The periods of Mu Cephei

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A contribution from the Variable Star Section (Director: G. Poyner)

Period analysis of BAA-VSS and JAS/SPA observations of Mu Cephei shows that two main periods of 850 and 4400 days were present during the years 1959–1993. This result is in good agreement with periods found in earlier data. It is likely that the 730-day period given in the GCVS and that of 920 days found in subsets of the data are aliases of the 850-day period.

Introduction

According to Humphreys,[1] Mu Cephei is one of the most luminous red supergiant stars in our galaxy. Its absolute visual magnitude (Mv) of –8.0 makes it significantly more luminous than the other naked-eye red supergiants Betelgeuse (Mv= –5.9), Antares (–5.3) and Alpha Herculis (–2.9). Its position and reddening are consistent with its being a member of the Cepheus OB2 Association at a distance of 800 parsecs (this association includes the stars Nu, Lambda, 14, 19 and VV Cephei, as well as the open cluster Trumpler 37). However, its radial velocity is rather more positive than the rest of the association, so there does remain some doubt about its membership.

Like many other red giant and supergiant stars, Mu Cephei shows variations in brightness that are loosely periodic. In the *General Catalogue of Variable Stars*[2] (GCVS) it is listed as a semiregular supergiant variable (type SRc) with a spectral type of M2eIa, a visual range of 3.43–5.1, and periods of 730 and 4400 days.

Previous period studies

The variability of Mu Cephei was discovered in 1848 by J. R. Hind and confirmed soon afterwards by F. W. A. Argelander. For the next three decades only fragmentary observations are available, but from 1881 onwards the coverage is almost complete, thanks largely to an epic 55-year run of 5275 observations by the German visual observer Joseph Plassmann. Such a long continuous run of observations by a single person has been particularly valuable to later researchers because it is largely free from the observer-dependent errors that normally plague visual observations of red stars. As a consequence, Plassmann's data has been central to practically all subsequent studies of the periodicity of this star.

One of the earliest of these studies was that published in 1939 by Hassenstein.[3] He collected all available observations from 1848 to 1937, 'homogenised' them by applying observer corrections, and found semiregular variations with periods of about 730, 904 and 5000 days. Perhaps more important, as well as publishing his conclusions, Hassenstein also published the data on which they were based, thus allowing later researchers to repeat the analysis as more advanced techniques became available.

Ten years later, Balasoglo[4] carried out a restricted form of periodogram analysis (using only 38 trial periods from 70 to 5000 days) on the subset of Hassenstein's data from the years 1881–1935 (the interval covered by Plassmann). He found periods of approximately 700, 900, 1100 and 4500 days. Balasoglo's analysis was later criticised by Ashbrook

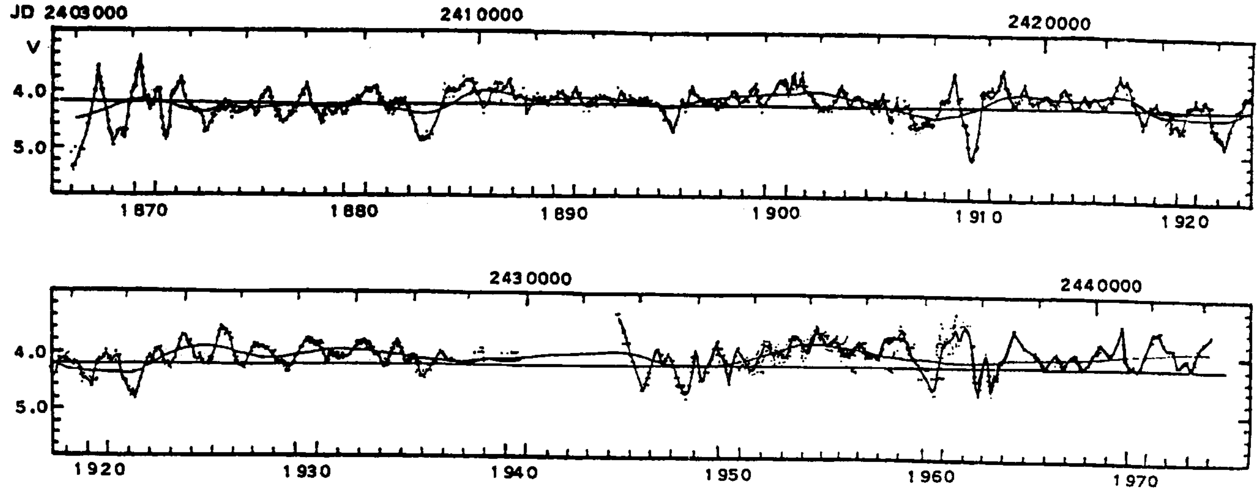

Figure 1. Polyakova's lightcurve of Mu Cephei for 1866–1973. Freehand lines have been drawn through the observational points to delineate the course of the short-term and longterm variations. The horizontal line at mag 4.2 marks the mean brightness of the star.

et al.[5] on the grounds that periodogram analysis assumes that any periodic variations present are stable. This need not be so, for example if the variations were due to randomly-triggered damped pulsations, or to randomly-occurring spots on a rotating star. Ashbrook *et al.* instead used autocorrelation to analyse the data and, finding only the 900-day period, they dismissed most of Balasoglo's results as 'without significance'.

Table 1. Observer totals

Observer	*No. of obs.*	*Observer*	*No. of obs.*
S K Abrol	137	J E Isles	491
J E Agar	113	R A Kendall	62
S W Albrighton	237	J W Kent	20
L Baker	75	D Lenihan	74
B J Beesley	72	T Markham	223
D B P Beglan	152	L R Matthews	56
M Beveridge	67	A D Mayer	76
P Bowers	286	I A Middlemist	58
T Brelstaff	167	B R M Munden	185
J S Bullivant	88	I P Nartowicz	102
R J Butler	110	J Phillips	711
A Chapman	50	G Pointer	154
A Cook	83	H Robinson	73
G Cowie	152	D Stott	757
H G Duncan	227	M D Taylor	374
R B I Fraser	128	N Taylor	117
R J Geddes	156	J Toone	283
M A Hapgood	56	B Wells	66
M Harris	166	G Winstanley	55
I D Howarth	57	103 others*	1653
D Hufton	145	Total	8503

*The 103 other observers, each of whom contributed fewer than 50 observations, were: A F Alexander, C M Allen, M Bailey, N R Baker, M Barrett, S Beaumont, R Belcher, P Bibbings, R Billington, A Brown, G B Chaplin, M Clarke, P R Clayton, J Corey, M J Currie, I Dutton, D Duvall, S J Evans, M Foulkes, V J Freeman, D Fry, A Gardner, P J Garner, F W Gibson, S Godwin, B A L Green, F Gribbin, F Hamilton, R S Henderson, C Henshaw, J C Hilder, A J Hollis, P W Hornby, E H Horsley, A Horton, I Howard-Duff, A C Hughes, G M Hurst, S Jenner, A Jennings, D B Kazansky, D Keir, B Kelly, R S Kirby, P G Kitchingman, S Koushiappas, P Langridge, G H Lindop, Loh Chee Seng, R S Lomas, M Long, T Lubek, G Manuell, R J Marshall, D Martin, J W Mason, M J Maunder, J McCue, R McKay, R H McNaught, A O Miller, R Minty, P A Moore, P D Noble, B O'Halloran, C Panayiotides, J Peck, H Perkins, R D Pickard, D A Pickup, H S Piper, I Poplett, R Potter, A R Pratt, M J D Price, P Quadt, C V Reeves, M Reynolds, N Richardson, S G Ridley, M J Ring, J A Roberts, M Robinson, M G Savage, A M Savill, A Smeaton, A Smith, A L Smith, R M Steele, R Steer, N Stern, P G Sutherland, S Taylor, G Thompson, A J Thomson, G Tiller, M Walsh, S T Wanstall, G S Warbey, J M Webb, W O Williams, J D Wise, D Young.

However, the use of periodogram analysis on semiregular variables was rehabilitated in 1966 by Sharpless *et al.*[6] who pointed out that the analysis of Ashbrook *et al.* only covered periods up to 2000 days. When they computed the correlogram up to 16000 days, they found good evidence for stable periodicity at around 4400 days. They then used this to justify a detailed periodogram analysis which revealed significant peaks corresponding to a main period at about 4836 days and its first 6 harmonics (that is, at 1/2, 1/3, 1/4, 1/5, 1/6 and 1/7 of the main period). Though Sharpless *et al.* did not mention it, the presence of these harmonics indicates that the main variation is significantly non-sinusoidal.

More modern observations were included in the 1975 study by Polyakova[7] which covered the years 1866–1973. Figure 1 shows the resulting lightcurve. She found periods of 730, 920 and 1280 days, and her periodogram also shows one at about 4800 days. Polyakova found that the values and the amplitudes of these periods varied between different sections of the lightcurve. She concluded that the light variations were probably not related to the rotation of the star.

In 1978, Kluys[8] used linear digital filters to reanalyse Hassenstein's data and found periods of 720 and 4400 days. He interpreted the shorter period as the axial rotation period of the star and the longer one as due to cyclic global variations in local disturbances in the star's atmosphere.

When Mantegazza[9] re-examined all of the available visual observations to 1982, he found that the post-1935 data contained 'large systematic errors between different observers which are nearly of the same order as the searched variations'. He therefore restricted his analysis to only the 1881–1935 data that had already been analysed by Balasoglo, Ashbrook *et al.* and Sharpless *et al.* He found two periods, 880 and 4850 days, and also noticed that the harmonics found by Sharpless *et al.* make the longer period variation cepheid-like, with a sharp rise and a slower fade. This, he suggested, meant that the longer period was probably the star pulsating in the fundamental mode, rather than being the timescale of the turnover of convective cells in the star's atmosphere, as had been proposed by Stothers and Leung.[10] The 880-day variation would then be the first overtone. Mantegazza also found peaks in the periodogram that may be due to non-linear coupling between the two main periods.

The observations

Mu Cephei was added to the observing program of the BAA Variable Star Section (BAA-VSS) in 1974. Before then it had already been under observation by members of the Variable Star Section of the Junior Astronomical Society (JAS) since 1959. The JAS has since been renamed the Society for Popular Astronomy (SPA) and its observations of Mu Cephei have been incorporated into the BAA-VSS Computer Archive where they form a significant proportion of the data on this star. Also included in the Archive are a smaller number of observations by the Binocular Sky Society from the years 1971–1974. In all, 8503 visual observations covering 1959–1993 were available for analysis. The names of the observers and their observation totals are listed in Table 1.

One difficulty with the observations in the BAA-VSS Computer Archive was that the various organisations adopted different values for the comparison star magnitudes at different times in their histories. This problem was partially overcome by re-reducing the magnitudes using a coherent set of comparison star magnitudes. These magnitudes were derived from photoelectric B and V measures using the formula of Howarth and Bailey:[11]

$$v = V + 0.16(B-V)$$

These v-magnitudes are better than V-magnitudes for visual work because they more closely match what is seen by the dark-adapted eye. Among the comparison stars used, the difference v–V ranges from –0.02 to +0.25 mag.

Unfortunately, most of the pre-1975 JAS observations do not include light estimates and so could not be re-reduced in this way. However, up to 1969 the JAS used Harvard magnitudes and these match the derived v-magnitudes quite well. But, from 1969 to 1975 the JAS used V-magnitudes and, as a consequence, their observations in this interval probably contain a small systematic difference with respect to the other observations. It would have been possible to try to correct these observations in some other way, but this would have involved guessing the comparison stars used in each observation, or else averaging the corrections in some way, so this was not attempted.

The observations were then used to plot the lightcurve shown in Figure 2. Variations on a timescale of several years are clearly visible. However, the observations do contain rather a large amount of scatter. The usual errors and biases associated with visual observations are probably

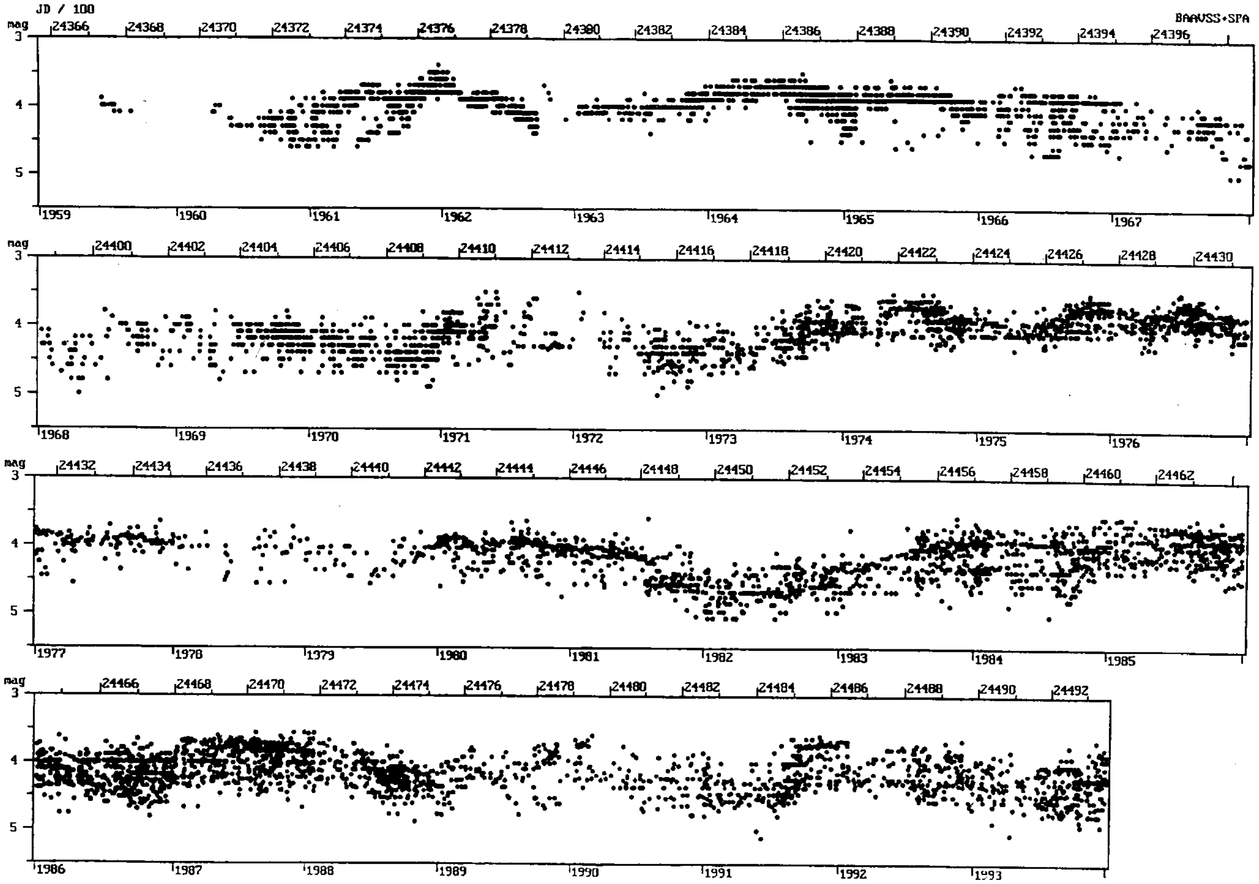

Figure 2. Lightcurve of the individual raw observations.

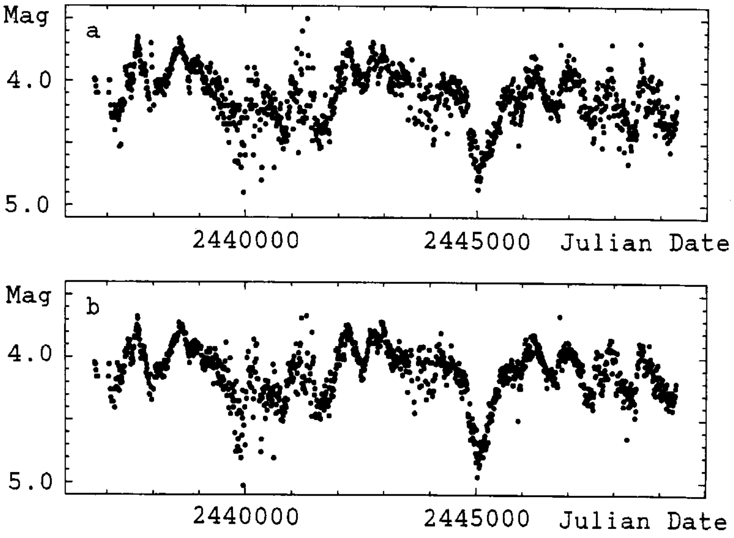

Figure 3. **a).** Lightcurve of 10-day means of the raw observations. **b).** Lightcurve of 10-day means of the observations corrected for observer bias.

accentuated by the strong red colour of the star. The long-term variations show up more clearly when 10-day means are plotted instead (see Figure 3a).

In previous studies a significant part of the scatter was found to be due to systematic differences between observers. In an effort to reduce this, all the observations have been corrected for the observer-dependent personal bias. The procedure is a relatively crude device for removing what is a subtle and complex effect. In essence it is assumed that each observer makes estimates systematically bright or faint, and their observations are allowed to shift up or down to minimise the scatter in the lightcurve. Correcting the observations in this way can significantly improve the quality of the lightcurve and thereby, the detection of periodic variations.[12] The corrected mean lightcurve, given as 10-day means, is shown in Figure 3b and is a distinct improvement on the uncorrected mean lightcurve (Figure 3a) for most of the period covered by the observations. Interestingly, the only exception is during the period 1969–1975 when, as mentioned earlier, the observations could not be re-reduced to a common scale. The histogram of personal biases applied to the observations is shown in Figure 4. Although the maximum bias reaches about 0.5 mag, more than 85% of the observers have corrections of less than 0.25 mag.

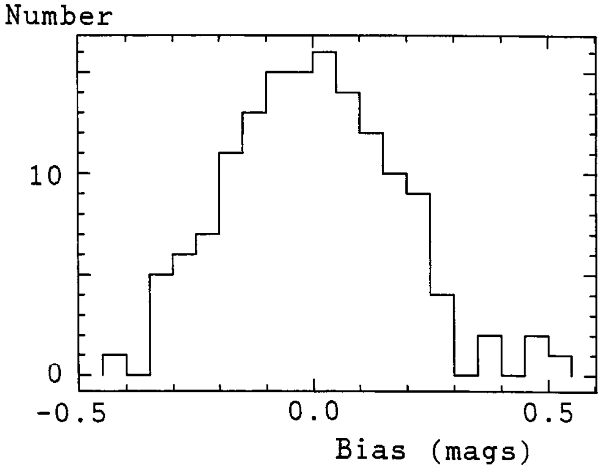

Figure 4. Frequency distribution of the observer bias.

The mean lightcurve

The most obvious periodic feature of the lightcurves in Figure 3 is the sawtooth wave associated with the shorter period. On occasions this is extremely well defined and has all the appearance of a pulsational variation. However, for some cycles this variation is lost in the noise or apparently absent. The longer period is not so clear but the sharp eclipse-like minimum visible in Figure 3 may be traced back, with some variation, through many cycles in Figure 1. According to Mantegazza the longer period is vaguely Cepheid-like in shape, with considerable overlying structure, but none of this is apparent from the lightcurve. In multi-periodic stars the component periods are often difficult to identify in the lightcurve as the complex interaction distorts their shapes.

Periodogram analysis

In order to investigate the periodicity of these variations a periodogram was computed by the classical Fourier transform method of Deeming.[13] Frequencies from 0.00 to 0.02 cycles per day (equivalent to periods from infinity down to 50 days) were covered. The periodogram was computed using the individual raw observations (not the 10-day means, nor the observations corrected for observer bias). Fisher's method of randomisation, as described by Nemec and Nemec,[14] showed that the 5% significance level of the periodogram was 0.018 mags. This level is the semiamplitude that the highest peak in the periodogram would be expected to reach in 5% of trials if the observed variation in the magnitudes was solely due to random noise. It does not mean that all peaks that exceed this level must necessarily correspond to true periods, but it does mean that any below this level can be discounted as there is a significant

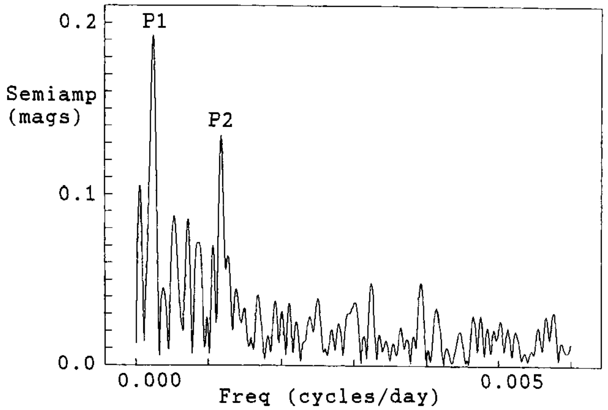

Figure 5. Periodogram of the individual raw observations.

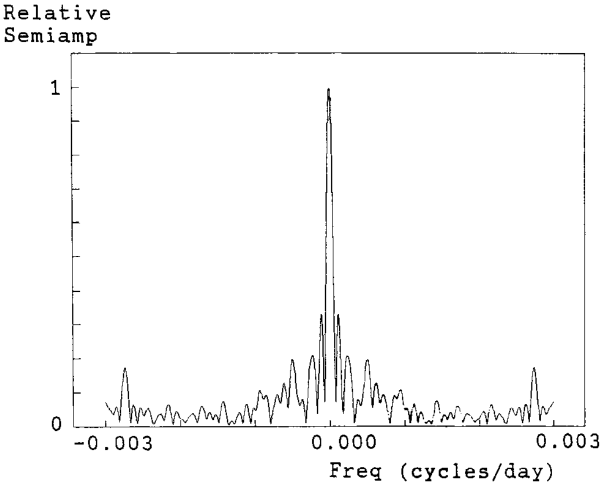

Figure 6. The spectral window function of the individual observations.

chance (more than 1 in 20) that they could have arisen from the noise alone.

Practically all of the peaks significant at the 5% level were at frequencies of less than 0.006 cycles per day (there were two significant peaks above this frequency but these were only just significant). The periodogram in Figure 5 therefore shows only frequencies below 0.006 cycles per day. The two main peaks, labelled P1 and P2, correspond to sine waves with periods of 4403 and 850 days, respectively. The properties of P1 and P2, determined simultaneously, are listed in Table 2. The numbers in parentheses are standard errors.

The spectral window function of the observations, which is shown in Figure 6, has side peaks at ±0.0027 cycles per day which reflect a tendency for the observations to clump at one year intervals. Generally there are fewer observations each year in the spring and early summer, when Mu Cep is low in the sky from the UK, and the nights are short. There are also side-peaks closer in to the main peak which are due to longer-term variations in the density of the observations over the years. The spectral window function can be used to identify alias peaks in the periodogram. For example, from it one can deduce that the 0.048 mag peak at 0.0039 cycles per day in Figure 5 is probably a 1-year alias of P2 and that P2 is not an alias of P1.

However, a better way of identifying aliases is to prewhiten the observations by subtracting the main sine waves from them, and then to recompute the periodogram. Those peaks which are aliases of the main peaks should then disappear or else be greatly reduced. Figure 7 shows the periodogram of the observations prewhitened in this way. The reduction in the peak at 0.0039 cycles per day confirms that it is indeed an alias on one of the main peaks. Several other peaks have also been removed but there still remain many 'significant' peaks at low frequencies. Some of these may represent true periods but it could be that they are just 'modelling' irregularities in the lightcurve. In the present data only P1 and P2 can be identified with any confidence. The time span of the present observations limits the resolution of the periodogram so that it is difficult to comment on Mantegazza's findings of harmonic and non-linear terms. However, it is possible that the main features remaining after the subtraction of P1 and P2 could be identified as P1/3 and P1/4.

Table 2. Properties of the two highest peaks in the periodogram

Peak	*Frequency (cycles/day)*	*Semiamplitude (magnitudes)*	*Epoch of min (JD Hel)*	*Period (days)*
P1	0.0002271	0.168	2445022	4403
	(0.0000009)	(0.004)	(16)	(19)
P2	0.0011769	0.110	2443270	849.7
	(0.0000014)	(0.004)	(5)	(1.1)

Autocorrelation

Another way of searching for periodic variation in the lightcurve is to calculate the autocorrelation function (ACF) of the observations. The ACF works by comparing the lightcurve with a copy of itself which is progressively shifted in time. Where repeated features match, the degree of correlation will be high and where they do not, it will be low. In Figure 8 the ACF of the 10-day means of the observations corrected for observer bias is shown for shifts (lags) up to 10000 days, that is about two-thirds of the time-span of the data. The increase in correlation with the two periods at about 800 and 4000 days is clear although their

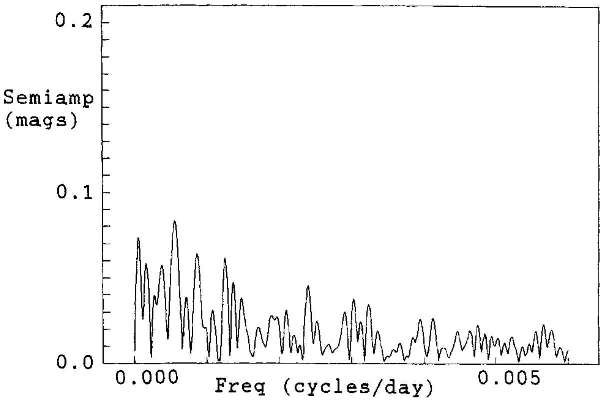

Figure 7. Periodogram of the individual raw observations prewhitened by the removal of the two main periods, P1 and P2.

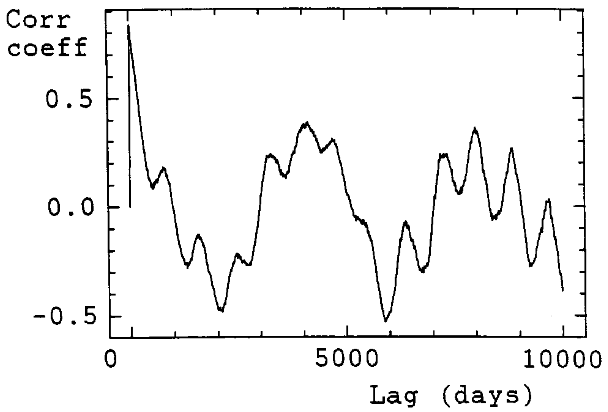

Figure 8. Autocorrelation function of the 10-day means of the observations corrected for observer bias.

interaction makes precise measurement of the periods difficult (but this is not what the ACF is for). The persistence of the correlations over 10000 days suggests a strong degree of coherence, particularly for the shorter period, but also for the longer period where at best only three cycles are covered.

Discussion

In Table 3 the periods found in this study are compared with those found by previous investigators. A colon is used to indicate uncertain values. Some care must be exercised in interpreting this data as most of the results are not independent; they are simply reanalyses of the same data using different techniques. The present work uses observations that have not been used in previous studies although there is some time overlap with Polyakova's data.

The different analyses have produced a range of results but there are many features in common. There is good agreement between the two periods found here and those of Mantegazza. The longer period of 4400–4800 days appears in all the data although it is not always mentioned. A period of 840–920 days is also found in all the studies except that of Klyus who found 720 days. The 720-day period also appears in addition to the 840–920 day period in three other analyses. Polyakova and Balasoglo both found a period around 1200 days which may be identified as P1/4. This period may also appear weakly in the present data and, of course, as a harmonic in Mantegazza's and Sharpless *et al.*'s studies.

There can be little doubt that the longer period represents a coherent variation in all the data even though it is not precisely defined. The problem lies with the shorter periods in the 720–920 day range. It is tempting to try to calculate the shorter period from the times of maximum light where they are well defined in Figure 3. However, several maxima are missing, either through observational noise, or because they are lost in the deep longer-period minimum, or because they are just not there. The missing maxima combined with the distortion introduced by the longer period variation create uncertainty in the period to such an extent that it is possible to fit the maxima with periods of about 730, 790, 850 or 920 days. These bear a striking resemblance to the various short periods that have been found in the past. It therefore seems likely that these periods are aliases, probably of the 850-day period, and which one dominates depends on the timespan of data and the technique used. Polyakova, for example, found different short periods in subsets of the data whereas Mantegazza found only the 850-day period. Despite the apparent ambiguity, the 850-day period emerges strongly and coherently in the present data and is probably the only short period present in the earlier data.

Table 3. Summary of the period studies of Mu Cephei

Study	*Years covered*		*Periods found*		
Hassenstein	1848–1937	730	904		5000:
Balasoglo	1881–1935	700:	900:	1100:	4500:
Ashbrook *et al.*	1881–1935		840		
Sharpless *et al.*	1881–1935		880		4836
Polyakova	1866–1973	730	920	1280	(4800)
Klyus	1848–1937	720			4400
Mantegazza	1881–1935		880		4850
This paper	1959–1993		850		4400

Conclusions

In a star of such slow and complex variations as Mu Cep the present observations can give only a snapshot of its behaviour. Nevertheless the present analysis clearly indicates the presence of two coherent periods at about 850 and 4400 days which are consistent with those seen in earlier data. It further suggests that the other short periods around 730 days, as given by the GCVS, and 920 days are artefacts produced by distortion of the 850-day period. To firmly establish the nature of both periods will require a complete re-examination of all the observations from 1848 to the present.

The results further demonstrate that observations by visual observers provide a valuable and often unique record of the behavior of long-period variables, which are essential for the study of these stars.

Acknowledgments

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