Teaching Addition and Subtraction Facts: A Chinese Perspective

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In its *Principles and Standards for School Mathematics,* the NCTM suggests that fluency with basic addition and subtraction number combinations is a goal in teaching whole-number computation (NCTM 2000, p. 84). A mastery of lower-order skills instills confidence in students and facilitates higher-order thinking. The ability to automatically recall facts strengthens mathematical ability, mental mathematics, and higher-order mathematical learning. Without this automation, students have difficulty performing advanced operations.

How teachers can help children master the basic addition and subtraction facts is an important, long-standing issue in every country in the world. Educators in different countries have developed unique approaches to teaching basic addition and subtraction facts. This article examines how Chinese mathematics educators deal with these facts.

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Differences in Language Structure

Researchers have found that children's spoken language affects how they think and, thus, can affect the learning of the basic facts (Miura et al. 1994). For instance, compare the counting sequence in English with that in Chinese, as shown in **table 1.** Unlike the English, Chinese clearly and consistently highlights the grouping-by-ten nature of our numeration system. In Chinese, *fourteen* is *ten-four, eighteen* is *ten-eight,* and *thirty* is *three-ten*. The structure of the language easily leads Chinese children to view two-digit numbers as tens and ones (Cao 1994). They can readily think of 12 both as one group of ten items plus two ungrouped items and as a collection of twelve ungrouped items. English counting terms are less explicit and consistent in revealing the base-ten nature of our numeration system. For example, *twelve* is not *ten-two* and *twenty* is not *two-ten*.

Furthermore, Yang and Cobb (1995), in their study of children's conception of number, found that American mothers rarely interpreted numbers in the teens as composites of one 10 and some 1's when they interacted with their children. Instead, they usually initiated and guided learning activities in which children completed tasks involving numbers in the teens by counting by ones orally or with manipulatives. This practice reinforces the view that 12, for example, represents only a collection of twelve items. As a result, Chinese children are more inclined than children in the United States to use tens and ones to represent numbers and, subsequently, to use 10 as a bridge when performing addition and subtraction.

Differences in Teaching

American teachers often use counting in a one-to-one correspondence to introduce addition and subtraction of whole numbers. This strategy is based on the "one more than" relationship between consecutive whole numbers; for example, 4 is one more than 3 and 9 is one more than 8. As a result, when two-digit addition and subtraction are introduced, American children rely heavily on counting-based and collection-based concepts; for instance, 13 is treated as a collection of thirteen objects.

Chinese teachers use a three-step method to teach addition and subtraction. Children first develop an understanding of number concepts, the meanings of addition and subtraction, and the relationships between addition and subtraction. Next, children master addition and subtraction facts in three substeps. First, they learn sums and related subtraction facts to 10, then they learn facts between 11 and 20, and finally, they learn facts between 20 and 100. In the third overall step, students are introduced to the addition and subtraction algorithms. Each step is the foundation for the next step. Making sure that children successfully complete one level before moving to the next is important to the teachers. If children acquire a solid foundation at each of these three steps, they can easily extend the process to even larger numbers.

When sums up to 10 are first introduced in Chinese elementary schools, counting skills are emphasized to help children understand the relationships among these sums. When sums between 11 and 20 and related subtraction facts are introduced, rather than rely on counting, children are usually encouraged to create collections of tens and ones to represent the number; this approach is consistent with the linguistic structure of the Chinese counting sequence. For example, to teach 8 + 3, Chinese children are often asked to take two objects from a collection of three and put them together with eight to make a 10; thus, they see that the whole becomes a collection of ten and one, or eleven. The "make ten" thinking strategy is demonstrated in the following examples:

TABLE 1

Number names in two languages

	Verbal Counting Sequences	
Numeral	**English**	**Chinese**
1	one	yi
2	two	er
3	three	san
4	four	si
5	five	wu
6	six	liu
7	seven	qi
8	eight	ba
9	nine	jiu
10	ten	shi
11	eleven	shi-yi
12	twelve	shi-er
13	thirteen	shi-san
14	fourteen	shi-si
15	fifteen	shi-wu
16	sixteen	shi-liu
17	seventeen	shi-qi
18	eighteen	shi-ba
19	nineteen	shi-jiu
20	twenty	er-shi
21	twenty-one	er-shi-yi
22	twenty-two	er-shi-er
30	thirty	san-shi
40	forty	si-shi
50	fifty	wu-shi
60	sixty	liu-shi
70	seventy	qi-shi
80	eighty	ba-shi
90	ninety	jiu-shi

a) 9 + 4 = ? Think:
- 9 + ? = 10.
- 9 + 1 = 10.
- ```
   4
  / \
  1  3
  ```
- Therefore,
  9 + 1 = 10;
  10 + 3 = 13.

b) 8 + 7 = ? Think:
- 8 + ? = 10.
- 8 + 2 = 10.
- ```
   7
  / \
  2  5
  ```
- Therefore,
  8 + 2 = 10;
  10 + 5 = 15.

TABLE 2

Differences between American and Chinese teaching

American Textbooks 14 (Unknown) Family		Chinese Textbooks 6+ (Known) Fact Table	
1 + 13	13 + 1	6 + 1	6 + 6
2 + 12	12 + 2	6 + 2	6 + 7
3 + 11	11 + 3	6 + 3	6 + 8
4 + 10	10 + 4	6 + 4	6 + 9
5 + 9	9 + 5	6 + 5	
6 + 8	8 + 6		
7 + 7			

The Chinese numerical language shown in **table 1** plays an essential role in this strategy. Moreover, Chinese teachers believe that students should use 10 as a bridge because of its importance in the base-ten numeration system.

Chinese teachers strongly emphasize using addition facts to do subtraction. By doing so, they not only encourage students to apply their previously learned knowledge in the new situation but also help students see how addition and subtraction are related. Consider the following examples:

a) 13 – 5 = ? Think:
- 5 + ? = 13.
- 5 + 8 = 13.
- Therefore, 13 – 5 = 8.

b) 15 – 8 = ? Think:
- 8 + ? = 15.
- 8 + 7 = 15.
- Therefore, 15 – 8 = 7.

Differences in Thinking Strategies

Thinking strategies are emphasized in both America and China in teaching the basic facts (see, e.g., Baroody [1998]), but the way in which these facts are presented is quite different. Many American textbooks arrange the basic facts as families, such as sums of 12, sums of 13, and so forth. The textbooks introduce a variety of strategies, such as counting up, learning doubles, or recognizing double-plus-one and double-minus-one situations (see, e.g., Addison-Wesley Mathematics [Menlo Park, California], Houghton Mifflin Math Central [Dallas, Texas], Harcourt Brace Math Advantage [Orlando, Florida]). To some extent, basic facts are viewed as associations to be memorized through hands-on activities, then recalled on demand. Chinese textbooks arrange the basic facts using fact tables (Curriculum and Teaching Materials Research Institute 1999), and the primary strategy taught is "make 10." Chinese teachers generally introduce the basic facts in units, such as the 6+ unit, the 7+ unit, and so on. These units are categorized by the known entity (addends) instead of the unknown entity (sums). This difference between American and Chinese teaching can be seen in **table 2.**

The fact families from 2 to 18 contain 153 additional facts that American students need to study; the fact table, in contrast, contains only

TABLE 3

Addition facts

1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 5	1 + 5 = 6	1 + 6 = 7	1 + 7 = 8	1 + 8 = 9	1 + 9 = 10
2 + 1 = 3	2 + 2 = 4	2 + 3 = 5	2 + 4 = 6	2 + 5 = 7	2 + 6 = 8	2 + 7 = 9	2 + 8 = 10	2 + 9 = 11
3 + 1 = 4	3 + 2 = 5	3 + 3 = 6	3 + 4 = 7	3 + 5 = 8	3 + 6 = 9	3 + 7 = 10	3 + 8 = 11	3 + 9 = 12
4 + 1 = 5	4 + 2 = 6	4 + 3 = 7	4 + 4 = 8	4 + 5 = 9	4 + 6 = 10	4 + 7 = 11	4 + 8 = 12	4 + 9 = 13
5 + 1 = 6	5 + 2 = 7	5 + 3 = 8	5 + 4 = 9	5 + 5 = 10	5 + 6 = 11	5 + 7 = 12	5 + 8 = 13	5 + 9 = 14
6 + 1 = 7	6 + 2 = 8	6 + 3 = 9	6 + 4 = 10	6 + 5 = 11	6 + 6 = 12	6 + 7 = 13	6 + 8 = 14	6 + 9 = 15
7 + 1 = 8	7 + 2 = 9	7 + 3 = 10	7 + 4 = 11	7 + 5 = 12	7 + 6 = 13	7 + 7 = 14	7 + 8 = 15	7 + 9 = 16
8 + 1 = 9	8 + 2 = 10	8 + 3 = 11	8 + 4 = 12	8 + 5 = 13	8 + 6 = 14	8 + 7 = 15	8 + 8 = 16	8 + 9 = 17
9 + 1 = 10	9 + 2 = 11	9 + 3 = 12	9 + 4 = 13	9 + 5 = 14	9 + 6 = 15	9 + 7 = 16	9 + 8 = 17	9 + 9 = 18

81 facts. When students understand the commutative property of whole numbers, the number of addition facts that they need to know is reduced to only 45 (see **table 3**). Although Chinese and American textbooks arrange addition facts differently, they both use relationships to minimize the amount of information that must be memorized.

When Chinese children learn the basic facts, their task involves not only memorizing but also using logical thinking and reasoning based on relationships among the numbers. Encouraging children to examine a visual aid similar to **table 3** and to look for patterns and relationships can help them devise thinking strategies that can aid in mastering the basic facts (see, e.g., Baroody [1998]).

Chinese teachers also teach different strategies that are not introduced in the textbooks but that can help children see the patterns among the addition facts. Consider the following examples, in which *n* is a whole number:

- For $n + 1$, the sum is the next whole number, that is, the number after *n* in the counting sequence (Baroody 1998).
- For $n + 2$, the sum is the next odd or even whole number.
- The sum of $n + 9$ can be found by adding 10 to *n,* then subtracting 1. This strategy is a shortcut for the make-10 approach discussed previously.

Because they rely on such thinking strategies, Chinese children rarely use manipulatives to figure out facts.

Two other strategies for subtraction are often seen in Chinese classrooms. One is to use 10 as the bridge number in a subtraction equation. Consider these examples:

a) 14 – 9 = ? Think:
- 10 = 9 + (1).
- 14 – 10 = 4.
- 4 + 1 = 5 (because you subtract one more, you need to add one back).
- Therefore, 14 – 9 = 5.

b) 15 – 8 = ? Think:
- 10 = 8 + (2).
- 15 – 10 = 5.
- 5 + 2 = 7 (because you subtract two more, you need to add two back).
- Therefore, 15 – 8 = 7.

The other strategy also uses 10 as a bridge, but it requires students to recall simple addition facts. The following are examples:

a) 13 – 4 = ? Think:
- 13 = 10 + **3.**
- 10 – 4 = **6.**
- **6** + **3** = 9.
- Therefore, 13 – 4 = 9.

b) 16 – 9 = ? Think:
- 16 = 10 + 6.
- 10 – 9 = 1.
- 1 + 6 = 7.
- Therefore, 16 – 9 = 7.

Summary

When using thinking strategies to perform addition and subtraction, students reinforce their understanding about the facts that they have learned by using those facts repeatedly. By the time they finish learning single-digit addition and related subtraction, they can easily recall the addition and subtraction facts and are more than ready to learn the formal algorithms of addition and subtraction. Chinese teachers introduce these strategies as early as first grade (Curriculum and Teaching Materials Research Institute 1999). Students may not be expected to master these strategies in a short time, but if the foundation is laid early, students can apply their knowledge of the basic facts and these strategies to other mathematical content that they will study later.

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