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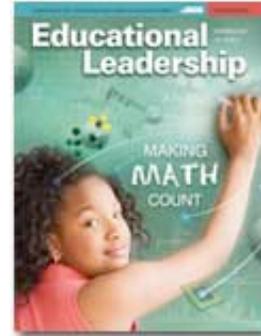
**Making Math Count** Pages 32-36

## Learning from Singapore Math

**The United States could benefit from looking at five elements driving the program's success.**

*Steven Leinwand and Alan L. Ginsburg*

As the United States strives to improve student performance in mathematics, it is not surprising that the small country of Singapore—with its highly regarded mathematics program familiarly known as Singapore Math—has attracted so much attention. After all, Singapore students have the enviable record of scoring first in the world in mathematics proficiency on the past three Trends in International Mathematics and Science Studies (TIMSS), soundly beating their U.S. counterparts.



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But this world-class performance did not happen by accident. It is proof that taking a few crucial actions can pay rich dividends in terms of significantly raising student achievement. In fact, Singapore's mathematics and science achievement in the early 1990s was comparable to the consistently mediocre achievement of the United States (International Association for the Evaluation of Educational Achievement, 1988). However, the country's poor performance served as the impetus for concerted national efforts that have resulted in its current success in mathematics.

Of course, all international comparisons need to be made with great care, given the vast cultural, governmental, and demographic differences among countries. In the case of Singapore and the United States, however, these differences are often overstated. Some claim that Singapore's student population is not as diverse as that of the United States. In fact, almost a quarter of Singapore's students are Malay or Indian, with the majority of its population being Chinese. Others argue that with a population of only 4.1 million people, Singapore cannot be reasonably compared with the United States, with its population of 300 million. Nevertheless, Singapore's population makes the country only a little smaller than Chicago and more populous than approximately one-half of U.S. states.

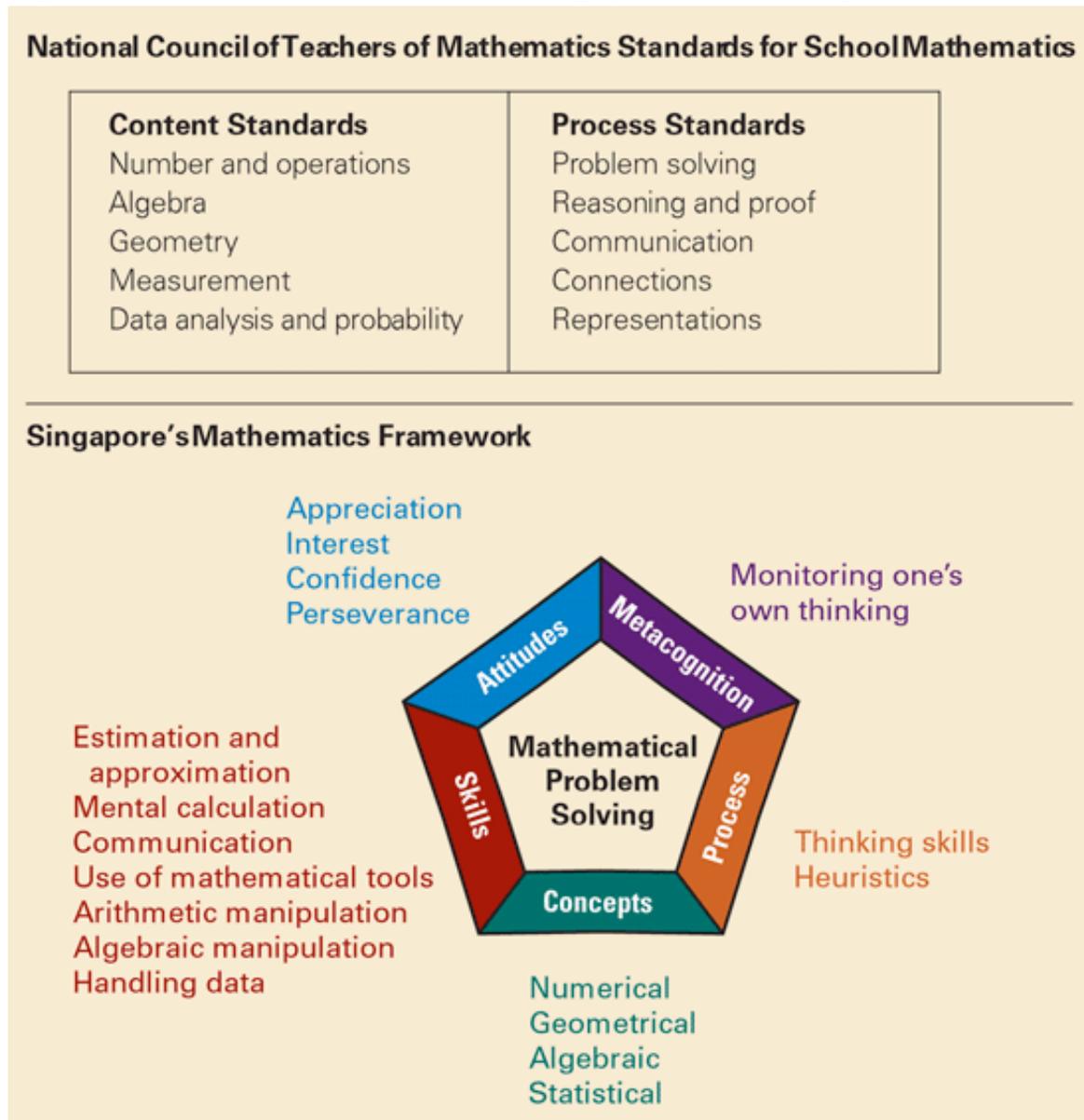
Winning the international horse race in mathematics should not receive as much attention as the specific elements of the program and the policies that support its implementation. These can be powerful models for a reasoned analysis of how to make U.S. mathematics programs far more productive. The following five elements have contributed to the success of Singapore Math.

### Element 1: An Organizing Framework

In the United States, a shared common vision for school mathematics does not exist. By default, the National Council of Teachers of Mathematics has come closest to articulating a national vision, built most recently in *Principles and Standards for School Mathematics* (2000) on two lists of standards—one for content and one for processes. In contrast, Singapore's guiding framework presents a balanced, integrated vision that connects and describes skills, concepts, processes, attitudes, and metacognition (see fig. 1). Instead of implicitly giving equal weight to content and process components and failing to make explicit the crucial connections between them—the

current situation in the United States—Singapore places problem solving at the center of the framework and uses a pentagon to represent the connections and integration of program goals.

**Figure 1. The United States' and Singapore's Differing Math Frameworks**



Source: From *Mathematics Syllabus Primary*, p. 6, 2001, Singapore: Curriculum Planning and Development Division, Ministry of Education, Singapore. Copyright 2001 by the Curriculum Planning and Development Division. Adapted with permission.

It is interesting to note the similarity between the Singapore framework and the five strands of mathematical proficiency—conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition—that are presented in the National Research Council's *Adding It Up: Helping Children Learn Mathematics* (2001), a source too infrequently consulted by designers of mathematics programs.

The lesson to be learned is that the very narrowness and disconnected nature of the organizing structure of mathematics education in the United States prevents us from developing a stronger and more effective instructional program.

## Element 2: Alignment

One of the most significant differences between the two countries has to do with alignment: The elements of Singapore Math *deliberately* align with one another. It is no secret that in the United States, teachers face a fragmented array of “masters,” including the textbook, the local curriculum, the state curriculum, and, most powerfully, state tests. Not only are these elements poorly aligned, but also their competing and often conflicting demands make programmatic focus and coherence nearly impossible to attain.

The wide range of expectations among the 50 states concerning mastery of key mathematics skill standards illustrates this lack of alignment. For example, according to the Center for the Study of Mathematics Curriculum, one state expects fluency in addition and subtraction of fractions in 4th grade; 15 states expect this of their students in 5th grade; 20 states, in 6th grade; and 6 states, in 7th grade (Reys et al., 2006).

In Singapore, each element of the system—the framework, a common set of national standards, texts, tests, and teacher preparation programs—is carefully aligned to clear and common goals. The United States is unlikely to achieve world-class status in mathematics as long as the patchwork of 50 different state frameworks and assessment instruments remains misaligned with the major textbooks, which are designed to be all things to all people.

## Element 3: Focus

The degree to which the often-used phrase “a mile wide and an inch deep” applies to the typical U.S. mathematics curriculum becomes clear when analyzing Singapore Math. Typical U.S. state mathematics frameworks delineate many more topics and outcomes per grade level than Singapore does. For example, regarding mathematics standards in grades 1–6, Singapore covers an average of 15 topics per grade level (Ministry of Education, Singapore, 2001), compared with Florida's 54 and New Jersey's 50. Singapore's success suggests that reducing the repetition of topics from year to year, particularly within the number and geometry strands, would enable the curriculum to progress at a faster pace and with better results. It is interesting to note that the two states that delineate the fewest number of topics per grade level—North Carolina, with 18, and Texas, with 19—are recognized for their successful performance in the National Assessment of Educational Progress.

An analysis of 3rd grade textbooks (Ginsburg, Leinwand, Anstrom, & Pollock, 2005) shows that the most commonly used elementary textbooks in the United States compound the problem of a lack of coherence and focus. For example, the textbook commonly used in Singapore at that grade level has 496 pages, 14 chapters, 42 lessons, and an average of 12 pages per lesson. Compare that with the 3rd grade Scott Foresman textbook, with its 729 pages, 32 chapters, 164 lessons, and 4 pages per lesson. Singapore's success is likely attributable, in part, to the substantial number of pages it allots per lesson (12 as opposed to 4 in the Scott Foresman textbook) and to the fact that it focuses on far fewer lessons per grade level than U.S. textbooks do (42 lessons as opposed to 164). This has resulted in much greater mathematical focus at each grade level.

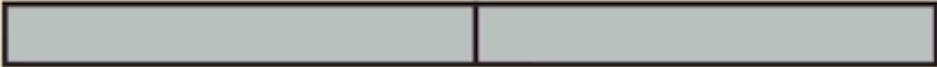
## Element 4: Multiple Models

Many U.S. textbooks are notable for their four-color photographs that relate only tangentially, if at all, to the mathematics at hand. In contrast, Singapore textbooks are notable for their multiple representations and for simple cartoons (see fig. 2), in which an illustrated figure suggests a strategy (“Divide 18 by 2”); notes an equation; provides information about the problem; or poses questions.

**Figure 2. How the Bar Model Is Used for Word Problems in Singapore Math**

1. Robert has \$18.  
He has twice as much money as Sarah has.  
How much money does Sarah have?

\$18

Robert 

Sarah  ?

$18 \div 2 = \underline{\quad}$

Sarah has \$         



The development of fraction understandings, for example, consistently blends concrete representations (four of six balls are striped), pictorial representations (six of nine congruent regions in a rectangle are shaded), and abstract representations ( $2/3 = 4/6 = 6/9$ ). Work with multiplying and dividing by 6 is presented with " $\times 6$ " and " $\div 6$ " input-output "machines." An input of 5 in the  $\times 6$  machine results in an output of 30; an input of 30 in the  $\div 6$  machine results in an output of 5. This representation is followed by a 5-unit-by-6-unit rectangular array accompanied by a think bubble noting that  $5 \times 6 = 6 \times 5$  and by a column of 5 blocks " $\times 6$ " resulting in 6 columns of 5 blocks that provide still another pictorial representation of  $5 \times 6$ . Thus, the textbook provides students and teachers with multiple representations on which to build skills and conceptual understanding. In this manner, Singapore textbooks systematically support instruction consistent with research on how the progression from concrete to pictorial to abstract enhances learning (Ginsburg et al., 2005).

In addition, most U.S. mathematics programs superficially jump around from counters to number lines to base-10 blocks to teach number concepts and from area models to strip models to number line models to teach fractions. In contrast, Singapore's program consistently uses the bar or strip model as a pictorial model for parts and wholes in addition, subtraction, multiplication, division, fractions, ratios, and percentages (see fig. 2). This consistent use of a single powerful model provides a unifying pedagogical structure entirely missing in U.S. mathematics programs.

**Element 5: Rich Problems**

We know that in reading instruction, higher-order questions and more advanced text help develop stronger comprehension. Analogously, more complex multistep problems support stronger mathematical development. However, the vast majority of problems in U.S. textbooks are one-step exercises that rarely demand anything more than recall and routine application.

Consider this problem from Singapore Math, grade 4:

Meredith bought  $\frac{2}{5}$  kg of shrimps. Courtney bought  $\frac{1}{10}$  kg of shrimps less than Meredith.

Find the weight of the shrimps bought by Courtney.

Find the total weight of shrimps bought by both girls.

Or this problem from Singapore Math, grade 6:

The ratio of the number of blue beads to the number of red beads in a jar was 2:5 at first. Ian removed  $\frac{1}{4}$  of the blue beads and  $\frac{2}{5}$  of the red beads from the jar.

Find the new ratio of the number of blue beads to the number of red beads.

If there were 12 more red beads than blue beads left in the jar, how many beads were removed altogether?

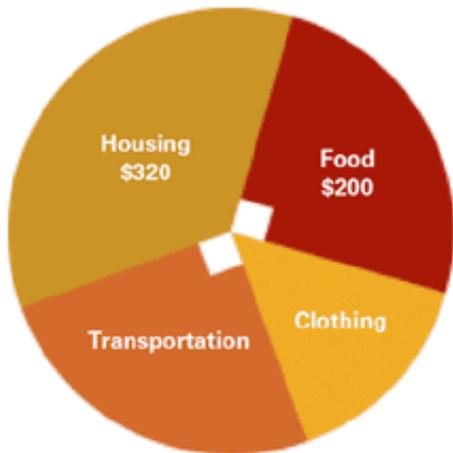
In both cases, as is typical in the Singapore texts, students are expected to complete multistep problems, apply a range of skills and concepts, and often find intermediate values to arrive at a solution. A steady diet of problems such as these enhances the development of broader and deeper mathematical understanding.

We can see the difference between the Singapore and U.S. approaches in two grade 6 problems about pie charts, one typical of Singapore Math and one typical of a Scott Foreman math textbook (see fig. 3). The Singapore problem requires about six steps and an understanding that a right angle is 90 degrees and that the two right-angle sectors must be equal and must represent half the total amount of money. The problem also illustrates how Singapore uses problems to reinforce a range of skills and understandings while teaching students how to interpret pie charts.

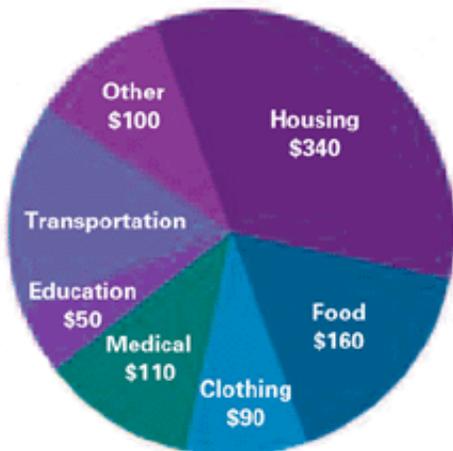
### **Figure 3. Pie Chart Problems in Singapore and the United States**

**A pie chart problem typical of Singapore Math:**

The pie chart represents the amount of money that a family spends each month.

**Monthly Family Budget**

- What fraction of the total budget went to transportation?
- What was the total amount of money the family budgeted for the month?
- How much money was budgeted for clothing?
- What was the ratio of the amount budgeted for housing to the amount budgeted for food?

**A pie chart problem typical of Scott Foresman-Addison Wesley Math****Annual Family Expenses  
(for each \$1,000)**

- How much does the family spend on transportation for each \$1,000?
- For each \$1,000, how much more does the family spend on housing than on food and medical expenses?
- TEST PREP. If the family spends \$4000, about how much is spent on clothing?  
1) \$100 2) \$200 3) \$400 4) \$1000
- For what category does the family spend about three times what it spends on medical expenses?

Compare the richness of this problem with the routine, low-level expectations required by the Scott Foresman problem. It is easy to get a sense of why U.S. students are often deprived of important opportunities to learn.

**A Lesson for Us All**

Too often in our search for quick fixes, we jump on the latest bandwagon or reach out for the latest fad. It would be unfortunate to react to the United States' problems in mathematics achievement by assuming that the widespread adoption of Singapore Math is the cure-all. Instead, the real value is in the Singapore Math *program*—in the fact that it helps us identify the essential features that the United States needs to adapt and adopt to remain mathematically competitive.

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**Steven Leinwand** is Principal Research Scientist at the American Institutes for Research, 1000 Thomas Jefferson St., NW, Washington, DC 20007; 202-403-6926; [SLeinwand@air.org](mailto:SLeinwand@air.org). **Alan L. Ginsburg** is Director of the Policy and Program Studies Service in the U.S. Department of Education, Maryland Ave., NW, Washington, DC 20024; 202-401-1270; [Alan.Ginsburg@ed.gov](mailto:Alan.Ginsburg@ed.gov).

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1703 N. Beauregard Street, Alexandria, VA 22311 USA • 1-800-933-2723 • 1-703-578-9600

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