *strict* definition of the comparison range. Notice that this range can typically be thought of as the *theoretically possible* range allowed by the background theories included in k' since typically the background theories will only tell us the range of possible values for the constant but will not give us any reason to prefer one value of the constant over any other in that range.

If we can find a suitable k' and R that meets criteria (i) and (ii), then R will by definition be the comparison range we should use. The key to determining the comparison range will be determining k'. This should be done by a plausible, nonarbitrary procedure which can be grounded in principles that apply more generally to determining the appropriate background information in cases of old evidence.

Roughly, our suggestion for determining k' is to "subtract-out" the relevant information, L_c , that the constant falls into the intelligent-life-permitting region from our background information k. More precisely, we start with the body of relevant background information k which includes all the laws of physics along with the values of all the constants of physics, with one exception: k does not contain the exact value for the constant k under consideration, but only the information that k falls into the intelligent-life-permitting region. (The justification for this exception will be presented below at the end of the next subsection.) Then, for the specific constant k under consideration, we pretend that we do not know that it falls into the intelligent-life-permitting region. This means that we "subtract-out" information k from the background information k, leaving us with a body of information k'. How exactly we are to subtract-out k will be considered more below in the context of the problem of old evidence.

As we will see below, in some cases the method of subtracting will not yield a unique k' but a whole set of possibilities. In such cases, all we can say is that there is not a unique comparison range, but rather a set of possible comparison ranges, the set consisting of all those comparison ranges given by the various possible k's. A constant C will then be said to be fine-tuned if the intelligent-life-permitting range is small compared to the lower bound of the comparison ranges in this set.

We can now see how defining the comparison region in the way specified above makes sense. k' captures all of our knowledge of the background laws and constants of physics except that pertaining directly to the value of the constant C under consideration. The comparison range is simply the range of values allowed by those constants and laws, or the range over which they imply that we should be epistemically indifferent. Is there a general justification for this procedure for determining k'? The issue of determining k' in our case is just the issue of how to determine k in the case of *old evidence*, extensively discussed in the literature on applying Bayesian methods to scientific inference, which we will now explain.

(ii) Justification of Method: The Problem of Old Evidence

In cases of inference in which a theory novelly predicts new data E - such as general relativity's prediction of the degree of bending of light around the sun - the background information k' is simply the information available right before the prediction was tested. Then, the idea behind the Bayesian approach is that we can ask ourselves what is our prior degree of belief in E given k' (i.e., P(E/k')) and compare that with P(E/k') & H), where H is the hypothesis, such as general relativity, that is being confirmed or disconfirmed by E. Then, E is said to confirm H if and only if P(E/k') & H) > P(E/k').

In other cases, however, a theory predicts a set of data that is already known. For example, general relativity accurately predicted the precession of the perihelion of Mercury, data that was known for fifty years but was inexplicable under Newton's theory of gravity. In this case, the problem for Bayesian methodology is finding the right background information. Clearly, if k' is simply the background information available at the time general relativity was developed, then it would include this information E about the precession of the perihelion of Mercury. But, then P(E/k') = 1 = P(E/k' & H), and hence it follows from Bayesian confirmation theory that E will not confirm general relativity, which is clearly false.

Although the problem of old evidence is almost always discussed in the context of subjective Bayesianism, it arises for any account of confirmation in which e's confirming H is explicated in terms of the degree to which the knowledge of e should raise the credibility of h. That is, we would like to say that e confirms or supports h if h is more credible given k & e than given k alone, where k is our background information. But, for cases in which evidence e is already known, k & e = k, and hence this initially plausible account of confirmation entails that e can never confirm h when e is already known.

In the literature, there are two major attempted solutions to the problem of old evidence (Howson, 1991). The first solution is learning that h explains e, not knowledge of e, that boosts the credibility of h. As pointed out by Howson and others, perhaps the most severe problem with this account is that even in those cases in which an hypothesis h was explicitly constructed to explain some body of data e (such as Newton constructing his inverse square law to explain Kepler's laws of planetary motion), we still regard e as confirming h. In such cases, the knowledge that e explains h was learned at the same time as h. Further, as John Earman notes, we want to say that the observed value of Mercury's perihelion advance is good evidence for Einstein's theory of general relativity, even though for most of us, the "first thing we may have learned about the theory before hearing any details about the theory itself, was that it explains the perihelion advance." (Quoted in Howson, p. 545) Thus, it seems to me this is not an adequate solution to the problem of old evidence. Even if one adopts this solution, however, I do not believe it would lead to different comparison ranges, though I will not argue for it here.

The second solution to the problem of old evidence is subtract-out our knowledge of e from the background information K available at the time a theory was developed, and then relativize confirmation to this new body of information $k' = k - \{e\}$. As Colin Howson explains, "when you ask yourself how much support e gives h, you are plausibly asking how much a knowledge of e would increase the credibility of h"(P. 548). As Howson points out, however, this is "the same thing as asking how much e boosts the h relative to what else be know" (P. 548). This "what else" is just our background knowledge k minus e, or symbolically, k - $\{e\}$. (P. 548). So, intuitively, relativizing confirmation to $k' = k - \{e\}$ seems the correct way to proceed. That is, e could be said to confirm h if and only if e boosts the credibility of h if added to knowledge k'; or put differently, if h is more credible given k = k' & e then given k' alone. As appealing as this method seems, it faces a major problem: there is no unambiguous way of subtracting e from k. To illustrate, consider the case of fine-tuning, and let e be the claim that the constant C falls within some range of experimentally determined values x. Now, the fact that a constant C falls into the range x, along with the laws of physics, the initial conditions of the universe, and the other values for the constants of physics, entails certain facts F_1 about the large-scale structure of the universe, and further renders other facts F_2 highly probable. So, our question is, when subtracting e from k, do we also subtract all of our knowledge that deductively and probabilistically depends on e? If yes, then things get very murky: among other problems, what level of probability counts as probabilistically dependent? That is, to what degree must e render facts F₂ probable in order to say that we should subtract F₂ out? If no, then k will effectively contain e. For example, certain facts F_1 about the large-scale structure will entail e, and other facts F_2 will make e likely. For example, the fact that life exists on some planet - which is a fact about the large scale structure of the universe -- might render it likely that all the constants must fall into the *intelligent*-life-permitting region. Thus, if we include this fact in our background information k, we will get little confirmation.

Howson recognizes these and other problems and attempts to solve them by claiming that we should regard k as "in effect, an independent axiomatization of background information and k-{e} as the simple settheoretic subtraction of e from k." (P. 549). That is, Howson proposes that we axiomatize our background information k by a set of sentences A in such a way that e is logically independent of the other the other sentences in A. Then k' would simply consist of the set of sentences A -{e}. One serious problem with this method is that there are different ways of axiomatizing our background information. Thus, as Howson recognizes, the degree to which e confirms h is becomes relative to our axiomatization scheme. (P. 550). In practice, however, this is not a serious as one might expect since in many cases our background information k is already represented to us in a partially axiomatized way in which e is logically isolated from other components of k. As Howson notes, "the sorts of cases which are brought up in the literature tend to be those in which the evidence, like the statements describing the magnitude of the observed annual advance of Mercury's perihelion, is logically isolated component of background information." (1991, p. 549]. In such cases, when we ask ourselves how much e boosts the credibility of h with respect to "what else we know," this what else we know is a well-defined by how we represent our background knowledge. Of course, in those cases in which there are alternative ways of axoimatizing k that are consistent with the way our background knowledge is represented to us, there will be corresponding ambiguities in the degree to which e confirms h. I agree with Howson that this is not necessarily a problem unless one thinks that the degree of confirmation e provides h must be independent of the way we represent our background knowledge. Like Howson, I see no reason to make this assumption: confirmation is an epistemic notion and

thus relative to our epistemic situation, which will include the way we represent our background information.

In the case of fine-tuning, our knowledge of the universe is already presented to us in a partially axiomatized way. Assuming a deterministic universe, the laws and constants of physics, along with the initial conditions of the universe, supposedly determine everything else about the universe. Thus the set of propositions expressing these laws, constants, and initial conditions, constitute an axiomatization of our knowledge. Further, in scientific contexts, this represents the *natural* axiomatization. In fact, I would argue, that this is the natural axiomatization of our knowledge is part of our background knowledge, at least for scientific realists who want scientific theories to "cut reality at its seams." [5] Furthermore, it should be noted, we have a particularly powerful reason for adopting this axiomatation in this case. The very meaning of a constant of physics is only defined in terms of a particular framework of physics. Saying that the strong force constant has a certain value, for instance, would be meaningless in Aristotelean physics. Accordingly, the very idea of subtracting out the value of such a constant only has meaning relative to our knowledge of the current set of laws and constants, and hence this constitutes the appropriate axiomatization of our relevant background information k with respect to which we should perform our subtraction.

Using Howson's method, therefore, we have a straightforward way of determining k - $\{e\}$ for the case of the constants of physics: we let k be axiomatized by the set of propositions expressing the initial conditions of the universe and the laws and fundamental constants of physics, where the constants of physics can be considered as given by a list of numbers in a table. To obtain k', therefore, we simple the proposition expressing the value of C from that table. To summarize, the problem of determining k' in our case is simply the problem of determining k' in the case of old evidence. The procedure we are using for determining k' is a variation of a general procedure advocated for such cases.

Finally, as mentioned above, we will not include in our original axiomatized background information k the exact value of the constant C that we are considering, but only that it falls into the known intelligent-lifepermitting range. The reason is that it makes the argument conceptually simpler. The reason it is legitimate to make this exclusion is that all we need to include in k is all information relevant to the Bayesian style confirmation argument we are interested in - that is, all information that makes a difference in the ratio P(Lc/T & k')/P(Lc/As & k') in steps (2) and (3) of our quasi-Bayesian argument presented in section I. Additional information regarding the experimental value of C - or more precisely, the range of values for C within experimental error - presumably will not affect our judgement of this ratio. (1) Thus, for purposes of this argument, it can be excluded from our background information, just as other irrelevant information, such as the location in which I live, can be excluded. On the other hand, even if it were relevant, we would have arrived at an identical k' and hence an identical comparison range by initially including the experimentally determined value (within experimental error) of C in k, and then subtracting it out to determine k'. If we followed this procedure, we would have had to consider the experimental value of C as our relevant evidence, instead of the mere fact that it fell into the intelligent-life-permitting range. Although one could run the probabilistic version of the fine-tuning argument this way (e.g., by comparing the probability of a universe existing with C falling within the experimentally determined range under the design or many-universes hypothesis and the atheistic single-universe hypothesis), it seems less natural to do it this way.

IV. Determining the Comparison Range: Three Case Studies:

a. Fine-tuning of Gravity And Other Force Strengths.

In this section, we will show how to apply the procedure outlined above to delimit the range of force strengths. In criticizing the fine-tuning argument, Ian Hacking argues that advocates of this argument assume what he calls a Galilean view of laws, in which the laws of nature were in some sense given prior to the existence of the universe itself (1987, pp. 128-131). Instead, he argues, the laws of nature only make sense as laws of some universe or another. Instead of hurting the fine-tuing argument, I will argue that if Hacking is correct, all that follows is that the comparison range for each force of nature is limited to a very large finite range, though not necessarily the same large range for each force.

Given that the very idea of a constant of physics only makes sense within a set of laws of nature, and a set of laws only make sense as instantiated in some universe, it makes no sense to talk about varying a constant beyond its universe-permitting range. In other words, possible law structures can only exist if there is a possible universe to instantiate them. The range of possible law structures, therefore, cannot exceed the range of possible universes allowed by the background laws of physics included in k'. Thus, the universe-permitting range forms the absolute bounds for the value of the constants. Since there seem to be no other inherent restrictions on the force strengths, it seems natural to choose this as the theoretically possible range of values. For example, consider the gravitational force. Although it is unclear exactly what the upper bound of the "universe-permitting" strength of the gravitational force is, certainly if gravity were, for example, a factor of 10¹⁰⁰ larger, a viable universe would be impossible: the gravitational attraction that a single particle exerted on itself would result in a black-hole.

Another way of thinking about this issue is in terms of our background information k'. k' includes the information about the laws of nature, initial conditions of the universe, and the values of the other constants of physics. All such information, however, entails the existence of a universe. Thus, k' entails that every constant C - such as the strength of some force - must fall into the universe-permitting range. Thus, given k', at most we could only be epistemically indifferent with regard to values of C within this range; all other values would be assigned zero probability.

A tricky question is what constitutes a universe. Does empty space-time qualify? Does our "black hole universe" consisting of no-space time qualify? Here I think the answer partly depends on the account of the laws of nature that one adopts. Nonetheless, under most of the major views of the laws of nature, it is difficult to see how the "black-hole universe," consisting merely of a black-hole singularity, could qualify. Under a regularist view of the laws of nature, for instance, the laws of nature are simply ways of describing the basic regularities in the world. Certainly a black-hole universe could hardly be said to qualify under this view, since there would not be any nontrivial regularities to speak of in such a "universe." To elaborate, a black-hole universe would consist of a singularity in space-time, where the curvature of space-time would be infinite, and thus strictly speaking it is mathematically undefined. In cosmology, black holes are only describable in terms of as what happens in the limit as the singularity is approached. Thus, to even make sense of the idea of a black hole instantiating some laws of nature requires that the black-hole be surrounded by regions of space-time with finite curvature. On the other hand, under most necessitarian views of the laws of nature, the laws of nature cannot exist apart from some universe that instantiates them. Under the necessitarian view of the laws of nature adopted by David Armstrong, for instance, the laws of nature consist of relations between universals, and these universals must be instantiated. A "black-hole universe." however, could hardly be said to significantly instantiate any universals. Finally, it is difficult to see how a "black-hole" universe could have initial conditions. Since the initial conditions - for some spacelike hypersurface - are included in our background information k', it follows that such universes would be outside of the theoretically possible range dictated by k'.

What about those accounts of the laws of nature in which the laws in some sense exist apart from being instantiated by some universe? The two major accounts along these lines are the theistic account, in which the laws of nature are the standing will of God, and the neo-platonic account presented by Michael Tooley according to which the laws of nature are relations between universals, where the universals are considered to exist in some platonic realm. As pointed out by Ian Hacking, if one adopts the theistic account, the fine-tuning argument to design becomes superfluous since one already is assuming the conclusion of the argument. On the other hand, if we adopt Tooley's account, we could assume that since the basic laws would exist no matter what the forces strengths. If we neglected the fact that k' includes the initial

conditions of the universe, this view and the theistic view would allow the range of possible force strengths would be infinite, from minus infinity to infinity. Since the initial conditions of the universe, however, are included in k' - and presumably black hole universes could not have initial conditions since they do not instantiate laws - it seems that once again the range of force strengths allowed by k' turns out to be finite.

Finally, as an illustration, we will try to provide a rough estimate of the upper bound for the degree of fine-tuning Wr/WR in the case of gravity. As I show in chapter 2, if the strength of gravity is increased by more than 3000 fold from its current value $_{\rm G}$ (expressed in standard dimensionless units) then no stable stars could exist with life-times longer than a billion years. This would certainly drastically decrease the probability of intelligent life evolving on any earthlike planet. Thus, the intelligent life-permitting or life-optimizing region could be taken to be 0 to 3000 $_{\rm G}$. On the other hand, it seems clear that a universe could exist with gravity a trillion times larger than $_{\rm G}$, though only detailed calculations could establish this for sure. Thus, R could be taken to be at least 10^{12} $_{\rm G}$, and hence Wr/WR < $3000/10^{12}$ 3 x 10^8 . On the other hand, if gravity were increased to the strength of the strong force, which in standard dimensionless units is considered 10^{40} $_{\rm G}$, then probably no universe could exist, and hence no law structures under non-platonic and non-theistic accounts of the laws of nature. Accordingly, under non-platonic and non-theistic accounts of the laws of nature, this would not be part of the possible range of values allowed by our background information k' and hence 0 to 10^{40} $_{\rm G}$ is not a candidate for our comparison range R.

B. Fine-tuning of Carbon/Oxygen Production

The first significantly discussed, and probably most famous, case of fine-tuning involves the production of carbon and oxygen in stars. Since both carbon and oxygen play crucial roles in life-processes, the conditions for complex, multicellular life would be much less optimal without the presence of these two elements in sufficient quantities. (For a fairly complete presentation of these reasons, see Michael Denton 1998: ch.s 5 and 6). Yet a reasonable abundance of both carbon and oxygen appears to require a fairly precise adjustment of the strong nuclear force. A quantitative treatment of the effect of changes in either the strong or electromagnetic force on the amount of carbon and oxygen produced in stars has been performed by three astrophysicists - H. Oberhummer, A. Csótó, and H. Schlattl (2000a). Using the latest stellar evolution codes, they calculated the effects on the production of carbon and oxygen in stars of a small decrease, and a small increase, in either the strength of the strong or electromagnetic force. Based on this analysis, the authors conclude that

a change of more than 0.5% in the strength of the strong interaction or more than 4% in the strength of the Coulomb [electromagnetic] force would destroy either nearly all C or all O in every star. This implies that irrespective of stellar evolution the contribution of each star to the abundance of C or O in the ISM [interstellar medium] would be negligible. Therefore, for the above cases the creation of carbon-based life in our universe would be strongly disfavored. (Oberhummer *et al.* 2000a: 90)

The exact amount by which the production of either carbon or oxygen would be reduced by changes in these forces is thirty- to a thousand-fold, depending on the stellar evolution code used and the type of star (Oberhummer *et al.* 2000a: 88).

One limitation in the above calculation is that no detailed calculations have been performed on the effect of further increases or decreases in the strong and electromagnetic force that go far beyond the 0.5 and 4 per

cent, respectively, presented by Oberhummer *et al.* For instance, if the strong nuclear force were decreased sufficiently, new carbon resonances might come into play, thereby possibly allowing for new pathways to become available for carbon or oxygen formation. In fact, an additional 10 per cent decrease or increase would likely bring such a new resonance of carbon into play. A 10 per cent increase could also open up another pathway to carbon production during Big Bang nucleosynthesis via ⁵He or ⁵Li, both of which would become bound. Apart from detailed calculations, it is difficult to say what the abundance ratio would be if such resonances or alternative pathways came into play (Oberhummer *et al.* 2000b). We can say, however, that decreases or increases from 0.5 per cent to 10 per cent would magnify the disparities in the oxygen/carbon ratios by magnifying the relevant disparities in the rate of carbon synthesis and oxygen synthesis. Thus we have a small island of life-permitting values with a width of 1 per cent, with a distance of 10 per cent between it and the next nearest possible life-permitting island.

How should we choose the comparison range in this case? We should choose the comparison range R to be equal to what I shall call the *epistemically illuminated region*: that is, that region, IL, for which we can make determinations of whether or not the value of the strength of the strong force constant will be intelligent life permitting, or optimal for intelligent life occurring. If we let the current strength of the strong force be S_0 , then this range is $IL = [S_0 - 0.1S_0, S_0 + 0.1S_0]$, since beyond this range we do not know whether new pathways become available for carbon and oxygen production that would yield optimal, or near optimal, values for the abundance of carbon and oxygen. For R = IL, therefore, Wr/WR = 0.1 for the fine-tuning of the strong force for the joint production of carbon and oxygen in stars.

Although this seems intuitively to be the right choice, can we more rigorously justify this choice in terms of our procedure mentioned above? To do this, we will have to divide our evidence Lc into two parts: Lc = Lc & E, where E is the claim that the constant fell within the epistemically illuminated region IL. Then, P(Lc/k' & As) = P(Lc & E/k' & As) = P(Lc/k' & As) = P(Lc/k' & As). Since under k' we should be epistemically indifferent over region R = IL, it follows by the principle of indifference that P(Lc/k' & As & E) = Wr/WR, and hence that P(Lc/k' & As & E) < Wr/WR. Thus, choosing R = IL as our comparison range allows us to infer from the smallness of Wr/WR the smallness of P(Lc/k' & As & E) as required by step (2) of our quasi-Bayesian argument, and thus serves as an *effectively* adequate choice of the comparison range for our purposes even though it does not meet our original *strict* definition.

So, although k' does not give us a comparison range under our *strict* definition of the comparison range, it does give us an *effective* comparison range that can be used to ground the second step involved in the probabilistic version of the fine-tuning argument mentioned above. (8)

C. Fine-tuning of the Cosmological Constant

The smallness of the cosmological constant is widely regarded as the single greatest problem confronting current physics and cosmology. The cosmological constant, L, is a term in Einstein's equation that, when positive, acts as a repulsive force, causing space to expand and, when negative, acts as an attractive force, causing space to contract. Apart from some sort of extraordinarily precise fine-tuning or new physical principle, today's theories of fundamental physics and cosmology lead one to expect that the vacuum - that is, the state of space-time free of ordinary matter fields - has an extraordinarily large energy density. This energy density in turn acts as an effective cosmological constant, thus leading one to expect an extraordinarily large effective cosmological constant, one so large that it would, if positive, cause space to expand at such an enormous rate that almost every object in the universe would fly apart, and would, if negative, cause the universe to collapse almost instantaneously back in on itself. This would clearly make the evolution of intelligent life impossible.

What makes it so difficult to avoid postulating some sort of highly precise fine-tuning of the cosmological constant is that almost every type of field in current physics - the electromagnetic field, the Higgs fields associated with the weak force, the *inflaton* field hypothesized by inflationary cosmology, the *dilaton* field hypothesized by superstring theory, and the fields associated with elementary particles such as electrons - each contributes to the vacuum energy. Although no one knows how to calculate the energy density of the vacuum, when physicists make estimates of the contribution to the vacuum energy from these fields, they

get values of the energy density anywhere from 10^{53} to 10^{120} higher than its maximum life-permitting value, L_{max} .

(Here, L_{max} is expressed in terms of the energy density of empty space.)

Although each field contributes in a different way to the total vacuum energy, for purposes of illustration, I will look at just one example here. According to the widely accepted Weinberg-Salem-Glashow electroweak theory, the electromagnetic force and the weak force acted as one force prior to symmetry breaking of a postulated Higgs field in the very early universe when temperatures were still extremely high. Before symmetry breaking, the vacuum energy of the Higgs field had its maximum value V_0 . This value was approximately 10^{53} L_{max} . After symmetry breaking, the Higgs field fell into some local minimum of energy density, V_{min} , which theoretically could be anywhere from zero to 10^{53} L_{max} , being solely determined by V_0 and other free parameters of the electroweak theory.

Since obviously the energy of the local minimum must be less than the initial energy density, V_0 , at symmetry breaking, V_0 could be thought of as providing an upper bound for the local minimum of the energy of the Higgs field. Assuming that the vacuum energy of the Higgs field cannot be negative, zero is a lower bound.

Thus, the range of theoretically possible values for V_{min} - that is, the range of possible values allowed by the physical theories in our background information k' - appears to be 0 to V_0 , where $V_0 = 10^{53} \, L_{max}$. Our comparison range, therefore, is 0 to $10^{53} \, L_{max}$, with the range of intelligent-life-permitting values being 0 to L_{max} , yielding a fine-tuning of Wr/WR = $L_{max}/10^{53} \, L_{max}$, or one part in 10^{53} .

Unfortunately, when we attempt to make the argument more precise using the procedure discussed above, the above way of calculating the comparison range constitutes only one of three possible ways, each of which yield different answers. To apply our more precise method, we first note that our initial background information k will include the electroweak theory, with the various values for its free parameters. We now notice that the value of V_{min} appears to be fine-tuned for intelligent life. Hence, the fact that V_{min} falls into the intelligent-life-permitting range can be considered our relevant evidence Lc. We now construct a new set of background information k' by subtracting Lc from k using the method outlined above.

Now as we will see, there is no unique way of subtracting-out our knowledge that Vmin falls into the intelligent-life-permitting region. Rather, there are three obvious ways with differing results. The reason for these three separate ways is that in terms of the equations of the electroweak theory, V_{min} is given by the equation $V_{min} = V_0 - u^4/4l$, where the values of the experimentally determined free parameters of the electroweak theory yield a value for $u^4/4l$ of approximately 10^{53} L_{max} (Sahni and Starobinsky 1999: section 6). The fact that $V_{min} = V_0 - u^4/4l$ means that we can subtract-out our knowledge of V_{min} either by effectively subtracting out our knowledge of $u^4/4l$ while retaining V_0 as part of our background knowledge, or by subtracting out our knowledge of V_0 while retaining knowledge of $u^4/4l$, or possibly by not retaining knowledge of either V_0 or $u^4/4l$. We will explore each of these methods in turn.

The first way of subtracting-out the knowledge of the actual value of V_{min} - that of excluding any knowledge of $u^4/4l$ while retaining knowledge of V_0 -- is the way which we implicitly employed in our initial calculations above. Since $V_{min} = V_0 - u^4/4l$, and the intelligent life permitting range of V_{min} is 0 to V_{max} (assuming Vmin cannot be negative), it follows that the intelligent-life permitting range of $u^4/4l$ must be $v_0 = V_0 - V_{max}$, $v_0 = V_0$. Thus, for a given $v_0 = V_0$, the fine-tuning of $v_0 = V_0$ is equivalent to the fine-tuning of $v_0 = V_0$. Assuming it is physically impossible for $v_0 = V_0 = V_0$, the value of $v_0 = V_0 = V_0 = V_0$. Further, since this constraint flows from our general knowledge of the laws of physics, it follows that it would be included in our background information $v_0 = V_0 = V_$

The second way of excluding knowledge of V_{min} involves retaining the actual value of $u^4/4l$ as part of our background information, while excluding knowledge of V_0 . Within current theory, there is no obvious upper bound for the initial energy density, V_0 , at which symmetry breaking occurs, though current theory is vague on this point. Given that Vmin cannot be negative, $V_0 u^4/4l = 10^{53} L_{max}$. Hence, it seems that V_0 could be anything from $10^{53} L_{max}$. to infinity, making the theoretically possible range of V_{min} be zero to infinity. So, this method of choosing k' yields [0, infinity] as the range of theoretically possible values for V_{min} , and thus our comparison range R.

On the other hand, there might be some further restrictions on the total initial energy V_0 , such as some restriction arising out general relativity or quantum theory regarding the total energy carrying capacity of empty space, or a restriction arising from superstring theory regarding the maximum initial energy density of the universe. Let V_{max} represent the this hypothetical maximum value. We know that $V_{max} > V_0$. Unless V_{max} is exceedingly close to V_0 - which would itself involve some sort of fine-tuning - Vmax will still allow an enormous range of variation in V0, and hence the degree of fine-tuning of the cosmological constant will turn out to be very large. For example, suppose that V_{max} is only twice as large as V0, that is, $V_{max} = 2V_0$. Then the range of theoretically possible values of V_0 would $V_{max} - V_0 = 10^{53} L_{max}$, which yields a fine-tuning for the cosmological constant of one part in 10^{53} . To make the fine-tuning less than one part in a hundred, for instance, Vmax would have to be within one part in 10^{50} of V_0 , which itself would involve an extremely precise fine-tuning.

Finally, it is easy to see by a similar analysis that the third method of subtracting-out knowledge of V_{min} , that of subtracting-out knowledge of both V_0 and $u^4/4l$, will yield a similar result. Given this ambiguity in determining the comparsion range, all we can say is that there is a range of values for the width of the comparison range, namely 10^{53} L_{max} to infinity. This means that by our definition of fine-tuning, we can confidently say that the cosmological constant is fine-tuned to at least one part in 10^{53} .

D. Subtracting out more than one Constant at a Time

As a final issue, we should deal with the question of what happens if we subtract-out knowledge of more than one constant at a time. In this case, we apply Howson's method once again to determining k' as we did above, and then determine the joint theoretically possible range allowed by k' for the constants in question. In general, his method will be more difficult to implement than in the simple case in which we subtract out one constant at a time.

Appendix: The Purported Problem of Infinite Ranges

Timothy McGrew, Lydia McGrew, and Eric Vestrup (2000) have recently argued that the comparison ranges R for the various cases of fine-tuning are all infinite, and that such infinite ranges pose fatal problems for the fine-tuning argument. As mentioned above, in developing their argument, they assume, with little argument, that the comparison range is just the range of conceivable values that the parameters could have, and since we can conceive of the values being anything, this range is infinite.

As argued above, the comparison range should not be taken as the conceivable range, but rather as the range determined by the background information k'. Nonetheless, in some of the cases above, the upper bound for the comparison range was infinity, or more precisely, the upper bound for the range of comparison ranges was infinity. This might be thought to present a problem even for my method of

choosing the comparison range. In this appendix, I will argue that even if the proper comparison range were infinite, their objection to the fine-tuning argument fails.

Two problems have claimed to arise if the comparison range is infinite, namely that it imply that the probabilities in the in the fine-tuning argument are not normalizable and second that it would entail the soundness of what they call the coarse-tuning argument. We will consider the normalizability problem first.

(i) The So-Called Normalizability Problem

Vestrup and McGrew argue that if the comparison range is infinite, and if we consider all sub-ranges of equal width equally probable - which is they claim is the only possible non-arbitrary assignment of probability --then the probability assignments are not normalizable:

Probabilities make sense only if the sum of the logically possible disjoint alternatives adds up to one -- if there is, to put the point more colloquially, some sense that attaches to the idea that the various possibilities can be put together to make up "one hundred percent" of the probability space. But if we carve an infinite space up into equal finite-sized regions, we have infinitely many of them; and if we try to assign them each some fixed positive probability, however small, the sum of these is infinite." (2001, p.)

Hence, they argue, since non-normalizable probabilities do not make sense, any probabilistic version of the fine-tuning argument fails.

Of course, an immediate response to this objection is that we can assign each finite subrange a zero probability. To see this, note that any infinite range can be divided into a countably infinite set of finite subranges of equal length that cover the entire range. If we apply the principle of indifference, we must assign a probability of zero to the parameter landing in any given sub-range. To this, McGrew and Vestrup could argue that such an assignment of probability violates the axiom of countable additivity, an issue they unfortunately do not address. The axiom of countable additivity says that the countable sum of mutually exclusive classes of events must be equal to the probability of an event occuring in the union of the classes. That is, sum over i, i goes from 1 to infinity, $P(x_i) = P(x_1 \text{ or } x_2 \text{ or } x_3 \dots) = P(\text{constant falling in range R}) = 1$, where the x_i are mutually exclusive events. [Countable additivity is a generalization of finite additivity: that is, sum over i, i = 1 to k, $P(xi) = P(x1 \text{ or } x2 \dots xk)$.] Thus, since the ranges are mutually exclusive and exhaustive, the probability of a parameter the entire range should be the sum of respective probabilities for each of the infinite number of sub-ranges. Since the parameter must have some value, however, the probability for the entire infinite range is 1, whereas the sum of the probabilities of each range is zero.

There are several responses to this objection. First, one could save countable additivity simply by assigning each region and infinitesimal probability such that the countable sum of these infinitesimals added up to one. But second, and more importantly, this objection confuses the mathematical definition of probability with the sort of probability used in science and everyday life, particularly *epistemic* probability which is the type of probability I am claiming occurs in the fine-tuning argument. (14) As mentioned above, epistemic probability has to do with rational degrees of belief. But, it is certainly seems rational to be certain that any given member of an infinite class of events will not occur, and at the same time believe that one of the events will nonetheless occur. Or, at the very least, such a set of beliefs is not obviously irrational.

Consider, for example, a situation in which there is an infinite number of discrete alternatives at that are mutually exclusive and exhaustive, and for which we are epistemically indifferent between the alternatives. Under this circumstance, it certainly seems rational to be certain, for each alternative ai, that ai will not be

the case: that is, to assign each ai zero epistemic probability. Indeed, in such a situation, this is the only non-arbitrary degree of belief one could have in any given ai. Any uniform *finite* degree of belief over the ai would violate the axiom of finite additivity. Countable additivity, on the other hand, would demand that we either assign some ai different probabilities than other ones, or claim that there is no probability in this case. Clearly, however, it cannot be rationally obligatory to assign different ai different rational degrees of belief, when there is no epistemic distinction between them. As De Fineti points out, such a principle would require me to treat as epistemically unequal what I regard as epistemically equal: "even if I think they are equally probable, [...] I am obliged to pick a convergent series which, however I choose it, is in absolute contrast to what I think." (de Finetti, 1970, p. 123.). Worse, such a distribution would force me to be practically certain (99.999999% epistemic probability) to some finite set of alternatives that I believe is epistemically on par with each of an infinite class of sets of other alternatives. As de Finetti remarked, "Should we force him, against his own judgement, to assign practically the entire probability to some finite set of events, perhaps chosen arbitrarily?" [de Finetti, 1972, pp. 91 -2] It is difficult to see how such a principle could be rationally obligatory! It seems on the face of it to be completely irrational.

One might respond to this sort of case that the most rational alternative when the only non-arbitrary distribution of degrees of belief violate the axiom of countable additivity is to remain agnostic. After all, one need not assign epistemic probabilities to all propositions. I do not believe this is an adequate response, since I think in some of the type of cases under consideration it would be irrational to remain agnostic. To illustrate, suppose that God created an infinite lottery by creating a truly random number generator that randomly picks the winning number, which we will call Q, from the range from one to infinity of the integers. (Never mind for now whether you think the creation of such a generator is possible.) Further suppose that you have no reason to believe that this number falls into one range instead of any other and that each ticket sells for \$20, with a \$100, 000 jackpot. (15) Finally, suppose that your sister decided to buy lottery tickets, spending \$20 a month on them. I am sure any of us would try to persuade her not to. We would not merely be agnostic about her winning; we would be sure that she would lose. And this certainty could be substantiated with significant arguments. For example, we might reason that if the lottery had a 100 trillion, trillion, trillion tickets, it would be a waste of money to buy tickets, since the chance of winning would be so small one could say with confidence that she could be certain of not winning. Moreover, the larger the number of tickets, the more we would be certain that she should not buy a ticket. Certainly, in the limit as the number of tickets becomes infinite, we should not merely become agnostic about whether she would win. We would consider such a bet completely foolish. Compare this with the case in which there is a lottery, but one is given no idea of how many tickets there are. In such a case, one would be truly agnostic about winning. On might simply buy the ticket and hope one would win, without any idea of what one's chances are. In the infinite lottery case, it seems clear that one should have no hope whatsoever about winning, which is not the attitude of agnosticism.

Here is another argument against requiring agnosticism in these cases. Suppose one adopted the agnosticism alternative. Then for any finite range [M, M+N], one should be agnostic about whether the winning number is in that range, where M and N are positive integers. Even though agnosticism does not commit one to a degree of belief, to be agnostic is to be less than certain. Now consider some given number k that is in the range [0, M], and let k^* be the proposition that k is the winning number. If one were certain that the number was in the range [0, M], but did not have any information about which member of the range it was, then one would be every member of this range an epistemic probability of 1/M for being the winning number. If one were agnostic about whether it fell into that range, then one's probability would obviously be less. Thus, one could reason as follows: if I were sure it the number was in that range, then it would have an epistemic probability of 1/M. But, since I don't even know that the winning number is in the range, I have even less reason to believe the winning number is k - that is, my epistemic probability $P(k^*)$ should be less than 1/N. Now, for any positive integer N, k will be a member of some range [0, M+N]. Thus, for every N, the probability of k being the winning number will have to be less than 1/(M+N). The only consistent value we can assign to the probability of k^* , therefore, is 0 or an infinitesimal, since $P(k^*)$ is less than 1/(M+N) for all N.

Of course, one might object that no such random number generator could ever be constructed, even by God. As Williamson notes, however, this objection is irrelevant (P. 407). The issue is not whether some

objective random number generator could be made, but whether the agent in question believes that such a random number generator exists, since epistemic probability is a relation between propositions held by some agent in a set of specified epistemic circumstances. Further, many of the arguments would go through even without having a random number generator: one could simply believe that God picked some finite number, and that God picked the number independently from any foreknowledge of what number you would guess, and that your choice was causally independent of what God chose.

Given the formidable set of arguments above for rejecting countable additivity for epistemic probability, the burden is on those who insist on countable additivity to show that distributions that violate this axiom are rationally incoherent. Thus, in order for this objection stand, McGrew and Vestrup would have to show that such a set of beliefs is irrational, which they have not even attempted to do.

To elaborate further on this point, although countable additivity is usually required within the mathematical conception of probability, when it comes to considering actual philosophical interpretations of probability for example, the subjective theory, the epistemic theory, or the frequency theory - countable additivity is controversial. For example, according to Burno de Finetti, one should only introduce axioms for probability insofar as they can be justified in terms of the meaning of probability in one's interpretation. [Gilles, 67]. Yet, as De Finetti stated in 1970, "no one has given a real justification of countable additivity (other than just taking it as a 'natural extension' of finite additivity)" (de Finetti, 1970, p. 119).

Further, A. N. Kolmogorov noted in his mathematical axiomatization of probability theory, the axiom of countable additivity could not be justified for the frequency interpretation, but he nonetheless adopted this axiom because "it has been found expedient in researches of the most diverse sort." (P. 66 of Gilles). Indeed, as Jon Williams points out, the frequency notion of probability "must admit uniform distributions over countable partitions ... and hence fail to satisfy countable additivity." (1999, P. 407). As an example of this, Williamson presents an example of a factory that produces car engines, each with a different number. If this factory never stops producing engines, then the frequency of any particular number is zero, while the frequency of cars having some number occurring is one. (1999, p. 407). Williamson cites other authors who concur about the controversial nature of the axiom of countable additivity. For example, he cites P. C. Fishburn, another advocate of countable additivity tells us that "the present wisdom seems to be that countable additivity can keep one out of trouble that arise in its absence even if it is arbitrary, or at best uncompelling, as a principle of rational choice." (Reference) Similarly, Williamson cites a recent historical survey of probability by von Plato, who notes with regard to countable additivity that "At present it seems that the foundations of the topic remain as open as ever" (von Plato, 1994, p. 278). The point here is that one cannot simply assume this axiom without argument, as McGrew and Vestrup have done; McGrew and Vestrup simply treat the axiom as countable additivity as a given, in no need of argument. One must present an argument for its applicability to the interpretation of probability being considered. As we saw in the case of epistemic probability, the arguments against its applicability are very strong. (18)

(ii) The Coarse Tuning Argument

The second argument presented by McGrew and Vestrup goes back to similar argument presented by theoretical physicist Paul Davies (1992), and also repeated by Manson (2000). According to this argument, which McGrew and Vestrup call the *coarse-tuning argument (CTA)*, if the comparison range is infinite, then no matter how large the range r of intelligent-life-permitting values, as long as it is finite, the ratio of it to the conceivable range will be zero, which means that the narrowness of the range becomes irrelevant to our assessment of degree of fine-tuning. This gives rise to what they call the *coarse-tuning argument*, which is the argument from the finiteness of the intelligent-life-permitting range to (the confirmation of) design or many-universes. Thus, McGrew and Vestrup conclude that "if we are determined to invoke the Principle of Indifference regarding possible universes, we are confronted with an unhappy conditional: if the FTA [fine-tuning argument] is a good argument, so is the CTA. And conversely, if the CTA is not a good argument, neither is the FTA." (2001, p.).

Purportedly, this is supposed to provide a reductio of the fine-tuning argument, though they never even attempt to explain why we should think that the CTA would not be a good argument if the comparison range is infinite. Davies presents a similar objection to the coarse-tuning argument, but instead of concluding that the fine-tuning argument is unsound, Davies concludes that the comparison range cannot be infinite. Says Davies,

"From what range might the value of, say, the strength of the nuclear force... be selected? If the range is infinite, then any finite range of values might be considered to have zero probability of being selected. But then we should be equally surprised however weakly the requirements of life constrain those values. This is surely a *reductio ad absurdum* of the whole argument. (Davies, *The Mind of God : The Scientific Basis for a Rational World*. New York, Simon and Schuster, 1992, pp. 204-205).

One response to this argument is to agree with Davies that we should be "equally surprised however weakly the requirements fo life constrain those values." If we truly found that for some constant C that: (i) the range of possible values for C were infinite, (ii) the range of intelligent-life-permitting values were finite, and (iii) we should be epistemically indifferent over the range of possible values, then we should conclude that the fact that the constant falls into the intelligent-life-permitting range does strongly confirm design (or multiple universes) over the atheistic single-universe hypothesis. That is, we should conclude that CTA is a sound argument. I see nothing odd or counterintuitive about this. CTA is certainly not obviously absurd, as Davies and McGrew and Vestrup assume without argument. Physics would still be required in constructing CTA. For example, physics would still be needed both to show that the intelligent-life-permitting range was indeed finite and that the total theoretically possible range was infinite, and to show that we have no physical reason for preferring one value of a parameter over any other in the range.

The main reason Davies has trouble with the coarse-tuning argument is that he claims that we are more impressed when "the requirements of life are more restrictive." According to Davies, it is the smallness of the range of intelligent-life-permitting values that appear to ground our temptation to infer to design or many universes. If, however, we had good reason to believe that the comparison ranges were truly infinite, and that we should be epistemically indifferent over that range, and that the principle of indifference was sound, then we would just have to conclude that our initial impressions were wrong that it was the smallness of the range, not its finiteness, that gives the fine-tuning argument its force. Although there is a slight presumption in favor of such initial impressions, they are by no means indefeasible.

That said, we can rationally reconstruct why we are impressed with the relative smallness of the intelligent-life-permitting region instead of merely its finiteness. The reason we are impressed with the smallness, I suggest, is that we actually do have some vague finite range that we are comparing the intelligent-life-permitting range to. We do not, as a matter of fact, think that all values for a parameter are equally likely, just those in some limited range around the actual value of the parameter in question. This vague sense of a comparison range is given a rigorous foundation when we actually apply our method to the cases of fine-tuning since it allowed for finite comparison ranges.

This leads us to our second response to the objection based on the coarse-tuning argument. In order for the CTA to be a good argument, we would have to unequivocally say that the comparison range is infinite. Given our method of defining the comparison range, we cannot unequivocally say that the comparison range is infinite, as shown by the three case studies examined above. The fact that some of the ranges within our set of viable comparison ranges are finite is sufficient to undercut any coarse-tuning argument for design or many universes based on the mere finiteness of the intelligent-life-permitting range.

To concretely illustrate, suppose that all we knew was that the intelligent-life-permitting range, r, for the cosmological constant was some large finite range, say 0 to 10^{53}_{max} , making $Wr = 10^{53}_{max}$. Now, although one of the comparison ranges in our set of ranges is infinite, there is also one whose width is finite - with a value of 10^{53}_{max} . Accordingly, because of the existence of this value, we cannot confidently say that the parameter is fine-tuned, since to say that it is fine-tuned means that the Wr/WR' is small, where WR' is the width of the comparison range with the minimum width in our set of possible comparison ranges generated

by the different possible ways of subtracting Lc from our background information. As we saw above when we considered the fine-tuning of the cosmological constant, however, at least one way of generating our background information k' yields a comparison range of 0 to 10^{53}_{max} . Hence, given that $Wr = 10^{53}_{max}$, Wr/WR = 1 for this way of subtracting Lc. Thus, because one of the viable comparison ranges is finite, the coarse-tuning argument fails in this case. Moreover, since in all the cases above the set of comparison ranges includes a finite range, the coarse-tuning argument also fails for all the cases considered above. Thus, the presumption of the whole objection of McGrew and Vestrup that the comparison range is unequivocally infinite is false.

Finally, I will present an argument that it is counterintuitive to reject the coarse-tuning argument for the reasons McGrew and Vestrup present. To see this, we will start by assuming that the fine-tuning argument would have probative force if the comparison range were finite. Although McGrew and Vestrup might not agree with this assumption, it will allow us to consider whether, as they claim, having an infinite instead of finite range R is relevant to the cogency of the fine-tuning argument. Now imagine increasing the width of the comparison range, while keeping it finite. It seems that the more WR increases, the stronger the fine-tuning argument gets. Indeed, if we accept the principle of indifference, as WR approaches infinity, P(Lc/As) will converge to zero, and thus that P(Lc/As) = 0 in the limit as WR approaches infinity. Accordingly, if we deny the soundness of the coarse-tuning argument *because* WR is purportedly infinite, we must draw the counterintuitive consequence that although the fine-tuning argument gets stronger and stronger the larger WR gets, magically when it becomes actually infinite, the argument loses all probative force. (Further, we would have to claim that even though the in the limit as WR approaches infinity, P(Lc/As) = 0, if WR is actually infinite, it is undefined.)

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1. As Manson (2000) points out, "the most common way of stating claims of fine-tuning for life is in terms of counterfactual conditionals, wherein expressions such as 'slight difference', 'small change', 'delicate balance', 'precise', 'different by n%', ..." (P. 342). This very simple counterfactual method of defining fine-tuning is a manifestly inadequate way of defining fine-tuning, however, as can be seen from the case of the fine-tuning of the strong force for carbon/oxygen production. As Oberhummer, et. al., note (2000a), a 0.4% change in the strength of the strong force - that is, a relativity slight change in its value --would radically decrease either the total amount of carbon or the total amount of oxygen in the universe, thereby severely decreasing the chances of intelligent observers forming. Thus, there is a narrow "island" of values of the

strength of the strong force that are optimal for life. On the other hand, Oberhummer, et. al., point out (2000b), a change of 10% in the strength of the strong force might land one on another island that allowed for significant quantities of both carbon and oxygen to form. This illustrates that one could have fine-tuning in the sense of a narrow island of intelligent-life-permitting values (that is, fine-tuned in the "slight difference" sense) and yet have many narrow islands right next to each other, which would render it entirely unsurprising that the constant in question had an intelligent-life-permitting value.

- 2. For those familiar with the probability calculus, a precise statement of the degree to which evidence counts in favor of one hypothesis over another can be given in terms of the odds form of Bayes's Theorem: that is, $P(H_1/E)/P(H_2/E) = [P(H_1)/P(H_2)] \times [P(E/H_1)/P(E/H_2)]$. The general version of the principle stated here, however, does not require the applicability or truth of Bayes's theorem. For a more indepth discussion of the PPC, see Chapter _____.
- 3. A credence function is simply a function that either describes or rationally dictates one' degree of belief in a proposition over a certain domain such as one's degree of belief of the value of a constant being in any given subregion of R
- 4. An outstanding issue that needs to be addressed is how much "weight" we should attach to assignments of epistemic probability obtained by this method. As Keynes's claims (1921, chapter 6), not all assignments of epistemic probability are to be placed on the same level; rather some are to be given more weight than others. Says Keynes,

"The magnitude of the probability of an argument, in the sense discussed in chapter 3, depends upon a balance between what may be termed the favourable and he unfavourable evidence; a new piece of evidence which leaves the balance unchanged, also leaves the probability of the argument unchanged. But it seems that there may be another respect in which some kind of quantitative comparison between arguments is possible. This comparison turns upon a balance, not between the favourable and the unfavourable evidence, but between the *absolute* amounts of relevant knowledge and relevant ignorance respectively. (1921, P. 77)"

Thus, Keynes goes on to tell us, when the absolute amounts of relevant evidence increases, we "have a more substantial basis upon which to rest our conclusion [of probability.]" (p. 77). Keynes call the substantiality of the basis the weight we should assign to the probability judgement, and argues that such an assignment of weight cannot simply be collapsed into a reassessment of our original probability. An example of such a situation might be a coin for which we do not know whether it is weighted on one side or another. By the principle of indifference, we might then attach it a probability of coming up ½ on heads and ½ on tails, since even if it is weighted, we have no more reason to believe it would be weighted in favor of heads instead of tails. Now suppose that upon further careful and extensive analysis, we determined that there were no relevant differences between the two sides; further suppose that we flipped it a thousand times and it came up approximately 50% heads and 50% tails. It seems that we would then have a more substantial basis for our probability judgement and thus in some sense should assign more weight to our judgement of epistemic probability. I will not further discuss this issue here, but in chapter 3 I argue that, given the validity of this concept of weight, that we should attach significant weight to our estimates of epistemic probability for the case of the fine-tuning when we use the method outline in this chapter.

- 5. One might object that this procedure is only justified under the assumption that we live in a determinstic universe, since otherwise the K we have chosen is not a true axiomatization of our knowledge. This is true, but it is difficult to see how the thesis that the world is indeterministic could be relevant to the legitamacy of the fine-tuning argument.
- 6. One might ask at this point, why worry about how to pick background information k' as long as k' & e = k? After all, according to the odds form of Bayes's theorem, $P(h1/e \& k')/P(h2/e \& k') = P(h1/k')/P(h2/k') \times P(e/h1 \& k')/P(e/h2 \& k')$, and hence as long as we can evaluate the probabilities P(e/h & k') and P(e/-h & k'), we can construct a legitimate confirmation argument. In particular, in the case of the fine-tuning

argument, the odds form of Bayes's theorem states that P(T/k' & Lc)/P(As/k' & Lc) = P(T/k')/P(As/k') x P(Lc/k' & T)/P(Lc/As & k'). So any comparison range R we construct relative to such a k' will allow us to apply the principle of indifference to attain P(Lc/As & k'), and hence the ratio P(Lc/T & k')/P(Lc/As & k'). The problem with this argument is that it requires that we be able to assess the prior probabilities P(T/k')/P(As/k'), something we want to avoid. Further, even if we could assess these prior probabilities, we have no way of guaranteeing that the end result, P(T/k)/P(As/k) = P(T/k' & Lc)/P(As/k' & Lc), will be the same for different choices of k'. One might further ask, however, whether one couldn't simply directly apply the prime principle of confirmation to this case - which would imply that Lc confirms T over As if and only if P(Lc/As & k') > P(Lc/k' & T)? But then the degree of confirmation will depend on how we choose k': for example, if we choose k' = k, then there will be no confirmation whatesoever, which is just the problem of old evidence. Thus to speak of confirmation in these contexts apart from appealing to prior probabilities, we must find some legitimate method of determining k'.

- 7. To see this more clearly, consider the theistic hypothesis. It is reasonable to suppose that the theistic hypothesis gives us no reason to favor one value for a constant of physics within the intelligent-life-permitting range over any other, thus giving rise to a uniform probability distribution over the intelligent-life-permitting region. Similarly, the atheistic single-universe hypothesis would also give rise to a uniform probability distribution over the intelligent-life-permitting range (along with a uniform probability distribution over the total comparison range R). Thus, since the probability distributions over the intelligent-life-permitting region are uniform in both cases, any further knowledge about exactly what subregion of the intelligent-life-permitting range the value falls into will be irrelevant for purposes of the argument for theism.
- 8. One might wonder about the difference between this case of the fine-tuning of the forces and the cases discussed in the last section, such as the case of the fine-tuning of gravity. The difference is that for gravity and most other cases of fine-tuning of force strengths, there is only one continuous intelligent-life-permitting range, whereas in the case of the fine-tuning of carbon/oxygen production, there might be several disjoint intelligent-life-permitting ranges which we do not know about. This adds some complication to the application of our method, requiring us to invoke the idea of an *effective* comparison range.
- 9. There are many good discussions of the cosmological constant problem. See, for example, Sahni and Starobinsky (1999: sections 5-7) and Cohn (1998: section II).
- 10. To be absolutely precise, all that the existence of life requires is that the total cosmological constant, L_{tot} , be within the life-permitting range. But, $L_{tot} = L_{vac} + L_{bare}$, where L_{vac} represents the contribution to the cosmological constant from the vacuum energy of all the field combined, and L_{bare} represents the "intrinsic" value of the cosmological constant apart from any contribution from the vacuum energy. Thus the contribution of any given field, such as the Higgs field, to the vacuum energy could be much greater than L_{max} , if such a contribution were almost cancelled out by the other contributions to the cosmological constant. But to get such a precise cancellation would itself require some sort of extraordinary fine-tuning or new principle of physics. To simplify our calculation and argument, we will assume that the Higg's field is the only contributor to the vacuum energy.
- 11. In many applications in physics, energy is relative to some standard reference point, and thus can take negative values. If, for example, we were to arbitrarily set zero as the gravitational potential energy of an object sufficiently far away from the earth, then any object on the surface of the earth would have a negative potential energy. Like the energy associated with the rest mass of a particle, however, the vacuum energy is an absolute magnitude: for example, it's absolute value determines, via Einstein's equation of general relativity, the expansion rate of empty space. It is difficult to see, however, how energy, in this absolute sense, could really be negative. Indeed, given Einstein's relation connecting mass and energy, it is as difficult to make sense of a negative vacuum energy as it is to make sense of a negative rest mass, except as a mere mathematical formalism.