Economics 6003
Quantitative Economics
Classic Static Panel Models // Random and Fixed Effects

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Monday/Thursday, 27/30 April 2009

What is panel data?

- Up to now, we have only considered either:
  - cross-sectional data(no natural order)
  - time-series data (naturally ordered)
- Panel data is characterized by **both** a cross-section and time element → an ordered series (usually time) exists for each cross-sectional unit.
- Let i index the cross-section dimension of the data, where $i = 1, \dots N$ (there are a total of N cross-sectional units).
  - Each i is sometimes referred to as a panel ⇒ panel data is a collection of panels.
- Let t index the time series dimension of the data, where $t = 1, \dots T_i$ for each cross-sectional unit i.
  - If $T_i = T \forall i$, then the sample is *balanced*. Otherwise, it is *unbalanced*.
  - We will assume that panels are balanced for simplicity.
- The sample size of a panel dataset is thus NT.

Examples of possible panels

- The critical characteristic of the set of cross-section units (panels) is that that do not have a natural order that we can leverage.
- Thus, example panels include:
  - individuals/people
  - families
  - households
  - firms
  - countries
  - relationships
    - individual-location
    - bilateral country import-export pairs
    - bilateral firm buyer-seller pairs
- *NB*: If there is some non-temporal ordering of the cross-section units (e.g., geographic proximities), we should incorporate that information into our approach, either directly through additional explanatory variables or structure on the error term.

A note on asymptotics

- The usual desiderata of estimators are *consistency* and *efficiency*. As asymptotic properties, they arise only as the sample size goes to infinity.
- With panel data, there are 2 dimensions which can go to infinity: N or T. Thus, we can outline 3 kinds of asymptotics:
    1. N asymptotics – we consider T fixed and ask how the estimator behaves as the number of cross-section units goes to infinity. Things behave like cross-sections, with a little extra information afforded by T.
    2. T asymptotics – we consider N fixed and ask how the estimator behaves as the number of time periods goes to infinity. Things behave like time series.
    3. NT asymptotics – both N and T go to infinity.
- Consistency and efficiency can only be evaluated relative to one of these asymptotic cases.

General single equation model

- Consider the most general, single equation linear panel data model:

$$\underset{(1\times 1)}{y_{i,t}} = \underset{(1\times K)}{\beta'_{i,t}}\underset{(K\times 1)}{x_{i,t}} + u_{i,t}$$

  where y and x are observables, β is a set of unknown parameters, and u is unobservable.
- In general, we would like to learn something about β. In the absence of additional restrictions though, we cannot estimate the model:
  - the number of parameters (elements of β) is equal to the size of the sample.
  - the relationship between the observables and the unobservables is unspecified; there could be confounding.
- First, let's suppose that $\beta_{i,t} = \beta \forall i, t$.
- Second, let's also assume that $u_{i,t} = \alpha_i + \varepsilon_{i,t}$.

Linear unobserved effects panel model

- Then, the linear panel data model has the form:

  $$y_{i,t} = \beta' x_{i,t} + \alpha_i + \varepsilon_{i,t}$$

  where α_i is an unobserved, cross-section unit-specific effect and $\varepsilon_{i,t}$ is an unobserved, idiosyncratic error term. α_i is also referred to as unobserved, time-invariant, cross-sectional heterogeneity.
- Consider the following assumptions on the relationship between the observables and unobservables:
  1. $E\left(\varepsilon_{i,t}|\alpha_i, x_i\right) = E\left(\varepsilon_{i,t}|\alpha_i, x_{i,t}\right) = 0 \forall t$.
     - There is no correlation between α_i and $\varepsilon_{i,t}$ conditional upon $x_{i,t}$. Furthermore, additional information on the time series of x_i is irrelevant. This is *strict exogeneity* of the observable regressors conditional on the unobserved effect.
  2. $E\left(\alpha_i|x_i\right) = E\left(\alpha_i\right) = 0 \forall t$.
     - The observable regressors contain no useful information about the unobserved effect. If this fails, then α_i is a relevant, omitted variable and inconsistency results if we consider N-asymptotics.

Random effects panel model

- Under these assumptions, α_i behaves just like a component of the unobserved error term.
- Such a model of α_i is known as a *random effects* model.
- We can use this structure to improve efficiency → Generalized-least-squares (GLS) allows us to incorporate the information in the variance/covariance matrix to improve the efficiency of our estimator.
- Assume that:

$$\begin{aligned} E\left(\varepsilon\varepsilon'\right) &= \sigma_{\varepsilon}^{2} I_{NT} \\ E\left(\alpha\alpha'\right) &= \sigma_{\alpha}^{2} I_{N} \end{aligned}$$

$\Rightarrow$ Each unobserved component exhibits homoskedasticity and is uncorrelated with the regressors and the other unobserved components.

Random effects panel model

- Then, the $(T \times T)$ variance/covariance block for each panel has the structure:

$$E\left[(\alpha_i + \varepsilon_i)(\alpha_i + \varepsilon_i)'\right] = \begin{bmatrix} \sigma_\alpha^2 + \sigma_\varepsilon^2 & \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\ \sigma_\alpha^2 & \sigma_\alpha^2 + \sigma_\varepsilon^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \sigma_\alpha^2 \\ \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 & \sigma_\alpha^2 + \sigma_\varepsilon^2 \end{bmatrix}.$$

  The full $(NT \times NT)$ variance/covariance matrix contains zeros for the off-diagonal, covariances across panels.
- If we impose such a structure on the variance/covariance matrix used by GLS, we can achieve an additional efficiency gain. Call this the RE estimator.

Fixed effects panel model

- Assumption 2 is strong. If we drop it, then the unobserved cross-section unit-specific effect may be correlated with the observed regressors.
- Such a model of α_i is known as a *fixed effects* model. In this case, we must address the presence of the effect more directly; it is a relevant, omitted variable.
- We consider three ways to accomplish this:
    1. Time de-meaning within panel and OLS estimation → the classic fixed effects estimator. Denote this FE.
    2. Panel dummy variables and OLS estimation → sometimes known as least-squares dummy variables. Denote this DV.
    3. Time differencing within panel and OLS estimation → usually done with the first difference. Denote this FD.

Time de-meaning within panel

- Time de-meaning is the classic method of estimating a fixed effects model.
  - Construct the panel-specific time average:

$$\bar{y}_i = \beta' \bar{x}_i + \alpha_i + \bar{\varepsilon}_i .$$

  - Subtracting the time average from the panel model equation, we have that:

$$\left(y_{i,t} - \bar{y}_i\right) = \beta' \left(x_{i,t} - \bar{x}_i\right) + \left(\varepsilon_{i,t} - \bar{\varepsilon}_i\right)$$

    where the fixed effect α_i vanishes since it is time-invariant.
- This is also known as the *within* transformation, since the OLS estimator of β only uses the within-panel time variation. It will be consistent, since assumption 1 ensures zero correlation of the observable regressors and the error term.

Panel-specific dummy variables

- A simple alternative to the within alternative (which can be shown to be equivalent) is the introduce a set of N panel-specific dummy variables into the regression. These effectively proxy for the unobserved fixed effects, acting as panel-specific intercepts. The OLS estimator of β is consistent under N-asymptotics.
  - The estimating equation for the regression is:

    $$y_{i,t} = \beta' x_{i,t} + \sum_{i=1}^{N} \widetilde{\alpha}_i D_i + \varepsilon_{i,t}$$

    where D_i is a dummy variable that takes the value of 1 if the observation is from panel i and zero otherwise.
  - Note that the $\widetilde{\alpha}$ coefficients for the dummy variables are not consistent estimators of the unobserved fixed effect under N-asymptotics.

Within-panel time differences

- The final alternative for estimating a fixed effects regression is to use within-panel time differences to eliminate the unobserved fixed effects.
    - The first difference transformation is the most common choice:

      $$\begin{aligned}(y_{i,t} - y_{i,t-1}) &= \beta' (x_{i,t} - x_{i,t-1}) + (\varepsilon_{i,t} - \varepsilon_{i,t-1}) \Rightarrow \\ \Delta y_{i,t} &= \beta' \Delta x_{i,t} + \Delta \varepsilon_{i,t}\end{aligned}$$

      where the fixed effect α_i vanishes since it is time-invariant. The OLS estimator for β will be consistent under the first assumption.
    - First-differencing is generally preferred since it only loses 1 observation per panel and requires somewhat weaker zero correlation assumptions for the regressors and the error term in the level specification than higher order differences.

Classic linear panel estimation

- The three approaches for fixed effects lead to *identical* estimation results for the coefficients. However, the standard errors are slightly different across each, but the usual canned statistical routines take account of this.
- Note that if we use fixed effects, we cannot include any time-invariant regressors, since they will either vanish under the FE or FD approaches or be exactly collinear with the dummy variables under the DV approach. We are able to include such regressors in an RE model.
- In general, it is advisable to include a set of time dummies in the model (FE or RE). These account for an arbitrary non-linear trend that is common across units ⇒ use `xi i.time` to generate a set.
- It is also often advisable to account for possible within-unit autocorrelation, by using a HAC-robust variance/covariance estimator for the standard errors.

Classic linear panel estimation

- In Stata, each of these estimators are readily implemented after you have read in your panel data and tsset it.
  - For random effects, you can use `xtreg` *depvar explvar*`, re vce(`*option*`)`.
  - For fixed effects:
    - The FE estimator (time de-meaning) ⇒ `xtreg` *depvar explvar*`, fe vce(`*option*`)`.
    - The DV estimator ⇒ `xi: regress` *depvar explvar* `i.panelid, vce(`*option*`)`.
    - The FD estimator ⇒ use the difference or lag operators, such as `regress d.`*depvar* `d.`*explvar*`, vce(`*option*`)`.
- Note that it is feasible to estimate the marginal effect of a time-invariant regressor in a fixed effects model by interacting it with a time-varying regressor. Obviously, the interpretation of the coefficient changes, since it represents the marginal effect of the time-invariant regressor evaluated at some level of the time-varying variable with which it is interacted.

Classic linear panel estimation

- A common test in linear, unobserved effects panel data models is the Hausman test of random versus fixed effects.
    - It essentially tests whether or not assumption 2 holds, using the usual Hausman specification test principle, which contrasts a consistent estimator under both the null and alternatives with an estimator that is consistent only under the null.
    - As usual, if random effects is correct, we have an efficiency gain relative to a fixed effects estimator.
    - In Stata, it is implemented via `estimates store` and then invoking the `hausman` command. See help for details.
- Note that have not considered the inclusion of lagged dependent variables. Lagged dependent variables pose special problems in a panel, since the strict exogeneity assumption is violated. We explore this more in the next lecture.

Related models

- Another common classic linear panel model is the so-called random coefficients specification. It takes the form:

  $$y_{i,t} = \beta_i' x_{i,t} + u_{i,t}$$

  In Stata, such models can be estimated using `xtrc` or `xtmixed`, with the appropriate options. See Wooldridge (2001) for details.

Currency unions and trade

- In a now famous paper, Rose (2000) presented empirical evidence that currency union membership led to a *tripling* of the size of trade flows amongst member nations. He argued that this was a causal effect.
- Interestingly, a fixed exchange rate regime with zero exchange rate volatility does *not* have the same effect as a currency union! Moving to zero exchange rate volatility would raise bilateral trade flows only by 2%.
- His analysis relied upon the use of the gravity model of trade in a panel context, extended to include bilateral indicators of currency union membership. The gravity model uses bilateral distances and economic size to predict bilateral trade flows.
- The finding is controversial, as the size of the effect is extremely large. Let's take a look at some of the results.

Baseline pooled OLS results

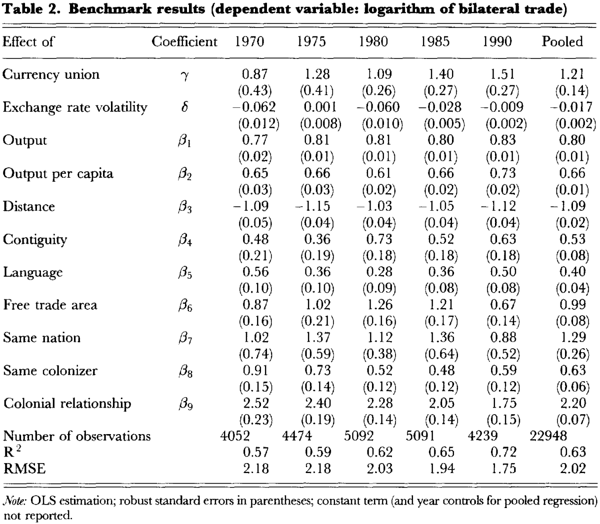

Table 2. Benchmark results (dependent variable: logarithm of bilateral trade)

Effect of	Coefficient	1970	1975	1980	1985	1990	Pooled
Currency union	γ	0.87	1.28	1.09	1.40	1.51	1.21
		(0.43)	(0.41)	(0.26)	(0.27)	(0.27)	(0.14)
Exchange rate volatility	δ	−0.062	0.001	−0.060	−0.028	−0.009	−0.017
		(0.012)	(0.008)	(0.010)	(0.005)	(0.002)	(0.002)
Output	β_1	0.77	0.81	0.81	0.80	0.83	0.80
		(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Output per capita	β_2	0.65	0.66	0.61	0.66	0.73	0.66
		(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.01)
Distance	β_3	−1.09	−1.15	−1.03	−1.05	−1.12	−1.09
		(0.05)	(0.04)	(0.04)	(0.04)	(0.04)	(0.02)
Contiguity	β_4	0.48	0.36	0.73	0.52	0.63	0.53
		(0.21)	(0.19)	(0.18)	(0.18)	(0.18)	(0.08)
Language	β_5	0.56	0.36	0.28	0.36	0.50	0.40
		(0.10)	(0.10)	(0.09)	(0.08)	(0.08)	(0.04)
Free trade area	β_6	0.87	1.02	1.26	1.21	0.67	0.99
		(0.16)	(0.21)	(0.16)	(0.17)	(0.14)	(0.08)
Same nation	β_7	1.02	1.37	1.12	1.36	0.88	1.29
		(0.74)	(0.59)	(0.38)	(0.64)	(0.52)	(0.26)
Same colonizer	β_8	0.91	0.73	0.52	0.48	0.59	0.63
		(0.15)	(0.14)	(0.12)	(0.12)	(0.12)	(0.06)
Colonial relationship	β_9	2.52	2.40	2.28	2.05	1.75	2.20
		(0.23)	(0.19)	(0.14)	(0.14)	(0.15)	(0.07)
Number of observations		4052	4474	5092	5091	4239	22948
R^2		0.57	0.59	0.62	0.65	0.72	0.63
RMSE		2.18	2.18	2.03	1.94	1.75	2.02

Note: OLS estimation; robust standard errors in parentheses; constant term (and year controls for pooled regression) not reported.

Sensitivity to distance

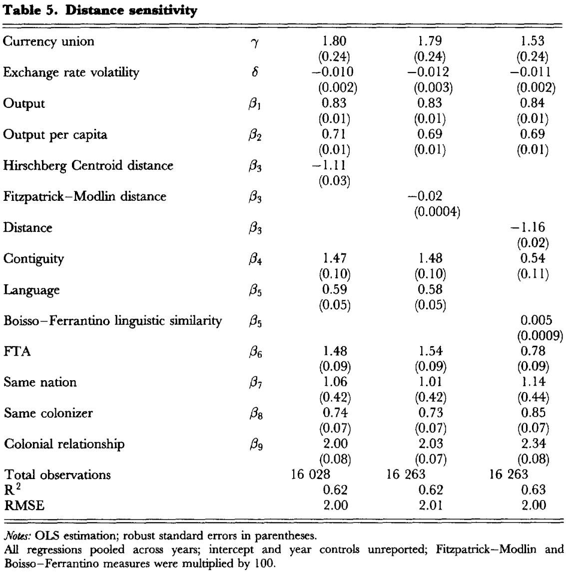

Table 5. Distance sensitivity

Currency union	γ	1.80	1.79	1.53
		(0.24)	(0.24)	(0.24)
Exchange rate volatility	δ	−0.010	−0.012	−0.011
		(0.002)	(0.003)	(0.002)
Output	β_1	0.83	0.83	0.84
		(0.01)	(0.01)	(0.01)
Output per capita	β_2	0.71	0.69	0.69
		(0.01)	(0.01)	(0.01)
Hirschberg Centroid distance	β_3	−1.11		
		(0.03)		
Fitzpatrick–Modlin distance	β_3		−0.02	
			(0.0004)	
Distance	β_3			−1.16
				(0.02)
Contiguity	β_4	1.47	1.48	0.54
		(0.10)	(0.10)	(0.11)
Language	β_5	0.59	0.58	
		(0.05)	(0.05)	
Boisso–Ferrantino linguistic similarity	β_5			0.005
				(0.0009)
FTA	β_6	1.48	1.54	0.78
		(0.09)	(0.09)	(0.09)
Same nation	β_7	1.06	1.01	1.14
		(0.42)	(0.42)	(0.44)
Same colonizer	β_8	0.74	0.73	0.85
		(0.07)	(0.07)	(0.07)
Colonial relationship	β_9	2.00	2.03	2.34
		(0.08)	(0.07)	(0.08)
Total observations		16 028	16 263	16 263
R^2		0.62	0.62	0.63
RMSE		2.00	2.01	2.00

Notes: OLS estimation; robust standard errors in parentheses.
All regressions pooled across years; intercept and year controls unreported; Fitzpatrick–Modlin and Boisso–Ferrantino measures were multiplied by 100.

Sensitivity to monetary regime

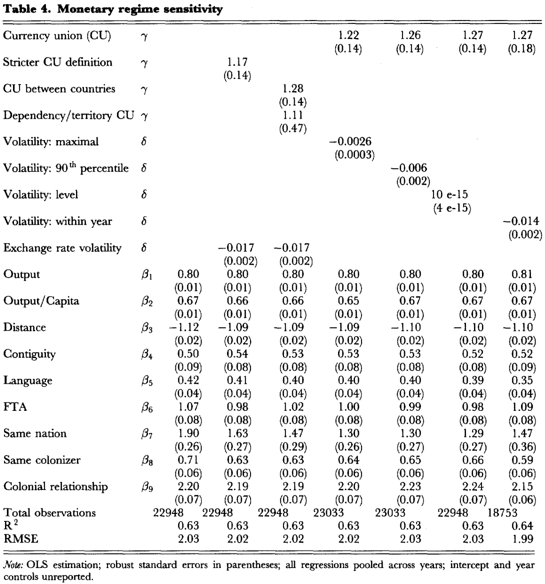

Table 4. Monetary regime sensitivity

Currency union (CU)	γ				1.22 (0.14)	1.26 (0.14)	1.27 (0.14)	1.27 (0.18)
Stricter CU definition	γ		1.17 (0.14)					
CU between countries	γ			1.28 (0.14)				
Dependency/territory CU	γ			1.11 (0.47)				
Volatility: maximal	δ				−0.0026 (0.0003)			
Volatility: 90th percentile	δ					−0.006 (0.002)		
Volatility: level	δ						10 e-15 (4 e-15)	
Volatility: within year	δ							−0.014 (0.002)
Exchange rate volatility	δ		−0.017 (0.002)	−0.017 (0.002)				
Output	β_1	0.80 (0.01)	0.80 (0.01)	0.80 (0.01)	0.80 (0.01)	0.80 (0.01)	0.80 (0.01)	0.81 (0.01)
Output/Capita	β_2	0.67 (0.01)	0.66 (0.01)	0.66 (0.01)	0.65 (0.01)	0.67 (0.01)	0.67 (0.01)	0.67 (0.01)
Distance	β_3	−1.12 (0.02)	−1.09 (0.02)	−1.09 (0.02)	−1.09 (0.02)	−1.10 (0.02)	−1.10 (0.02)	−1.10 (0.02)
Contiguity	β_4	0.50 (0.09)	0.54 (0.08)	0.53 (0.08)	0.53 (0.08)	0.53 (0.08)	0.52 (0.08)	0.52 (0.09)
Language	β_5	0.42 (0.04)	0.41 (0.04)	0.40 (0.04)	0.40 (0.04)	0.40 (0.04)	0.39 (0.04)	0.35 (0.04)
FTA	β_6	1.07 (0.08)	0.98 (0.08)	1.02 (0.08)	1.00 (0.08)	0.99 (0.08)	0.98 (0.08)	1.09 (0.08)
Same nation	β_7	1.90 (0.26)	1.63 (0.27)	1.47 (0.29)	1.30 (0.26)	1.30 (0.27)	1.29 (0.27)	1.47 (0.36)
Same colonizer	β_8	0.71 (0.06)	0.63 (0.06)	0.63 (0.06)	0.64 (0.06)	0.65 (0.06)	0.66 (0.06)	0.59 (0.06)
Colonial relationship	β_9	2.20 (0.07)	2.19 (0.07)	2.19 (0.07)	2.20 (0.07)	2.23 (0.07)	2.24 (0.07)	2.15 (0.06)
Total observations		22948	22948	22948	23033	23033	22948	18753
R^2		0.63	0.63	0.63	0.63	0.63	0.63	0.64
RMSE		2.03	2.02	2.02	2.02	2.03	2.03	1.99

Note: OLS estimation; robust standard errors in parentheses; all regressions pooled across years; intercept and year controls unreported.

Sensitivity to estimation (RE)

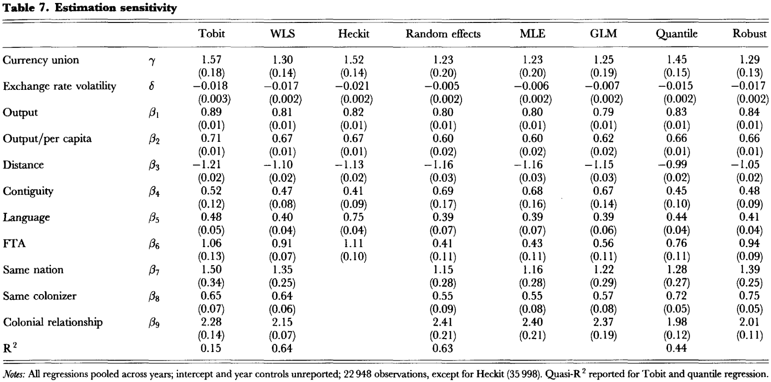

Table 7. Estimation sensitivity

		Tobit	WLS	Heckit	Random effects	MLE	GLM	Quantile	Robust
Currency union	γ	1.57	1.30	1.52	1.23	1.23	1.25	1.45	1.29
		(0.18)	(0.14)	(0.14)	(0.20)	(0.20)	(0.19)	(0.15)	(0.13)
Exchange rate volatility	δ	−0.018	−0.017	−0.021	−0.005	−0.006	−0.007	−0.015	−0.017
		(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Output	β_1	0.89	0.81	0.82	0.80	0.80	0.79	0.83	0.84
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Output/per capita	β_2	0.71	0.67	0.67	0.60	0.60	0.62	0.66	0.66
		(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)
Distance	β_3	−1.21	−1.10	−1.13	−1.16	−1.16	−1.15	−0.99	−1.05
		(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)
Contiguity	β_4	0.52	0.47	0.41	0.69	0.68	0.67	0.45	0.48
		(0.12)	(0.08)	(0.09)	(0.17)	(0.16)	(0.14)	(0.10)	(0.09)
Language	β_5	0.48	0.40	0.75	0.39	0.39	0.39	0.44	0.41
		(0.05)	(0.04)	(0.04)	(0.07)	(0.07)	(0.06)	(0.04)	(0.04)
FTA	β_6	1.06	0.91	1.11	0.41	0.43	0.56	0.76	0.94
		(0.13)	(0.07)	(0.10)	(0.11)	(0.11)	(0.11)	(0.11)	(0.09)
Same nation	β_7	1.50	1.35		1.15	1.16	1.22	1.28	1.39
		(0.34)	(0.25)		(0.28)	(0.28)	(0.29)	(0.27)	(0.25)
Same colonizer	β_8	0.65	0.64		0.55	0.55	0.57	0.72	0.75
		(0.07)	(0.06)		(0.09)	(0.08)	(0.08)	(0.05)	(0.05)
Colonial relationship	β_9	2.28	2.15		2.41	2.40	2.37	1.98	2.01
		(0.14)	(0.07)		(0.21)	(0.21)	(0.19)	(0.12)	(0.11)
R^2		0.15	0.64		0.63			0.44	

Notes: All regressions pooled across years; intercept and year controls unreported; 22 948 observations, except for Heckit (35 998). Quasi-R^2 reported for Tobit and quantile regression.

Critiques of the Rose (2000) result

- Lockwood argued that the result is driven by the CFA and ECCA, with little relevance for the Eurozone.
- Quah argued that the gravity equation plus currency union specification is *ad hoc* – the extended specification is not grounded in theory.
- Quah also argued that the currency union sub-sample only accounts for 1% of the observations (320) in the total sample. On statistical grounds, this should be fine given the other assumptions.
- Many have argued that currency union membership is endogenous. Thus, assumption 1 fails.
- Subsequent work (there has been lots) has whittled this giant effect down. I think the consensus is probably more like 15-20% trade creation, as opposed to 300%.

Summary

- Unobserved effects linear panel models allow us to incorporate panel-specific, time-invariant heterogeneity in means/intercepts. Note that slopes are still assumed to be common across panels.
- The random/fixed effects distinction depends upon whether or not we feel that the observed regressors are correlated with the unobserved effects. If they are not, then RE is fine. Otherwise, we should use FE.
- These methods now have a long history and are commonly used (easily implemented within Stata). However, it can be important to use HAC-robust standard error estimation to ensure that inference is correct. Moreover, we should always be wary of the strict exogeneity assumptions required in these models for inference to be accurate.