Red supergiants, neutrinos and the Double Cluster

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The Perseus Double Cluster is surrounded by one of the largest concentrations of red supergiant stars in the sky. As a consequence, the development of our understanding of the structure and evolution of these stars has been intimately connected with studies of this cluster. This paper traces the history of this connection from the end of the 19th century through to the early 1970s.

Early visual work

The presence of red stars in the Perseus Double Cluster (= h & Chi Per, = NGC 869 & NGC 884) was remarked on by several visual observers before the end of the 19th century. According to the Rev Thomas Espin,[1] there are references to them in the works of Herschel (presumably William), Heinrich d'Arrest, Admiral Smyth, John Birmingham, the Rev T. W. Webb, the Earl of Rosse, and the Rev T. T. Smith (who counted eight). However, it was Espin himself who carried out the definitive visual study of these stars in the winter of 1891–92.[1] Using a visual spectroscope attached to his 17-inch reflector at Tow Law in County Durham, he identified nine stars of Secchi class III (equivalent to modern spectral class M), all within one degree of the cluster. These were the stars now known as RS, SU, AD, BU, FZ, PR, V403, V439 and V441 Per (see Figure 1). Espin also suspected that one of them (SU) might be a variable star, as he found it to be over a magnitude brighter than it was listed in the *Bonner Durchmusterung*.

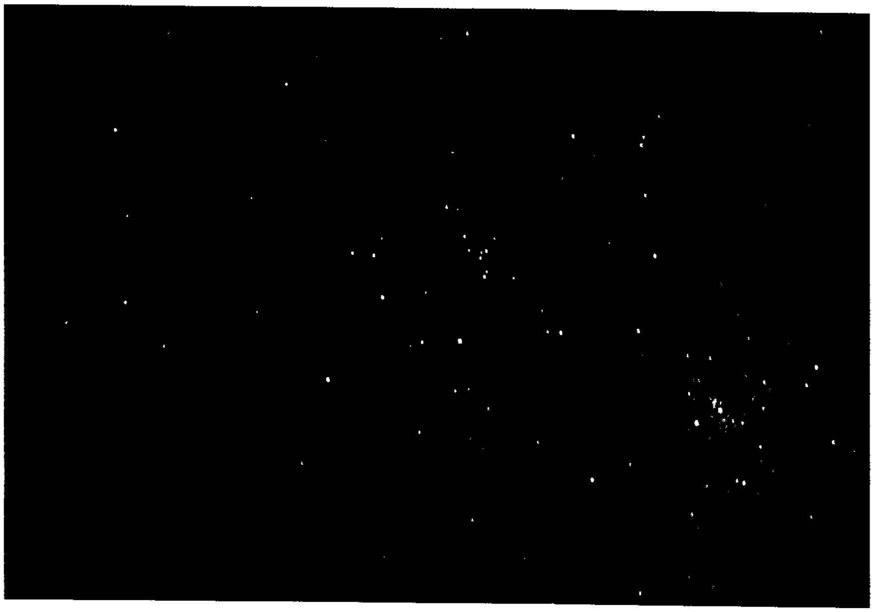

Figure 1. Photograph of the Double Cluster by Geoffrey Johnstone. A 10-minute exposure on hypered 5071 with a 12-inch (305mm) Newtonian reflector. Seven of Espin's nine red stars are shown (compare with Figure 5).

Recognition as supergiants

At the time of Espin's observations, supergiants had not yet been identified as a distinct class of star. This was first done by Antonia Maury in her spectral classification scheme of 1897.[2] As one of her classification parameters she used the thickness of the absorption lines in the spectrum. Stars with broad lines, she assigned to class 'a', stars with normal width lines to class 'b', and those with narrow lines to class 'c'. At that time, she did not know what caused these width differences but she speculated that they might reflect differences in some significant physical property of the stars. We now know that all 18 of the stars that she assigned to class 'c' are supergiants and that the narrowness of their lines is a consequence of their high luminosity. The unusually low densities of the atmospheres of supergiants means that there are fewer collisions between the light-absorbing atoms in them, so the spectral lines are not as smeared out as they are in less luminous stars. (The broad lines of class 'a' stars are not due to low luminosity, but either to rapid rotation or to the stars being unresolved binaries.)

The true significance of the narrow spectral lines was discovered in 1905 by Ejnar Hertzsprung.[3] He was then an amateur astronomer carrying out research in his spare time at an observatory in Copenhagen. In a statistical study of the relationship between the spectral type and the proper motion of stars in Miss Maury's classes, he found that c-line stars had on average the smallest proper motions. This implied that they were, on average, the most distant and, taking their mean magnitude into account, also the most luminous class. This was a tremendous discovery, probably the most important ever made by an amateur astronomer, but unfortunately Hertzsprung published it in a fairly obscure journal (devoted to scientific photography) and it went unnoticed by most astronomers at the time.

Early ideas on their evolution

Hertzsprung followed up his 1905 paper by studying the relationship between spectral type and luminosity. Similar work was also carried out in the USA by Henry Norris Russell using trigonometric parallaxes.[4] This led to the invention of the HR (Hertzprung–Russell) diagram, a plot of stellar luminosity against spectral type (or something closely correlated such as surface temperature or the colour index). The distribution of stars over this diagram, with a 'main sequence' running from hot bright stars down to cool faint ones, and a separate red giant branch, immediately suggested a scheme for stellar evolution. Assuming that gravity was the main energy source in stars (nuclear reactions were then unknown), Russell proposed that stars start out as red giants or supergiants and then move horizontally across the HR diagram contracting and getting hotter until they reach the main sequence. They then slowly cool and fade, moving down the main sequence until they eventually become too faint to be visible. This scheme was similar to one that had been proposed (on somewhat weaker evidence) several years earlier by Norman Lockyer.[5]

Spectroscopic absolute magnitudes

Luminosity-dependent effects in stellar spectra were rediscovered in a slightly different form in 1914 by Walter Adams and Arnold Kohlschütter.[6] They used the relative strengths of various lines in the spectrum to derive approximate 'spectroscopic' absolute magnitudes and hence distances. Initially this method was only applied to stars near to the Sun but it was later extended to cover more distant stars such as supergiants also. In 1926, Adams, Joy and Humason[7] identified seven of Espin's red stars as supergiants (BU and PR were not studied) and noted that their spectroscopic distances, radial velocities and proper motions were consistent with those of the other stars in the Double Cluster. However, the radial velocities of stars in the cluster do not differ very much from those of the background stars in that part of the sky, and their proper motions are very small, so these properties are not of much use in distinguishing members from non-members. For example, in a major survey of the Double Cluster published in 1937, Oosterhoff[8] was only able to exclude a relatively small number of stars from membership on these grounds.

The modern system of indicating luminosity classes by capital roman numeral suffixes to the spectral class (e.g. M2III) was introduced by William Morgan[9] in 1938. He placed the most luminous supergiants in class I (later split into Ia, Iab and Ib) and main sequence stars in class V. This scheme was based on spectral features thought to be sensitive to the surface gravity of the star, so that even though a B-type main sequence star is much more luminous than an M-type one they are both in luminosity class V because their surface gravities are comparable.

The red supergiant halo around the Double Cluster

The calibration of the Morgan luminosity classes for each spectral type against absolute magnitude was carried out by Philip Keenan,[10] one of Morgan's colleagues. In one of the first applications of this calibration, Keenan[11] surveyed the small-amplitude red variable stars in the northern Milky Way and found that there was a distinct clustering of the more luminous ones around the Double Cluster. The spectroscopic absolute magnitudes for five of them (T, RS, SU, YZ and AD) gave distances consistent with cluster membership. William Bidelman[12] emphasised that the diameter of this 'halo' of red supergiants, at about 5 degrees, was significantly larger than the accepted diameter of the cluster. In addition, this 'halo' appeared to coincide with a similar halo of luminous stars of spectral types O and B. This grouping of hot stars was later named the Perseus OB1 Association, the Double Cluster being recognised as a subgrouping within it.

Bidelman[13] went on to carry out a spectroscopic search for more red supergiant members and extended the total to thirteen (adding S, T, YZ and KK Per to Espin's original nine). In the 1950s Victor Blanco[14] and Stewart Sharpless[15] identified seven more amongst the outliers.

Red supergiants and spiral arms

The most spectacular result of Morgan's work on spectroscopic luminosities was his discovery of the spiral arms of our own galaxy.[16,17] The story of this discovery and its reception is told by Henbest and Couper in their recent book on the structure of our Galaxy.[18] One night in 1951, while walking to his observatory, Morgan looked up at the Milky Way in Perseus and Cassiopeiae and realised that the distances that he had obtained for the hot luminous stars in that region indicated that they were arranged in a band at right-angles to the line of sight, about 2kpc (kiloparsecs) distant. This band of hot stars included the Perseus Double Cluster and consequently became known as the Perseus Arm.

Nassau, Blanco and Morgan[19] later found that the cool, red supergiants can also be used to trace the spiral arms. Indeed, with infrared spectroscopy these stars can be identified out to even greater distances than the hot stars. This is because infrared radiation penetrates the interstellar clouds better than visual light does. This is the reason that many of the later studies of red supergiants were carried out in the infrared.

Red supergiants as post main sequence stars

Two developments during the 1940s had laid open the way for major advances in the understanding of stellar evolution. The first was the discovery that nuclear reactions were the main energy source in stars, and the second was the appearance of computers which would allow complicated stellar models to be calculated. Models of stars burning hydrogen in their cores were found to be located along the main sequence on the HR diagram, with the more massive ones towards the luminous end. These more massive stars burnt up their hydrogen more rapidly than the less massive ones. When the hydrogen in their cores was exhausted, their positions on the main sequence became unstable and it was

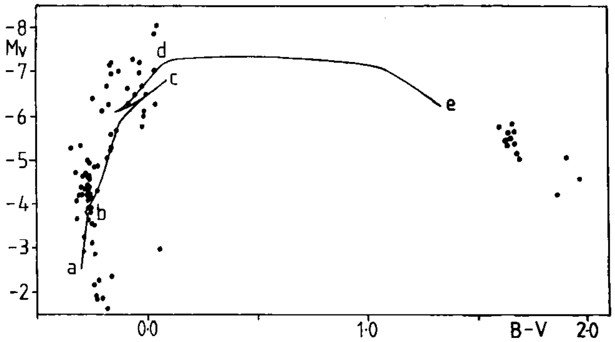

Figure 2. The evolutionary track of a star of 15.6 solar masses according to Hayashi and Cameron[25] plotted onto an HR diagram of the Double Cluster. The vertical scale, M_v, is the absolute magnitude and the horizontal scale is the B–V colour index. Both of these have been corrected for the effects of interstellar absorption. From a to b the star is burning hydrogen in its core, from c to d it is burning helium, and at e carbon burning starts. The times spent in each stage are as follows: a–b = 15.6My, b–c = 0.21My, c–d = 1.13My, d–e = 'very brief'. Note the clustering of stars around the parts of the track where core hydrogen and helium burning occurs.

assumed that they would then move off to some other part of the HR diagram, but where they would go to was not then known.

In the mid-1950s, Edwin Salpeter[20] made some simple assumptions and calculated that, of all the stars created in the solar region since the formation of the Galaxy, about 12 percent must have since exhausted their hydrogen and so left the main sequence. He estimated the average initial mass of these post main sequence stars to be about 3 solar masses, but there were no suitably massive candidates for the relics of these stars visible in the sky today. What could have become of them? Salpeter suggested that they might have evolved into white dwarfs, but he also pointed out that the maximum mass of white dwarfs is only 1.4 solar masses so that they could at best only provide part of the solution to this problem.

At about the same time, Armin Deutsch[21] was studying the spectrum of the red supergiant Alpha Herculis and found features that indicated it was losing mass at a very high rate in a stellar wind. Deutsch put two and two together and proposed[22] that when stars leave the main sequence they move over to the red giant or supergiant regions of the HR diagram, and there they lose about half of their mass in a stellar wind before settling down to become white dwarfs. He envisaged that the most luminous stars on the main sequence, those of spectral types O and B, move horizontally across to become red supergiants, while the fainter A-type main sequence stars become red giants. This idea was supported by the presence of both blue and red supergiants together in the Double Cluster and in Perseus OB1. It was also consistent with the then recent discovery of white dwarfs in the Hyades, a cluster which contained red giants at about the same luminosity as the top end of its main sequence.

The neutrino problem

Calculations of the evolution of stellar models tended to confirm Deutsch's scheme. For example, Chushiro Hayashi and co-workers published a series of papers[23,24,25] in which they followed the evolution of a star of 15.6 solar masses through core hydrogen exhaustion, on through core helium burning, and beyond to core carbon ignition. They found that the core helium burning occurs when the star is still a blue supergiant, a little above and to the right of the main sequence (see Figure 2). After helium exhaustion in the core, the star rapidly moves off towards the red supergiant region where carbon burning starts. Various simplifying assumptions had to be made, especially in the later stages, to make the computations tractable but a comparison of this track with the HR diagram of the Double Cluster strongly suggests that red supergiants must be either burning carbon in their cores or else at a later stage of evolution.

However, in spite of the apparently good fit of this track to the HR diagram, there was a major problem with it. A recent advance in particle physics made by Richard Feynman and Murray Gell–Mann[26] had suggested the existence of direct interactions between neutrinos and electrons. If these interactions were included in the evolutionary models they would speed up the carbon-burning and later stages so much that red supergiants would be very short-lived indeed, and hence much rarer than blue supergiants. The presence of almost equal numbers of red and blue supergiants in the Double Cluster directly contradicted this. This led Hayashi, Hoshi and Sugimoto[27] to suggest that the electron-neutrino interaction might somehow be 'forbidden' in nature. But over the next few years evidence for the interaction accumulated from other areas of physics and it became apparent that there must be something wrong with the red supergiant models.

The structure of Perseus OB1

Star clusters have been very important for testing theories of stellar evolution because one can usually assume that in

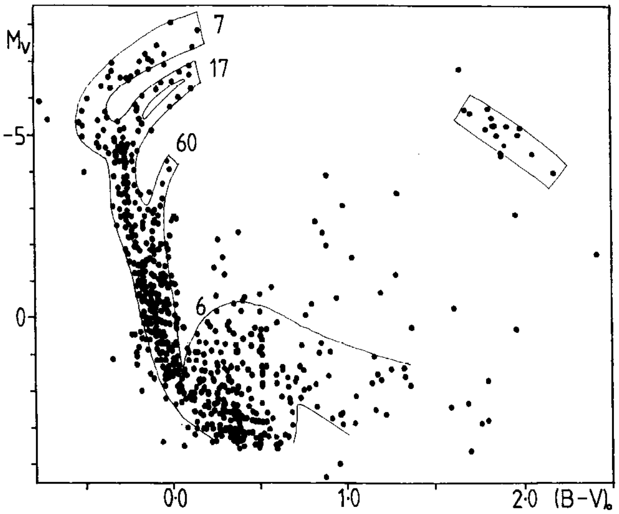

Figure 3. The HR diagram of the Double Cluster according to Wildey.[28] Wildey's estimates of the ages of the various branches on the upper main sequence are indicated (in millions of years). In addition the upper envelope on the stars still contracting onto the lower main sequence corresponds to an age of about 6 million years. The evolutionary track of Hayashi & Cameron[25] (see Figure 2) suggests that the band of red supergiants in the top right-hand corner is associated with the 17 My branch.

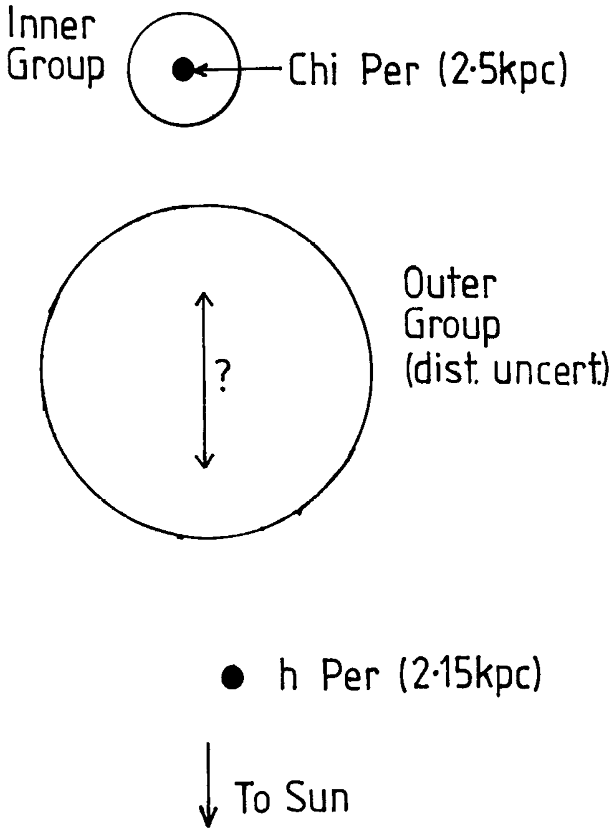

Figure 4. The structure for Perseus OB1 suggested by Schild.[30] The O-star population is about the same size and at about the same distance as the Outer Group.

Table 1. Stellar populations in the Double Cluster identified by Schild[30]

Name	*Age My*	*Diameter pc*	*Distance pc*
O-stars	'Very young'	200	2300?
Outer Group	6.4	200	2300?
h Per	6.4	15	2150
Inner Group	11.5	65	2500
Chi Per	11.5	20	2500

a given cluster all of the stars are about the same distance and age, and all started out with roughly the same chemical composition. Observed differences between members of the cluster must therefore be due to other factors such as differences in their initial masses. One might expect that the fact that the Double Cluster consists of two clusters would therefore make it unsuitable for evolutionary studies. However, the stars making up the two clusters are so similar and are so intermingled that up until the 1960s most researchers treated them as a single group. This was in spite of several studies having consistently found h Per to be slightly closer than Chi.

In 1964 Robert Wildey published[28] an extensive photometric study of the Double Cluster and interpreted the HR diagram he obtained as indicating at least three distinct ages for the stars: 7, 17 and 60 million years (see Figure 3). Soon afterwards, a study by Rudolph Schild[29] of Be stars in the nuclei of the two clusters suggested that h Per was slightly younger as well as slightly closer than Chi. Schild followed this up with a comprehensive review of all the data on the ages and structure of the stars in the Double Cluster.[30] He identified five distinct populations of stars of three different ages. The locations and properties of these populations are given in Table 1 and Figure 4. Schild found that the red supergiants were associated with Chi Per, the 'Inner Group' surrounding Chi, and also with the 'Outer Group'. However, those associated with the Outer Group tended to be more obscured by interstellar absorption than the others, which suggests that they are more distant and may actually be background stars in the Perseus Arm, at a distance of about 3.5kpc.

In the late 1960s Roberta Humphreys carried out a comprehensive survey[31] of the red supergiants in the Perseus Arm. From infrared spectra she derived spectral types, luminosity classes and radial velocites, and then she used these to assign the stars to the various stellar associations in the Perseus Arm. Those stars that she assigned to Perseus OB1 are listed in Table 2. These are distributed over quite a large area of sky (see Figures 5 and 6) and, as mentioned above, some may be background objects. Others may be members of neighbouring associations, for example, NSV 687 could be a member of Cassiopeia OB8 which includes NGCs 581, 654 and 663. Humphreys also suggested that Schild's Outer and Inner Groups were large enough to enclose both of the clusters (see Figure 7). Although she does not explicitly mention it, her data suggests that both of these groups are elongated in the line of sight, being about 200pc wide and 500pc long.

The solution of the neutrino problem

A solution to the 'neutrino problem' was proposed in the late 1960s by Richard Stothers and Chao-wen Chin.[32] They

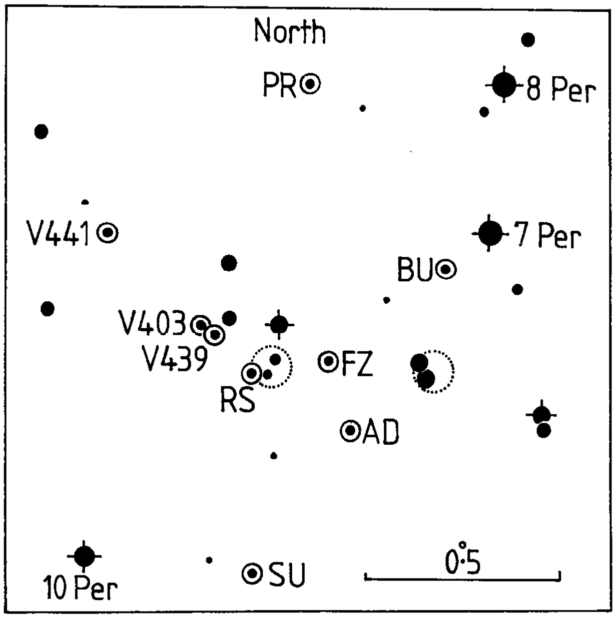

Figure 5. The inner red supergiants of Perseus OB1 according to Humphreys.[31] Chi Per is to the left and h is to the right.

Table 2. The red supergiants of Perseus OB1 according to Humphreys[31]

Name	*HD or BD*	*RA*	*(1950)*	*Dec*	*RA*	*(2000)*	*Dec*	*Magnitude*		*Type*	*Period*	*Spectrum*
		h	*m*	° ′	*h*	*m*	° ′				*d*	
	236915	01	55.0	+59 01	01	58.4	+59 16	8.3	v			M2Iab
NSV 687	+59.372	01	56.2	+60 01	01	59.7	+60 15	11.8	p	Cst?		K5–M0Ib
XX Per	12401	01	59.8	+55 00	02	03.1	+55 14	8.2 – 10.2	p	SRc	415	M4Ib + B
	236947	02	03.6	+58 33	02	07.2	+58 47	8.65	V			M2Iab–Ib
KK Per	13136	02	06.8	+56 19	02	10.2	+56 34	6.6 – 7.89	V	Lc		M1.0Iab–M3.5Iab
	13658	02	11.6	+57 54	02	15.1	+58 10	8.92	V			M0.5Ib
PP Per	+57.530a	02	13.6	+58 18	02	17.1	+58 32	9.10 – 10.30	V	Lc		M0.0–M1.5Iab–Ib
BU Per	+56.512	02	15.4	+57 12	02	18.9	+57 25	10.4 – 12.3	p	SRc	367	M3.5Ib
T Per	14142	02	15.8	+58 44	02	19.4	+58 58	8.34 – 9.7	V	SRc	2430	M2Iab
V605 Cas	14242	02	16.7	+59 27	02	20.4	+59 40	8.22 – 8.48	V	Lc		M2Iab
AD Per	14270	02	17.0	+56 46	02	20.5	+57 00	9.7 – 11.2	p	SRc	362.5	M2.5Iab
FZ Per	14330	02	17.6	+56 56	02	21.0	+57 10	9.8 – 10.77	B	SRc	184	M0.5Iab–M2.0Iab
PR Per	14404	02	18.1	+57 38	02	21.7	+57 52	9.8 – 10.8	p	Lc		M1Iab–Ib
SU Per	14469	02	18.6	+56 23	02	22.1	+56 36	9.4 – 10.8	p	SRc	533	M3.5Iab
RS Per	14488	02	18.9	+56 53	02	22.4	+57 07	7.82 – 10.0	V	SRc	244.5	M4Iab
S Per	14528	02	19.3	+58 22	02	22.8	+58 35	7.9 – 12.0	v	SRc	822	M3Iae–M7
V439 Per	+56.595	02	19.6	+56 58	02	23.2	+57 12	8.03 – 8.49	V	Lc		M0.5Iab
V403 Per	14580	02	19.9	+56 59	02	23.4	+57 03	8.31 – 8.50	V	Lc?		M0Iab
V441 Per	14826	02	21.8	+57 13	02	25.4	+57 26	8.19 – 8.53	V	Lc		M2.5Iab
NSV 824	+60.478	02	23.7	+60 29	02	27.4	+60 43	11.68 – 11.92	V	Var?		M2Iab
YZ Per	236979	02	34.8	+56 50	02	38.4	+57 03	10.01 – 11.9	B	SRc	378	M1Iab–M3Iab
GP Cas	+57.501	02	36.1	+59 23	02	39.8	+59 36	11.5 – 12.7	p	Lc		M2.0Iab
	+59.540	02	43.1	+59 25	02	46.9	+59 38	9.5	v			K5I? + AIII?
	17306	02	45.2	+53 58	02	48.7	+54 10	7.93	V			K3Iab + B?
NSV 929	237006	02	45.4	+57 48	02	49.1	+58 01	9.14 – 9.19	V	Var?		M1Ib + B?
W Per	+56.724	02	46.9	+56 47	02	50.6	+56 59	8.7 – 11.8	v	SRc	485	M3Ia–Iab–M7
V648 Cas	+57.647	02	47.3	+57 39	02	51.1	+57 51	11.8 – 12.6	p	Lc		M2Iab

used a more refined treatment of opacity and convection in their evolutionary models and discovered that massive stars could actually reach the red supergiant region while still burning helium in their cores, but only if their initial masses were not much more than 15 solar masses. (In Figure 2 this would mean that the cusp at point c would extend all the way across to the red supergiant region on the right-hand side). Burning helium in their cores, instead of carbon, these stars could stay in the red supergiant region long enough to explain the high ratio of red to blue supergiants in the Double Cluster. Stothers followed this up with a series of detailed studies[33,34,35] (one in collaboration with Kam-Ching Leung) of the properties of red supergiants in which he relied heavily upon observational data from the Double Cluster. These, and a study of the red supergiants in four southern clusters by Schild,[36] confirmed that only the lower mass supergiants spend a significant part of their lifetimes as red supergiants.

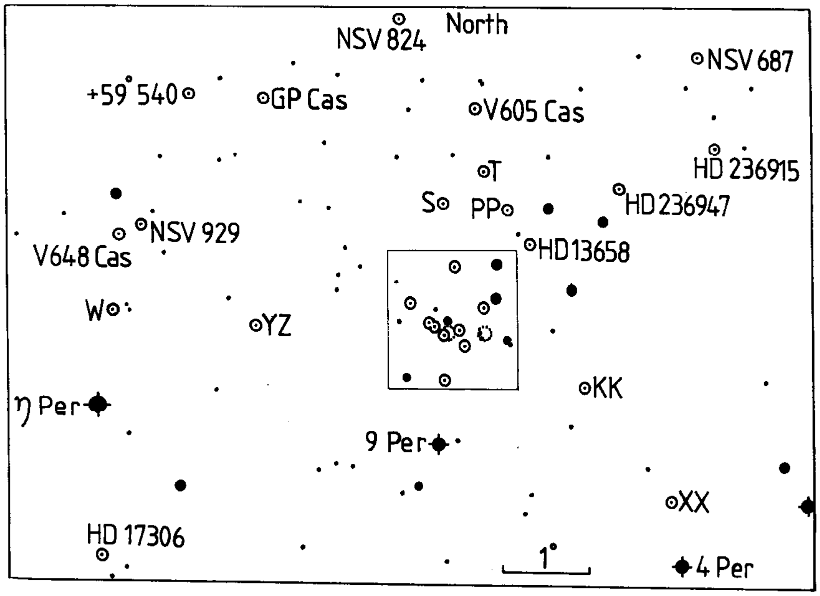

Figure 6. The outer red supergiants of Perseus OB1 according to Humphreys.[31]

In his paper, Schild presented the classification of stellar populations containing supergiants that is shown in Table 3. Note that what he calls the 'old' supergiant population is still relatively young on the scale of stellar ages. Clusters such as the Pleiades and the Hyades are much older, but they are not included because any supergiants they once contained have long since burnt themselves out. Schild's populations are characterised by the types of blue and red supergiants that they contain and by their 'turn-ups'. The turn-up is the point on the HR-diagram at which the line of stars leaving the main sequence becomes vertical. For example, the turn-up of the 7My branch in Figure 3 is at $M_v = -5.3$, B–V= –0.5. In Table 3 the turn-up is specified by the horizontal coordinate (in the form of the spectral type) alonc. Red supergiants are only common in the old and intermediate populations such as Chi Per, and are rare in younger ones such as h Per and the Perseus OB1 O-star population. But this is not to say that red supergiants never occur in the younger populations. Indeed, Schild suggests that YZ Per (the brightest red supergiant in

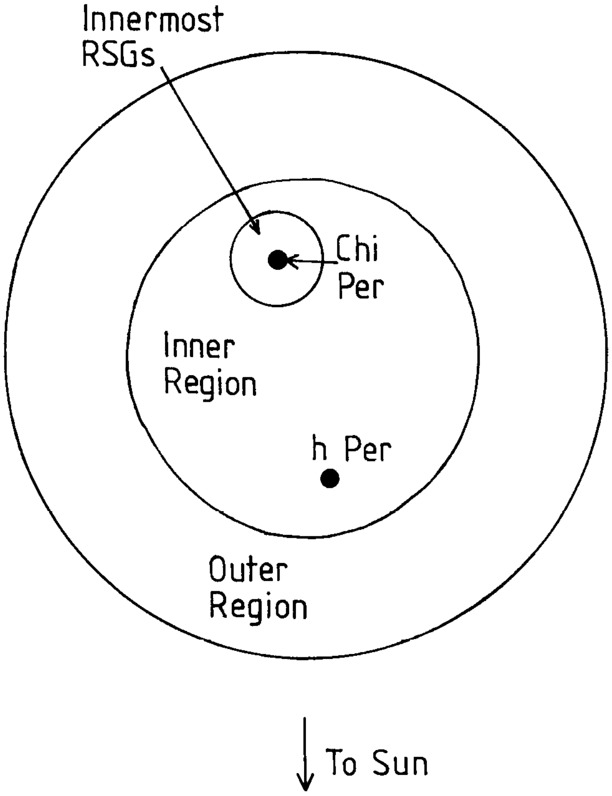

Figure 7. The structure for Perseus OB1 suggested by Humphreys.[31]

Figure 3) may be a member of the 'young' population in h Per and the Outer Group on the basis of its high luminosity.

Conclusion

By 1971, Stothers and Leung[35] could write that little doubt remained that most red supergiants were burning helium in their cores rather than carbon. In the next few years observational interest seems to have moved away from the Double Cluster and towards the red supergiants in the Magellanic Clouds and nearby galaxies. No doubt this was in part due to improvements in detectors which made these fainter objects accessible, but it also reflects the way that observation is driven by a 'good problem' and when that problem is solved then the motivation for observing disappears. In the 1960s the evolutionary status of red supergiants was a 'good problem' and observation of the Double Cluster was the best way to test possible solutions.

Table 3. Schild's classification of supergiant populations[36]

Name	*Age My*	*Turn-up*	*Blue supergiants*	*Red supergiants*	*Examples*
Early O-star	3.7	O4-O5	O8-O9Ia	Few or none	Per OB1 O-stars
			Scattered Ia+		Sco OB1 blue branch
Very young	5.4	O9-O9.5	B0-B0.5Ia	Few or none	Orion's Belt Sco OB1
Young	7	B0-B1	B0Iab B2-B3Ia	Few or none	h Per M29
Intermediate	13	B1-B2	B2Ib B5-A5Ia-Iab	M0-M5Ia-Ib	Chi Per NGC 4755
Old	20	B2-B3	B6-A0Ib-II	K5-M0Ib-II	NGC 3766 NGC 457?

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