The Sustainability of Government Deficit Spending

Suppose that the government consistently has a budget deficit, such that the nominal deficit-to-GDP ratio is held constant over time at level d. Assume that the interest rate on government debt is zero. Further, assume that nominal output (GDP) grows at constant rate g. In this case, what happens to the long run level of the debt-to-GDP ratio? Let nominal output be denoted by Y.

In continuous time, the assumptions listed imply that:

$$\begin{aligned}
\dot{DEBT}(t) &= DEF(t) \\
\frac{DEF(t)}{Y(t)} &= d(t) = d, \text{ for all } t. \\
\frac{DEBT(t)}{Y(t)} &= b(t) \\
\dot{b}(t) &= \frac{\dot{DEBT}(t)}{Y(t)} - \frac{DEBT(t)}{[Y(t)]^2}\dot{Y}(t) \\
&= \frac{DEF(t)}{Y(t)} - \frac{DEBT(t)}{Y(t)}\frac{\dot{Y}(t)}{Y(t)} \\
&= d - b(t)\, g.
\end{aligned}$$

If we look at a phase diagram for b, we can see that a steady-state exists which is stable and unique (graph $\dot{b}$ versus b). In the steady-state ($\dot{b} = 0$), we know that:

$$b_{SS} = \frac{d}{g}.$$

SS denotes the steady-state. The debt-to-GDP ratio stabilizes eventually at this value.

In discrete time, the assumptions listed imply that:

$$\begin{aligned}
DEBT_t - DEBT_{t-1} &= DEF_t \\
Y_t &= (1+g)\, Y_{t-1} \\
\frac{DEF_t}{Y_t} &= d_t = d, \text{ for all } t. \\
\frac{DEBT_t}{Y_t} &= b_t \\
b_t - b_{t-1} &= \frac{DEBT_t}{Y_t} - \frac{DEBT_{t-1}}{Y_{t-1}} \\
&= \frac{DEBT_t}{Y_t} - \frac{(1+g)\, DEBT_{t-1}}{Y_t} \\
&= \frac{DEBT_t - DEBT_{t-1}}{Y_t} - \frac{gDEBT_{t-1}}{Y_t} \\
&= d_t - \frac{gDEBT_{t-1}}{(1+g)\, Y_{t-1}} \Rightarrow \\
\Delta b_t &= d - \frac{g}{(1+g)} b_{t-1}
\end{aligned}$$

In the steady-state ($\Delta b = 0$), the debt-to-GDP ratio will be:

$$b_{\text{SS}} = \frac{(1+g)}{g} d.$$

Again, we can see that the debt-to-GDP ratio settles down to a constant level.

Is this debt-to-GDP ratio sustainable in the long run? Without more information, we do not know. A simple way to evaluate the sustainability of the debt-to-GDP ratio is to compare it to some benchmark. Suppose that there is some debt-to-GDP ratio which is no longer sustainable (people cease to be willing to purchase government debt when the debt-to-GDP ratio exceeds this level). Denote this level by $b_{\max}$. As long as $b_{\text{SS}} < b_{\max}$, then the government is able to engage in deficit spending forever. In this course, we have not modeled how $b_{\max}$ is determined. This is an interesting and important question that is continuing to be researched. There is a voluminous literature on sovereign debt which attempts to address this question.[1]

[1]There is an excellent introduction to these models in Obstfeld and Rogoff's advanced text *Foundations of International Macroeconomics* (1996).