Stimulated Brillouin scattering in liquid carbon disulfide

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Abstract

This paper covers the basic background of what Stimulated Brillouin scattering is and how it occurs. It will also briefly look at what can be expected from an incident laser beam of both 1.064 $\mu$m and 532 nm light as far as the gain coefficient, phonon lifetime, range and frequency, and the threshold power.
1. INTRODUCTION

This paper covers the basic background of what Stimulated Brillouin scattering is and how it occurs. It will also briefly look at what can be expected from an incident laser beam of both 1.064 µm and 532 nm light as far as the gain coefficient, phonon lifetime, range and frequency, and the threshold power.

2. A BRIEF LOOK AT STIMULATED BRILLOUIN SCATTERING

Stimulated Brillouin Scattering (SBS) occurs when the incident laser beam enters a substance, and is then scattered off a pressure wave caused by electrostriction or thermal absorption. For SBS induced by electrostriction, the incident laser being of sufficient intensity, will cause density changes in said material due to the electric field of the optical wave, which means there will be differences in the index of refraction at different points in the material causing the beam to scatter. The scattered wave, which as will be described as a stokes wave, then beats with the incident beam reenforcing the pressure or acoustic wave, at the same time the incident beam and the pressure wave beat causing the stokes wave to be reinforced. As this escalates it is possible to get well over 80% of the incident intensity scattered back in the opposite direction of the incident laser beam.

3. GENERAL THEORY

Stimulated Brillouin scattering turns out to be related to stimulated Rayleigh scattering, enough such that it should be included in the general theory discussion that follows where appropriate. There is two main types of both SBS and SRS, electrostriction and thermal, where electrostrictive is caused by the influence of the electric and magnetic fields from the incident beam and thermal which is caused by the absorption of some of the incident radiation and causes density variations in medium. The main difference between the SBS and SRS is that SBS scatters off pressure waves while SRS scatters off isobaric density fluctuations in the medium.
3.1. Equations Of Hydrodynamics

To begin one must first consider the three equations of hydrodynamics as follows.

3.1.1. Continuity

The equation of continuity is given as,

\[
\frac{\partial \tilde{\rho}_t}{\partial t} + \tilde{\mathbf{u}}_t \cdot \nabla \tilde{\rho}_t + \tilde{\rho}_t \nabla \cdot \tilde{\mathbf{u}}_t = 0,
\]

(1)

where \( \tilde{\rho}_t \) is the mass density of the fluid and \( \tilde{\mathbf{u}}_t \) is the velocity of some small volume of the fluid.

3.1.2. Momentum Transfer

The equation for momentum transfer is,

\[
\frac{\partial \tilde{\rho}_t}{\partial t} \frac{\partial \tilde{\mathbf{u}}_t}{\partial t} + \tilde{\rho}_t (\tilde{\mathbf{u}}_t \cdot \nabla) \tilde{\mathbf{u}}_t = \tilde{\mathbf{f}} - \nabla \tilde{p}_t + (2\eta_s + \eta_d) \nabla (\nabla \cdot \tilde{\mathbf{u}}_t) - \eta_s \nabla \times (\nabla \times \tilde{\mathbf{u}}_t).
\]

(2)

With

\[
\tilde{\mathbf{f}} = -\frac{\gamma_e}{8\pi} (\tilde{\mathbf{E}} \cdot \tilde{\mathbf{E}})
\]

(3)

and

\[
\gamma_e = \rho \frac{\partial \epsilon}{\partial \rho}.
\]

(4)

Where \( \tilde{\mathbf{f}} \) is the force per unit volume of any external forces imposed where for electrostriction, \( \tilde{\mathbf{E}} \) is the instantaneous value of the time varying electrical field, \( \gamma_e \) is the electrostrictive coupling constant, \( \tilde{p}_t \) is the pressure, \( \eta_s \) is the shear viscosity coefficient, and \( \eta_d \) is the dilation viscosity coefficient. In the case that the Stoke-Einstein relation \( (\eta_s = \frac{-2}{3} \eta_d) \) is valid, the terms containing \( 2\eta_s + \eta_d \) can be replaced as \( \frac{4}{3} \eta_s \).
3.1.3. Heat Transport

The equation for heat transport is,

\[
\tilde{\rho}_t c_v \frac{\partial \tilde{T}_t}{\partial t} + \rho_t c_v (\tilde{\mathbf{u}} \cdot \nabla \tilde{T}_t) + \tilde{\rho}_t c_v \left( \frac{\gamma - 1}{\beta_p} \right) (\nabla \cdot \tilde{\mathbf{u}}) = -\nabla \cdot \tilde{\mathbf{Q}} + \tilde{\phi}_\eta + \tilde{\phi}_{\text{ext}} .
\]

With

\[
\nabla \cdot \tilde{\mathbf{Q}} = -\kappa \nabla^2 \tilde{T}_t ,
\]

\[
\beta_p = \tilde{\rho}^{-1} \frac{\partial \tilde{\rho}}{\partial \tilde{T}} ,
\]

\[
\tilde{\phi}_\eta = \sum_{ij} \left( 2\eta_s d_{ij} d_{ji} + \eta_d d_{ij} d_{ji} \right) ,
\]

\[
d_{ij} = \frac{1}{2} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right)
\]

and

\[
\tilde{\phi}_{\text{ext}} = \alpha \frac{nc}{4\pi} \langle \tilde{E}^2 \rangle .
\]

Where \( \tilde{T}_t \) is the local value of temperature, \( c_v \) is the specific heat of the material at constant volume, \( \gamma = \frac{c_p}{c_v} \) is the adiabatic index of the material, \( \beta_p \) is the thermal expansion coefficient, \( \tilde{\mathbf{Q}} \) is the heat flux vector for thermal conduction, \( \kappa \) is the thermal conductivity of the material, \( \tilde{\phi}_\eta \) is the viscous energy deposited per unit time per unit volume, and \( d_{ij} \) is the rate of dilation tensor. Also \( \tilde{\phi}_{\text{ext}} \) is the energy per unit time per unit volume from external sources such as absorption of optical waves as the above equation (8) shows, and \( \alpha \) is the optical absorption coefficient.

Now we can derive acoustic equations by linearizing the hydrodynamic equations, assuming the following nominal conditions;

\[
\tilde{\rho}_t = \rho_0 + \tilde{\rho} \text{ where } |\tilde{\rho}| \ll \rho_0 ,
\]

\[
\tilde{T}_t = T_0 + \tilde{T} \text{ where } |\tilde{T}| \ll T_0 ,
\]

\[
\tilde{\mathbf{u}}_t = \tilde{\mathbf{u}} \text{ where } |\tilde{\mathbf{u}}| \ll v_0
\]

and

\[
\tilde{p}_t = p_0 + \tilde{p} \text{ where } |\tilde{p}| \ll p_0 .
\]

After the above assumptions, with \( v \) being the velocity of sound it is possible to simplify the equations of hydrodynamics as follows.
3.1.4. Continuity

The equation for continuity is fairly straight forward and linearizes as

\[
\frac{\partial \tilde{\rho}}{\partial t} + \rho_0 \nabla \cdot \tilde{\mathbf{u}} = 0.
\] (13)

3.1.5. Momentum Transfer

Before linearizing the momentum transfer equation it is necessary to simplify the equation for pressure as follows,

\[
\tilde{p} = \left( \frac{\partial p}{\partial \rho} \right)_T \tilde{\rho} + \left( \frac{\partial p}{\partial T} \right)_\rho \tilde{T},
\] (14)

\[
\tilde{p} = \frac{v^2}{\gamma} (\tilde{\rho} + \beta_p \rho_0 \tilde{T}).
\] (15)

Using the equation for pressure above, the momentum transfer equation becomes,

\[
\rho_0 \frac{\partial \tilde{\mathbf{u}}}{\partial t} + \frac{v^2}{\gamma} \nabla \tilde{\rho} + \frac{v^2 \beta_p \rho_0}{\gamma} \nabla \tilde{T} - (2\eta_s + \eta_d) \nabla (\nabla \cdot \tilde{\mathbf{u}}) + \eta_s \nabla \times (\nabla \times \tilde{\mathbf{u}}) = \tilde{\mathbf{f}}.
\] (16)

3.1.6. Energy Transport

Similarly the equation for energy transport becomes,

\[
\rho_0 c_v \frac{\partial \tilde{T}}{\partial t} + \frac{c_v (\gamma - 1)}{\beta_p} (\nabla \cdot \tilde{\mathbf{u}}) - \kappa \nabla^2 \tilde{T} = \phi_{\text{ext}}.
\] (17)

Taking the divergence of the equation of momentum transfer, replacing all the terms containing \( \nabla \cdot \tilde{\mathbf{u}} \), and substituting in the equation for \( \tilde{\mathbf{f}} \) it is possible to get the following equations for momentum transfer and energy transport,

\[
\frac{\partial^2 \tilde{\rho}}{\partial t^2} + \frac{v^2}{\gamma} \nabla^2 \tilde{\rho} + \frac{v^2 \beta_p \rho_0}{\gamma} \nabla^2 \tilde{T} + \frac{2\eta_s + \eta_d}{\rho_0} \frac{\partial}{\partial t} (\nabla^2 \tilde{\rho}) = \frac{\gamma_e}{8\pi} \nabla^2 \langle \tilde{E}^2 \rangle,
\] (18)

and

\[
\rho_0 c_v \frac{\partial \tilde{T}}{\partial t} - \frac{c_v (\gamma - 1)}{\beta_p} \frac{\partial \tilde{\rho}}{\partial t} - \kappa \nabla^2 \tilde{T} = \frac{n \alpha}{4\pi} \langle \tilde{E}^2 \rangle,
\] (19)

respectably.
3.2. Wave Propagation

These are now coupled equations in terms of variables \( \tilde{\rho} \) and \( \tilde{T} \) and show how they are driven by the optic field. Now we can look at solutions of freely propagating damped waves given as

\[
F(z, t) = Fe^{-\omega_0(A(tv - z))}e^{-\alpha_s z} + c.c. \tag{20}
\]

where \( F \) is a stand in variable to be replaced by either \( \rho \) or \( T \).

\[
\alpha_s = \frac{\omega_A^2}{2\rho_0 v^3} \left[ (2\eta_s + \eta_d) + (\gamma - 1)\frac{\kappa}{c_p} \right] \tag{21}
\]

is the sound absorption coefficient, valid when

\[
\omega_A \ll \frac{\rho_0 v^2}{2\eta_s + \eta_d}. \tag{22}
\]

The solution in the presence of driving terms, where the optical field is represented by

\[
\tilde{E}(z, t) = A_L e^{i(k_L z - \omega_L t)} + A_S e^{i(k_S z - \omega_S t)} + c.c. \tag{23}
\]

Now defining the frequency of the pressure wave as the difference of the frequencies of the laser and stokes waves (\( \omega_A = \omega_L - \omega_S \)) and that the wave vector of the pressure wave is the difference of the laser and stokes wave vectors (\( q = k_L - k_S \)). It is possible to get the following equations:

\[
\tilde{\rho}(z, t) = \rho e^{i(az - \omega_A t)} + c.c. \tag{24}
\]

and

\[
\tilde{T}(z, t) = Te^{i(az - \omega_A t)} + c.c. \tag{25}
\]

Making steady state approximations the wave equation becomes

\[
\frac{v^2 \beta_p \rho_0 q^2}{\gamma} T - \left( \omega_A^2 + i\omega_A \Gamma_B - \frac{v^2 q^2}{\gamma} \right) \rho
= \frac{\gamma q^2}{4\pi} A_L A_S^*, \tag{26}
\]

where

\[
\Gamma_B = \frac{1}{\tau_p} = \frac{(2\eta_s + \eta_d)q^2}{\rho_0}. \tag{27}
\]
Here $\tau_p$ is the phonon lifetime and $\Gamma_B$ is the Brillouin linewidth. These are valid as long as the system is sufficiently damped, which means that both

$$ q \gg \left| \frac{1}{\rho} \frac{\partial \rho}{\partial z} \right|, \quad \left| \frac{1}{T} \frac{\partial T}{\partial z} \right| \quad \& \quad q^2 \gg \left| \frac{1}{\rho} \frac{\partial^2 \rho}{\partial z^2} \right|, \quad \left| \frac{1}{T} \frac{\partial^2 T}{\partial z^2} \right| $$

are true.

For SRS, where $\tau_r = \Gamma_R^{-1}$ is the decay time of the isobaric density distributions,

$$ -(i\omega_A - \frac{1}{2}\gamma\Gamma_R)T + \frac{i(\gamma - 1)\omega_A}{\beta_p \rho_0} \rho = \frac{n c \alpha}{2 \pi c \rho_0} A_L A_S^* $$

and

$$ \Gamma_R = \frac{2 \kappa q^2}{\rho_0 c_p}. $$

By solving to remove $T$ it is possible to get

$$ \left[ \gamma e^{-i\gamma a q v \omega_A} \frac{q^2}{4\pi} \right] A_L A_S^* = 
\left[ - \left( \omega_A^2 + i \omega_A \Gamma_B - v^2 q^2 \gamma \right) - v^2 q^2 \omega_A (\gamma - 1) \right] \frac{(\omega_A + \frac{1}{2} i \gamma \Gamma_R) \gamma}{(\omega_A + \frac{1}{2} i \gamma \Gamma_R) \gamma} \rho, $$

where

$$ \gamma_a = \frac{2 \alpha n v^2 c \beta_p}{c_p \Omega_B} $$

is the absorptive coupling constant, and $\Omega_B = q v$ is the Brillouin frequency. Now as it turns out, for SBS $\omega_a^2 \approx \Omega_B^2 = v^2 q^2$ which means $\omega_A + \frac{1}{2} i \gamma \Gamma_R$ is non resonant signifying that SBS and SRS do not overlap so they can be considered separately. This means that the equation for density can be simplified to

$$ \rho = -\frac{\gamma e^{-i\gamma a q v \omega_A}}{4\pi (\omega_a^2 + i \omega_A \Gamma_B - v^2 q^2)} A_L A_S^*. $$

### 3.3. Optical Fields Amplitudes and Intensities

Looking at the influence of the optical fields on the system, it is necessary to define the non linear polarization which is given by:

$$ \tilde{p}^{NL} = p_L e^{i(k_L z - \omega_L t)} + p_S e^{i(-k_S - \omega_S t)} + c.c, $$
where

\[ p_L = \frac{\gamma_e}{4\pi \rho_0} \rho A_S \quad \text{and} \quad p_S = \frac{\gamma_e}{4\pi \rho_0} \rho^* A_L. \]

Introducing this into the wave equation results in,

\[
-\nabla^2 \left[ A_x (r e^{i k_x \cdot r}) - \frac{\epsilon \omega^2}{c^2} A_x (r) e^{i k_x \cdot r} \right] = \frac{4\pi \omega^2}{c^2} p_x e^{i k_x \cdot r} ,
\]

(35)

where the the subtext of \( x \) (\( \omega_x \)) is a stand in for either \( L \) or \( S \) (\( \omega_L \) or \( \omega_S \)). Now making the approximation of a slow varying amplitudes, the wave equation can be simplified into,

\[
\left( \frac{d}{dz} + \frac{1}{2} \alpha \right) A_L = \frac{2\pi i \omega}{nc} p_L
\]

(36)

and

\[
\left( \frac{d}{dz} - \frac{1}{2} \alpha \right) A_S = -\frac{2\pi i \omega}{nc} p_S.
\]

(37)

Here \( n = Re(\sqrt{\epsilon}) \) is the real refractive index and \( \alpha = \frac{2\epsilon}{\epsilon} Im(\sqrt{\epsilon}) \) is the optical absorption coefficient, with \( \epsilon \) is the dielectric constant. Since \( \rho \) is proportional to \( A_L A_S^* \) the above equations become:

\[
\frac{dA_L}{dz} = \kappa |A_S|^2 A_L - \frac{1}{2} \alpha A_L
\]

(38)

and

\[
\frac{dA_S}{dz} = \kappa^* |A_L|^2 A_S + \frac{1}{2} \alpha A_S ,
\]

(39)

where

\[
\kappa = \frac{q^2 \omega}{8\pi \rho_0 nc} \frac{i \gamma_e (\gamma_e - i \gamma_a)}{(\omega^2_A + i \omega_A \Gamma_B - v^2 q^2)}.
\]

(40)

The intensities for the interacting optical waves would then be given by

\[
I_i = \frac{nc}{2\pi} |A_i|^2
\]

(41)

resulting in the equations,

\[
\frac{dI_L}{dz} = -g_B I_L I_S - \alpha I_L
\]

(42)
and

\[ \frac{dI_S}{dz} = -g_B I_L I_S + \alpha I_S \]  \hspace{2cm} (43)

Where \( g_B = g_B^e + g_B^a \) is the Brillouin gain factor and \( g_B^e \) and \( g_B^a \) are the gains from electrostriction and absorption respectively given by,

\[ g_B^e = \frac{\omega^2 \gamma_e^2}{\rho_0 n v c^2 \Gamma_B} \left( \frac{\Gamma_B^2}{\Gamma_B^2 + (2\Delta \omega_A)^2} \right) \] \hspace{2cm} (44)

and

\[ g_B^a = \frac{-\omega^2 \gamma_e \gamma_a}{2\rho_0 n v c^2 \Gamma_B} \left( \frac{4\Delta \omega_A \Gamma_B}{\Gamma_B^2 + (2\Delta \omega_A)^2} \right) \] \hspace{2cm} (45)

With some simplification it is possible to get the general gain as,

\[ g_B = \frac{2\omega^2 \gamma_e}{2\rho_0 n v c^2 \Gamma_B} \left( \frac{\gamma_e \Gamma_B^2 - 4\gamma_a \Delta \omega_A \Gamma_B}{\Gamma_B^2 + (2\Delta \omega_A)^2} \right) \] \hspace{2cm} (46)

where

\[ \Delta \omega_A = \Omega_B - \omega_A , \]
\[ \Omega_B = qv = (k_L + k_S)v \] and \( \omega_A = \omega_L - \omega_S \).

For electrostriction, the max happens when \( \Delta \omega_A = 0 \) or the acoustic frequency is matched to the Brillouin Frequency. On the other hand the absorption max for gain happens when \( \Delta \omega_A = -\frac{\Gamma_B}{2} \) which means that the frequency of the acoustic wave is off from the Brillouin frequency by half of the Brillouin line-width. Also note, since the electrostrictive process tends to have a higher gain, \( g_B^e \) is commonly listed as \( g_0 \). The maximum value turns out to be

\[ g_B^e(\text{max}) = \frac{\omega^2 \gamma_e^2}{\rho_0 n v c^3 \Gamma_B} \] \hspace{2cm} (47)

and

\[ g_B^a(\text{max}) = \frac{\omega^2 \gamma_e \gamma_a}{2\rho_0 n v c^3 \Gamma_B} \] \hspace{2cm} (48)

It also turns out that the absorption gain is proportional to \( \gamma_a \) and therefore \( \alpha \), so if a chemical is added to the medium with a higher optical absorption coefficient, the gain would increase.
4. THE SBS GENERATOR, AND TRANSIENT LIMITS

It is common for an SBS amplifier to be used in many cases and situations. An SBS amplifier is designed so that a low energy laser counter propagates against the laser beam \((\omega_L)\) which is to be scattered. This low energy beam is chosen to be at the frequency of the scattered stokes wave \((\omega_S)\), thus causing the laser and stokes beam to beat which starts the fluctuations in density in the medium. Therefore an SBS amplifier allows SBS to be initiated at a lower level of intensity compared to the SBS generator. This paper is not going to consider the case of an amplifier at this time and instead will focus on the SBS generator.

4.1. Gain and Gain Threshold

The generator as it turns out is what was analyzed in the General Theory section of this paper. Following from that it can be expected that the stokes intensity such as \(I_S(L) = fI_L(L)\) where \(L\) is the \(z\) coordinate of the sample that is farthest from the incident beam. Consider the case where the reflection \(R = \frac{I_L(0)}{I_S(0)} \ll 1\). Since there is almost no scattering the intensity of the Laser beam is related to the stokes intensity by \(I_S(0) = I_S(l)e^G\) where \(G = g_BI_L(0)L\) and with the reflectivity being quite small the assumption is made that \(I_L(z)\) is constant the reflectivity can be stated as

\[
R \equiv \frac{I_S(0)}{I_L(0)} = fe^G \tag{49}
\]

By defining a threshold at \(R_{th} = 0.01\) it is then possible to define a gain threshold \(G_{th}\) which typically falls in the range of 25 - 30. Solving the above for \(f\) ends up with

\[
0.01 = fe^{G_{th}} \Rightarrow f \approx \frac{1}{e^{G_{th}}}
\]

\(f_{G_{th}=25} \approx 10^{-11}\) to \(f_{G_{th}=30} \approx 10^{-13}\)
For the case $G > G_{th}$ it is possible to get

$$\frac{I_S(L)}{I_L(0)} = \frac{R(1 - R)}{e^{G(1-R)} - R},$$

(50)

$$I_L(L) - I_S(L) = I_L(0) - I_S(0)$$

$$I_L(L) - I_S(L) \approx \frac{I_S(L)}{f}$$

$$\frac{I_S(L)}{I_L(0)} = f(1 - R)$$

substituting this result into the left side of equation (50), it is possible to get the following after substituting $G_{th}$ for $-\ln f$

$$\frac{G}{G_{th}} = \frac{\ln R + G_{th}}{G_{th}(1 - R)}.$$  

(51)

It happens that if the ratio of $\frac{G}{G_{th}}$ is greater then three (3), the reflectivity can be approximated by

$$R = 1 - \frac{G_{th}}{G} \quad \frac{G}{G_{th}} > 3$$

(52)

which for $G = 3G_{th}$ the equation above for the ratio $\frac{G}{G_{th}}$ (51) gives the approximate results $R=0.672$ or 0.671 for $G_{th}$ being 25 and 30 respectively, with the equation (52) $R=0.667$, and as the ratio goes higher the approximation gets better. Solving to get an equation for the intensity results in

$$I_L(L) = I_L(0)(1 - R),$$

(53)

$$I_L(L) = I_L(0)\frac{G_{th}}{G} = I_L(0)\frac{G_{th}}{g_B I_L(0)L},$$

(54)

$$I_L(L) = \frac{G_{th}}{g_B L}.$$  

This happens to hint at the transmitted beam, or part of the incident laser that is not scattered is capped at the threshold intensity for SBS to occur. If the assumtion is made that the laser beam has a Gaussian cross section the intensity is given by $I = \frac{P}{\pi w_0^2}$ with the beam waist diameter ($w_0$) given by $2w_0 = \left(\frac{\lambda}{2\pi}\right)\left(\frac{F}{D}\right)$, with $F$ being the focal length of the lens, and $D$ being the diameter of the beam. Also the length $L$ will be limited to the diffraction length $b = \frac{2\pi w_0^2}{\lambda}$.

$$I_L(L) = \frac{G_{th}}{g_B L} \Rightarrow G_{th} = g_B L \Rightarrow G_{th} = \frac{2\pi I g_B w_0^2}{\lambda}$$

$$G_{th} = \frac{2g_B P}{\lambda}$$

(55)
solving for power

\[
P_{th} = \frac{G_{th} \lambda}{2g_B}
\]  

(56)

which turns out to be the threshold power for SBS which will be looked at later in this paper with respect to CS₂

4.2. Transient SBS

Looking at the transient state, it is sufficient for now to look at the intensity of the stokes wave

\[
I_S(L, T) = \begin{cases} I_{Ne} & \Gamma_B T < \frac{g_B IL}{2} \\ I_{Ne}(g_B IL) & \Gamma_B T > \frac{g_B IL}{2} \end{cases}
\]  

(57)

where \( I_N \) is the effective noise input that initiates the SBS process, \( g_B IL \) is the gain on a single pass, \( \Gamma_B \) is the phonon damping rate and also the Brillouin line-width, and \( T \) is the length of the laser pulse. Now setting the single pass gain to the threshold value \( e^{G_{th}} \) the result is

\[
g_B I_{th} L = \begin{cases} \left( \frac{G_{th}^2 + 2 \Gamma_B T}{2 \Gamma_B T} \right) & \Gamma_B T < \frac{G_{th}}{2} \\ G_{th} & \Gamma_B T > \frac{G_{th}}{2} \end{cases}
\]  

(58)

It turns out that when in the range of transient scattering, or \( \Gamma_B T \sim \frac{G_{th}}{2} \), that most noticeable effect is the delay of the stokes pulse.\[2\] The laser pulse maximum occurs at \( \frac{t}{T} = 0 \) but as the single pass gain decreases the stokes delay, \( \Delta t \), approaches a maximum around \( \frac{\Delta t}{T} \approx 0.7 \). It should also be noted that if the linewidth of the laser becomes grater then the frequency shift of the stokes wave the threshold values become much greater, this occurs around 0.25cm⁻¹ for carbon disulfide in liquid form.\[3\]

5. CARBON DISULFIDE PROPERTIES AND CALCULATIONS

The Stokes-Einstein relation (\( \eta_s = \frac{2}{3} \eta_d \)) happens to be valid for CS₂, meaning the terms containing \( 2\eta_s + \eta_d \) can be replaced as \( \frac{4}{3} \eta_s \)[4, 5]. Since the specific volumes of solids and liquids are generally smaller, unless the pressure is extremely high, the work done by an applied pressure can be neglected. Therefore, if the enthalpy can be represented by the internal
energy component alone, the constant-volume and constant-pressure heat capacities can be said to be equal \( (c_p \approx c_v, \gamma = \frac{c_p}{c_v} \approx 1) \). The electrostrictive constant can be estimated using the Lorentz-Lorenz law resulting in \( \gamma_e \approx \frac{1}{3}(n^2 - 1)(n^2 + 2) \). The equation to used calculate the gain that can be expected below is slightly different from the Boyd version (46) but happens to be one from Pohl and Kaiser [6] and is given as,

\[
g = \frac{\gamma_e \omega^2}{n c^3 \rho_0} \frac{\gamma_B \Gamma_B + \gamma_a \Delta \omega_A}{\Delta \omega_A^2 + (\frac{1}{2} \Gamma_B)^2}.
\]  

(59)

5.1. 1.064 \( \mu \)m Light

The refractive index has been measured many times for \( \text{CS}_2 \), but in 2003 Samoc[7] used the experimental values to come up with three different formulas, resulting the following values at 1.064 \( \mu \)m; 1.59458, 1.59462, and 1.59490, though for the purpose of this paper the value of 1.595 will be used.

<table>
<thead>
<tr>
<th>CS2 Constants for 1.064 ( \mu )m[4, 7–14]</th>
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</thead>
<tbody>
<tr>
<td>( v ) [cm/( \mu )s]</td>
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<td>.115</td>
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| \( \gamma = \frac{c_p}{c_v} \) | \( \beta_p \) | \( \kappa \) [W/m K] | \( \epsilon \) | \( n \) |
| 1 | 0.001 005 8 | 0.161 | 2.641 | 1.595 |

The sound wave created has a frequency of 2.161 GHz and a lifetime of 18.67 ns or 10.7 nm. The total gain including both absorptive and electrostriction comes out as \( 6.65 \times 10^{-7} \frac{m}{W} \) meaning that the threshold power needed to start the process of SBS is in the range of 19 to 24 kW, which happens to be a bit larger then what was estimated in Boyd’s book, though the rest of the values are on the on the same order as his approximations.

5.2. 532 nm Light

Again Samoc[7] calculated the following values for 532 nm; 1.63937, 1.63936, and 1.63935, this paper will use the value of 1.639 will be used.
The sound wave created has a frequency of 4.323 GHz and a lifetime of 4.665 ns or 2.683 nm. The total gain including both absorptive and electrostriction comes out as $7.516 \times 10^{-7} \text{ m W}$ meaning that the threshold power needed to start the process of SBS is in the range of 8 to 10 kW, which is closer to what was expected than the result for 1.064 µm.

6. DISCUSSION

This paper has covered the basic background of what Stimulated Brillouin scattering is and how it occurs. As well as briefly looking at what can be expected from an incident laser beam of both 1.064 µm and 532 nm light. Most of the values came out in the range expected except that for the 1.064 µm light having a threshold power about twice the expected value, though it was expected that the threshold power would be lower for the 532 nm light since the threshold power is proportional to the wavelength of the light.

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