The 'Index Number Problem' and Poverty Monitoring: The Unique Advantages of a Capability Based Approach

Sanjay G. Reddy[1]

Version 1.1
April 18, 2003

The Problem:

An important use of index numbers, such as consumer price indices and purchasing power parities, is to describe differences in the "cost of living" across points in time or space.[2] It is evident that differences in the "cost of living" cannot be described without first specifying the "standard of living" in some particular way. The cost of living is the cost of attaining a particular standard of living. How should the cost of living be evaluated in view of this conceptual interdependence? We approach this problem in a general context and then with special attention to the exercise of monitoring poverty.

The Cost-of-Living in Utility Space:

How should the standard of living be described? The dominant approach to this problem in economics has involved specifying the standard of living in terms of the attainment of a particular level "utility", often interpreted as subjective preference satisfaction. The cost of living is then the cost of attaining a particular level of utility. For an individual with known preferences, whose satisfaction depends on market consumption, this cost can be identified given data on her income and on the prices that she faces, through the use of the standard machinery of microeconomics. This concept of cost of living [see Diewert (1976,1987)] gives rise to utility-referring index numbers of the Konus type:

$$K(\overline{U}, p_2, p_1) = \frac{E[\overline{U}(q), p_2]}{E[\overline{U}(q), p_1]} \tag{1.1}$$

[1] sr793@columbia.edu

[2] For their most helpful discussions and suggestions, I would like to thank Sudhir Anand, Christian Barry, Howard Nye, and Thomas Pogge. I am also extremely grateful for an insightful suggestion of Yuri Dikhanov at the Stockholm Conference of the IARIW which drew my attention to the significance in this context of the Konus index. I am also grateful to a seminar audience at the University of Florence.

where $E[\overline{U}, p]$ refers to the minimum monetary cost of attaining utility level $\overline{U}$, given the price vector p. We assume that the conditions for a minimum to exist are met. In this case $K(\overline{U}, p_2, p_1)$ refers to the minimum proportionate change in income that is required to enable an individual who had income just sufficient to attain utility level $\overline{U}$ at prices p_1 to achieve the same level of utility at prices p_2.

If the preferences of an individual are known and unchanging, then it is possible to establish the value of the Konus index, given a specific reference level of utility $\overline{U}$, and observed prices p_1 and p_2. Unfortunately, in practice it is doubtful that the preferences of individuals are unchanging, and they are never known to the degree required to establish the exact value of the Konus index. This informational limitation is the source of the "index number problem" in the context of utility-based living standards assessment.[3]

There are two possible approaches to overcoming the index number problem understood in this sense. The first, to identify what the preferences of individuals actually are, is difficult if not impossible to do, although it ay be attempted to infer them in an approximate fashion based on observed patterns of demand. The second approach, which has been widely pursued, is to "assume away the problem", by making the assumption that the preferences of individuals take a form such that a particular index number function $I(p_2,p_1,q_2,q_1)$, where q_2 and q_1 represent the quantities consumed given prices p_2 and p_1, is *exact*. We employ the following definition, following Diewert (1976,1987):

$I(p_2,p_1,q_2,q_1)$ is exact if $I(p_2,p_1,q_2,q_1) = K(U(q_1), p_2, p_1)$

Diewert shows there exist a number of functional pairs [$I(p_2,p_1,q_2,q_1)$, $U(q_1)$] such that the index number function $I(p_2,p_1,q_2,q_1)$ based on observable data, is exact for the unobservable utility function $U(q)$. These important results may be of limited value, however, if there is not reason to assume that individuals in fact have preferences that correspond to the postulated utility function $U(q)$. Certain well-known index numbers have the additional property of being "superlative", which is to say that they are "exact" for a utility function that provides a second-order approximation to an arbitrary (twice continually differentiable) utility function [Diewert (1976,1987)]. This is a useful property that may be especially useful in choosing index numbers for the purpose of capturing changes in the cost of living in a specific setting over short periods of time. However, it may not be of much help if the task at hand is to chose index numbers that reflect differences in the cost of living over larger intervals in time or space. Finally, this approach may be of limited help if the utility function for which an index number is exact does not adequately capture the evaluative judgments that are appropriate to a particular domain.

[3] "In order to numerically construct an individual's Cost-of-Living Index, it is necessary to know his or her preferences over economic goods. Since these preferences are essentially unobservable, it is necessary to construct approximations to the Cost-of-Living index." Diewert (1990).

The Cost-of-Living in a Generalized Value Space:

Utility is only one possible metric in which evaluation of the standard of living of individuals may take place. Other metrics, such as capabilities and resources have also been suggested in the literature. The choice of metric for the evaluation of the standard of living must ultimately be motivated by normative reasoning concerning the appropriate manner in which to evaluate the life circumstances of individuals. There is no escape from this dependence of the concept of the standard of living on the normative judgments of the evaluator. Even the decision to defer to information concerning individuals' subjective preference satisfactions represents a particular such evaluative judgment.

Let us suppose that the judgments of an evaluator concerning the life circumstances of an agent can be represented by a real-valued 'value function' V(s) describing the value attached by the evaluator to a particular state of affairs experienced by the agent, s. Suppose that $V(s_1) \geq V(s_2)$ iff the evaluator deems s_1 to be at least as valuable as s_2. We can note that in general the state of affairs, s, is influenced by a vector of parameters (π) that determine the choices available to the agent, as well as by the choices actually made by the agent. Let $E[\overline{V}(s), \pi]$ refer to the minimum monetary cost to the agent of bringing about a state judged by the evaluator to have value $\overline{V}$, given the vector of parameters (π).

We can then write:

$$K(\overline{V}, \pi_2, \pi_1) = \frac{E[\overline{V}(s), \pi_2]}{E[\overline{V}(s), \pi_1]} \tag{1.2}$$

A special case of a framework of this kind is the familiar setting in which π represents the prices, p, faced by an agent, and s represents the commodity bundle, q, consumed by the agent. An even more specialized (and familiar) case is that in which π represents the prices, p, faced by an agent, s represents the commodity bundle, q, consumed by the agent, *and* $V(s) = U(q)$, i.e. the value function describing the judgments of the evaluator is simply the utility function of the agent. In this special case, expression (1.2) reduces to (1.1). Therefore (1.2) is a generalization of (1.1).

The 'generalized Konus index' described by (1.2) is well-defined for value functions that describe evaluative judgments of many different kinds.

The Cost-of-Living in Capability Space:

Capability-based evaluation of living standards [as introduced by Amartya Sen and explicated in Sen (1995) etc.] is motivated by the idea that the appropriate way to conceive of the level of advantage experienced in a human life is through directly taking note of the set of beings and doings (or "functionings") that it is within a person's power

to achieve. The "elementary capabilities" such as the ability to be adequately nourished, to lead a long and healthy life, to be able to participate in collective decision-making, and so forth may be of particular interest when evaluating the capability set (or set of functionings that it is in a person's power to achieve) possessed by an individual. The particular capabilities to be emphasized and the extent of emphasis to be given to each must be determined by the application of normative reasoning.

Let C represent the capability set possessed by an individual. Let $V(C)$ denote a value function (as described above) representing an evaluator's judgments concerning the value ascribed to the possession of the capability set, C, by the agent whose life circumstances are being evaluated. We assume that the capability set, C, is influenced by a vector of parameters (π), representing such factors as the market prices that the agent faces. We assume in particular that $C = C(\pi)$. We can now describe another special case of the generalized Konus index, which we may call the capability-based Konus index for the reference capability set C:

$$K(\overline{V},\pi_2,\pi_1) = \frac{E[\overline{V}(C(\pi_2)]}{E[\overline{V}(C(\pi_1)]} \tag{1.3}$$

It may be observed that $K(\overline{V},\pi_2,\pi_1)$ is well defined as long as the mappings $V(C)$ (which describes the value attached to distinct capability sets) and $C(\pi)$ (which describes the capability sets attainable given specific conditioning parameters, such as prices) and the cost function $E[\overline{V}(C(\pi)]$ (which describes the minimum cost of attaining a capability set deemed to have a particular value, given specific conditioning parameters) are well-defined.

Poverty-Monitoring and the Unique Advantages of a Capability-Based Approach

The index number problem in the setting of utility-based evaluation arises as a result of the informational constraints that arise in the context of such evaluation. In contrast, capability-based evaluation suffers to a much lesser degree from this problem of informational constraints. The basic reason that this is so is that capability-based evaluation incorporates the external judgments of the evaluator concerning what forms of human flourishing (and in particular which elementary functionings) are of greatest importance. External judgments also play a central role in determining the commodity requirements of achieving particular functionings and therefore the minimum cost of achieving a particular set of capabilities. The central role of external judgments can limit the problem of informational constraints and enables capability-based assessment of the cost-of-living to go further than utility-based assessment in a wide range of circumstances.

These advantages of capability-based assessment are most evident in the context of those forms of living standards assessment that focus on the most elementary human capabilities. Assessments of the extent of absolute poverty are clearly of this kind.

Consider for example an elementary functioning such as the ability to be adequately nourished. The role that this functioning plays in assessing the extent of absolute poverty must clearly be central to any plausible capability-based account of absolute poverty. As a result of this external evaluative judgment, restrictions are placed upon the mapping $V(C)$ (which describes the value attached to distinct capability sets). In particular, $V(C)$ must reflect the great value attached to capability sets that make it possible to achieve this functioning. Similarly, $C(\pi)$ (which describes the capability sets attainable given specific conditioning parameters, such as prices) will also be conditioned by the factual and normative judgments of an external evaluator. For example, a nutrient and calorie content matrix describing the nutritional contents of available foods, can be combined with price data (contained in π) to determine a mapping from prices to nutritional capabilities (which for operational purposes an evaluator may wish to identify with calorie and nutrient requirements). However, information concerning the existing pattern of food consumption in a population is also likely to influence what capabilities are deemed to be achievable, given the set of conditioning parameters π. A "linear programming diet" that specifies the least-cost of achieving energy and nutrient requirements may not be appropriate in light of other relevant facts such as the norms that influence individuals' perceptions concerning what is an acceptable diet. Accordingly, existing efforts to determine the minimum cost of achieving a particular nutritional outcome defer to some degree to such perceptions[4]. On the one hand, the role of norms of this kind limits the ability to use a determinate methodology such as linear programming to identify the least-cost of attaining elementary capabilities. On the other hand, prevailing norms are also a form of externally observable information that can inform an evaluator's judgments concerning the commodities that are required in a specific context in order to achieve a particular set of capabilities.

The plausible range both of $V(C)$ and $C(\pi)$ are limited by externally observable information that enters into the formation of the judgments of an evaluator. For this reason the extent of informational indeterminacy associated with capability-based evaluation of the extent of absolute poverty is likely to be much less severe than is that associated with utility-based evaluation. This is not to say that capability based evaluation does not suffer from some of the problems of indeterminacy that beset utility based evaluation. This is certainly the case. For example, changes in relative prices will surely influence the commodity bundles that ought to be consumed in order to achieve a particular set of (e.g. nutritional) capabilities at least cost. Nevertheless, the focus on elementary capabilities, and (derivatively) on commodities that possess the relevant characteristics to promote these elementary capabilities allows for this problem to be addressed in two ways.

---

[4] See for example the careful methodology of the U.S. Department of Agriculture's 'Thrifty Food Plan', described on http://www.usda.gov/cnpp/FoodPlans/TFP99/TFP99Report.pdf .

First, the plausible range of substitution in response to changes in relative prices is limited by the need to achieve particular elementary capabilities. For example, although it is plausible that adequate nourishment can be achieved with different possible combinations of energy and nutrients, it is not plausible that adequate nourishment can be achieved if these are combined in very lopsided ways. We may make the following comparison. In utility-based evaluation, cost of living adjustments are uncertain because it is uncertain what the preferences of individuals are. Different preference maps can give rise to entirely different appropriate cost of living adjustments (or Konus indices). A case that is 'opposite' is that of a fixed commodity bundle. In this case, because the commodity bundle is known precisely, an exact evaluation of the cost of consuming this commodity bundle at different possible price vectors is feasible. Capability-based evaluation is mid-way between these two cases in the sense that the external information that they rely on helps to 'pin down' the plausible range of costs, although it does not enable this to be done with exactitude. A capability based Konus index reflects the resources required by individuals if they are to achieve a set of capabilities valued to a specific degree. In the context of poverty assessment, the elementary capabilities are of special importance. These elementary capabilities cannot be achieved in arbitrary ways. In particular, the range of plausible substitution is limited by what we know about how these capabilities are achieved.

Second, the range of plausible substitution across different achievable elementary capabilities is also limited by the value placed on achieving particular elementary capabilities. The importance attached to particular elementary capabilities limits the range of plausible substitution across commodities, as only particular commodities possess the characteristics that promote the valued elementary capabilities. Thus, it is unlikely that alcohol or tobacco would play an important role in achieving elementary nutritional capabilities, however important they may be to achieving utility.

A further advantage of the capability-based approach to poverty assessment is that it does not require the (plainly false) assumptions that people have the same preferences, or that the preferences of individuals are unchanging over time, that are adopted by the approach of identifying exact or superlative utility-based price indices. Since the ultimate focus of a capability-based approach is on the beings and doings that individuals can achieve, rather than on the subjective preference satisfaction that they experience, preference variation and preference change can be much more easily accommodated by a capability-based approach. This advantage may be especially great in the context of inter-country, inter-group and inter-temporal comparisons that must come to grips with significant divergences in preferences as well as circumstances.

These arguments suggest that the degree of uncertainty that is involved in assessing whether individuals possess valued elementary capabilities, and what costs are required in order to do so, can be limited in comparison with the "wide-open" case of utility based evaluation which must proceed in the absence of real information about the preferences that individuals have. This will be especially true if the evaluator is concerned with

judging whether and to what degree a narrow range of elementary capabilities (which may be furthered only in particular ways) are possessed by an agent.

Conclusion

We may say, not that capability-based evaluation "solves" the index number problem, but that it *limits* its severity. Nevertheless, the case for capability-based evaluation must ultimately rest primarily on whether it offers a *normatively well-founded* approach to assessing the level of advantage or disadvantage that persons experience, and not merely on its practical advantages. In the context of poverty assessment, the conceptual and normative merits of a capability-based approach are evident.

REFERENCES

Diewert, W.E. (1976), “Exact and Superlative Index Numbers”, in Diewert, W.E. and A.O. Nakamura (1993), *Essays in Index Number Theory: Volume I,* Amsterdam: North Holland.

Diewert, W.E. (1987), “Index Numbers”, in Diewert, W.E. and A.O. Nakamura (1993), *Essays in Index Number Theory: Volume I,* Amsterdam: North Holland.

Diewert, W.E. (1990), “The Theory of the Cost-of-Living Index and the Measurement of Welfare Change”, in Diewert, W.E. ed. (1990), *Price Level Measurement*, Amsterdam: North-Holland.

Sen, Amartya (1995), *Inequality Reexamined*. Oxford: Oxford University Press.