

# Trick or Truth: The Mysterious Connection between Numbers and Motion and Geometry

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## Abstract

The *ad hoc* invention of complex numbers is the gift that keeps on giving. However, that may not be a good thing, in the end, if our view of reality has to be a “vastly complicated mathematical structure,” inherent in string theory, as Sir Michael Atiyah has opined.

## Introduction

How can we understand Wigner’s “unreasonable effectiveness of mathematics” in physics, other than as a deep, mysterious and unexpected unity, which is observed between mathematics and physics? We can start by recognizing that there is an obvious and straight forward connection between numbers and geometrical magnitudes, after all.<sup>1</sup> When the former are properly understood, as rational numbers and this connection is extended to time, as well as space, the mystery disappears.

## Definitions

It was Sir Rowan Hamilton who most famously lamented the sorry state of the “science” of algebra, in his day, when compared to the science of geometry<sup>2</sup>, but with the advent of Dedekind’s and Cantor’s theories, mathematics would never be the same, and few would dare call it unscientific today.<sup>3</sup>

Nevertheless, it’s important to re-visit issues of scientific fundamentals occasionally, and it’s always important to start with fundamental definitions, and so we should define mathematics and physics, space and time, algebra and geometry, numbers and magnitudes, as well as dimensions and directions, from the beginning, before we delve into an examination of the topic of this essay. The trouble is, of course, it’s not always easy to obtain a consensus for these definitions, and there seems to be many exceptions to the rule, which is all the more reason we should define them here.

Fortunately, the basis of physics, if we can agree on its definition, as the science of the motion of massive and massless entities, makes this task fairly simple. We can define space, in one, two or three dimensions, in terms of a set of points, satisfying the postulates of geometry. The definition of dimensions is the minimum number of coordinates needed to specify any point within this

space, and we can define geometry, as the measurement and relationship of lines, angles, surfaces, solids and points, within this space.

To be sure, we find that to measure the space between any two of these points requires time. Regardless, of whether or not the measurement is made by physically extending a measuring rod between them, or directing an object, or a light wave, or a sound wave, to travel the distance between them, the measurement of the space (distance) between them cannot be made in the absence of time. Hence, we arrive at a definition of time, since the only known relation between space and time is reciprocal: Time is defined as the reciprocal of space, in the equation of motion.

Directions can be defined in two ways: One way to define them is in relation to the set of points in space. There is an infinite set of directions that can be specified outward away from any point in our set of points defining space. For each of these directions away from a given point, however, there exists another in the exact opposite direction from that point. Consequently, there are two “directions,” in or out, relative to any direction specified from a given point.

While there are two and only two exactly opposite directions relative to a point, each of which can be regarded as extending out from, or in toward, the point, in one dimension, there are an infinite number of these unique pairs, which can be specified in two, or three dimensions.

The definition of magnitude then follows from our definition of motion: magnitude is defined as a given change of space and time, in the direction or directions of a given dimension, or dimensions. Hence, we can define changes in magnitude, changes in dimension and changes in direction of motion in terms of space and time, which we have defined in terms of a set of points, satisfying the postulates of geometry, measured over time, the reciprocal of space, in the equation of motion.

Since the definition of mathematics is often most controversial, we will define it as simply as possible: It is defined here as the science of numbers, quantities, and shapes and the relations between them. However, we immediately run into the ancient problem of reconciling the concept of “quantity” or magnitude, with the concept of number, which brings us full circle back to the pre-Dedekind and pre-Cantor days; These are the days of Sir Hamilton, wherein he lamented the lack of a scientific basis for algebra, defined as a part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations.

Hamilton didn't like the fact that the algebra of numbers lacks the same philosophical foundation that geometry enjoys, namely that, if given right lines and circles from without, geometry satisfies the soul's hunger for the demonstration of eternal truth. He did not find it so with algebra. He wrote:

But it requires no peculiar scepticism to doubt, or even to disbelieve, the doctrine of Negatives and Imaginaries, when set forth (as it has commonly been) with principles like

these: that a greater magnitude may be subtracted from a less, and that the remainder is less than nothing; that two negative numbers, or numbers denoting magnitudes each less than nothing, may be multiplied the one by the other, and that the product will be a positive number, or a number denoting a magnitude greater than nothing; and that although the square of a number, or the product obtained by multiplying that number by itself, is therefore always positive, whether the number be positive or negative, yet that numbers, called imaginary, can be found or conceived or determined, and operated on by all the rules of positive and negative numbers, as if they were subject to those rules, although they have negative squares, and must therefore be supposed to be themselves neither positive nor negative, nor yet null numbers, so that the magnitudes which they are supposed to denote can neither be greater than nothing, nor less than nothing, nor even equal to nothing. It must be hard to found a Science on such grounds as these, though the forms of logic may build up from them a symmetrical system of expressions, and a practical art may be learned of rightly applying useful rules which seem to depend upon them.<sup>4</sup>

Some may regard this opinion as way outdated today, but the fact remains that consequences of these difficulties remain with us, as can be seen in the pathology of modern, multi-dimensional number systems. There even exists confusion in trying to describe the pathology itself, because mathematicians use the term “dimension” in a different way than others do. Students of geometry think of points simply as 0-dimensional entities, lines as 1-dimensional, areas as 2-dimensional, and volumes as 3-dimensional entities.

### **Mismatch between Physical and Mathematical Definitions**

However, for the mathematicians, the numerical concept of dimension is not limited to three (four counting 0). They regard 0D real numbers, corresponding to geometrical points, as 1-dimensional, because they take a single quantity to designate their dimension, and they are considered to lie, as a set of points positioned along a 1-dimensional “line” (the number “line”).

They regard complex numbers, corresponding to 1D geometrical lines, as 2-dimensional, because these lines take two quantities (one set of multiples of positive and negative units ( $2^1=2$ )) to specify their dimensions, as they are considered to lie, as a set of lines positioned within a unit plane (the argand plane).

They regard quaternion numbers, corresponding to geometrical areas, as 4-dimensional, because they take four quantities (two sets of multiples of positive and negative units ( $2^2=4$ )) to specify their dimensions, and they are considered to lie, as a set of planes positioned within a ball (the unit ball), while they regard octonions numbers, corresponding to geometrical volumes, as 8-dimensional, because they take eight quantities (three sets of multiples of positive and negative units ( $2^3=8$ )) to specify their dimensions, and they often are therefore thought of in terms of a set of two, 4D quaternions, because they cannot be regarded as positioned in a higher dimensional figure.

Summarizing then:

**Geometry:** points (0D); lines (1D); planes (2D); volumes (3D).

**Numbers:** real (1D); complex (2D); quaternions (4D); octonions (8D).

Naturally, the algebra of these multi-dimensional numbers gets to be quite complicated and obtuse, yet they have been used to explore physical phenomena with remarkable success, at least up to the 4D quaternions, which are also used very successfully to calculate 3D rotations, or motions in modern, computerized, mechanical applications.

Nevertheless, the use of these multi-dimensional numbers is problematic in many ways, in applications to theoretical physics, because their algebraic properties are pathological in a sense; That is to say, as the dimensions of these numbers increase, a certain property of the real number system is lost. To quote John Baez:

There are exactly four normed division algebras: the real numbers (R), complex numbers (C), quaternions (H), and octonions (O). The real numbers are the dependable breadwinner of the family, the complete ordered field we all rely on. The complex numbers are a slightly flashier but still respectable younger brother: not ordered, but algebraically complete. The quaternions, being non-commutative, are the eccentric cousin who is shunned at important family gatherings. But the octonions are the crazy old uncle nobody lets out of the attic: they are non-associative.<sup>5</sup>

What this means, on a certain level, is that, in reality, such higher-dimensional numbers, invented by the human mind, by adding new, *ad hoc* numbers (originally called imaginary numbers) to the set of real numbers, lose more than their correspondence to the geometric definition of dimension and direction, as defined above. They lose fundamental correspondence with reality. Nature does not have younger, non-ordered younger brothers, or eccentric cousins or crazy uncles.

Regardless, however, the invention has advanced science and technology forward to an unimagined degree, compared to the state of the art, when Hamilton, and later Kronecker, lodged their philosophical complaints against algebra and real numbers, respectively.<sup>6</sup>

What would Hamilton and Kronecker think of Lie groups, Lie algebras and their unitary representations, and their application to quantum mechanics through the principles of symmetry? Would they buy into the modern concepts of mathematics and physics, those indispensable concepts such as quantum spin and isospin, etc., and still wonder with Wigner over the mystery of it all? Or would it cause their reason to stare and their souls to ache, because, like Sir Michael Atiyah, they would see that, while the great advance to quantum mechanics, from classical mechanics, would have been impossible without the *ad hoc* imaginary numbers of modern mathematics (this pure invention of man that has seemingly turned up in Mother Nature), the

implications for truth are just too baroque and too distasteful, by the time we get to string theory?<sup>7</sup>

These imaginary mathematics indeed appear to be a gift, a gift that keeps on giving, whether we like it or not. Nevertheless, with the advent of string theory, and the prospect that it is a correct view of reality, hopefully leading to the so-called “final theory,” we “discover the possibility of a universe built on some fantastically intricate mathematics,” to put it in the words of Sir Atiyah.<sup>8</sup>

To say that this would be disappointing, if not depressing to Hamilton, Kronecker, Atiyah and many others, is obvious. Atiyah, for one, sees that such a prospect brings us back to the question, “What is reality? Is reality built out of this vastly complicated mathematical structure that the human brain, with the help of the physical world, has evolved? Is that the secret of reality?”<sup>9</sup>

His answer is that it is difficult to believe something like that could be true, and perhaps there are alternatives, “Is there a new paradigm needed? Is there a new way of looking at things?” he asks. “Perhaps the complicated mathematics we use is just in the eye of the beholder. Perhaps we use all this mathematics, because we got it and there is nothing else we can do. Perhaps there is an alternative way of looking at it, which will shed light and make great progress”<sup>10</sup>

Clearly, however, Sir Atiyah’s consternation is not of mathematics in general, and his suggestion of looking for a new paradigm is not a belief that mathematics is not that wonderful gift we think it is, but perhaps it’s an indictment of mathematical hubris, of convincing ourselves that what we know is so, when maybe it isn’t.

If we go back to fundamental set theory for instance, when the concept of real numbers was introduced, and has since evolved, we realize that the continuum hypothesis is the beginning of all the complication that so besets us today.<sup>11</sup>

## **Where is the “Zero-Point?”**

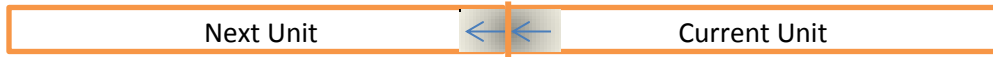
For the simple observation that the logic of Zeno’s paradox and mathematical concepts, such as Cantor’s continuum hypothesis, unlike physics, do not pre-suppose time, which is indispensable to the science of physics, as we have defined it above, we need only take the example of a clock.

The argument of how to divide the circle of the clock face into a number of equal units may become subject to the logic of Zeno and the math of Cantor, but when the hands of the clock begin to move, the mathematics of numbers must deal with magnitudes of time, as well as space.

If we pick a point on the face to which the hand is approaching, the number of equal units between it and the hand is diminishing, while for all points behind the hand, the number of units between them and the hand is increasing.

For those points for which the distance between them and the hand is less than one of a given unit division, the point arrived at, just after the hand leaves the first unit behind, and the point arrived at, just before the hand enters the next unit, give rise to important questions, regarding the location of the zero-point and the direction of the hand's motion.

When the unit division is fixed, can the transition from a location less than 1, to 1, in the current unit, and from a location of 1 to less than 1, in the next unit, be defined as the zero point? Is this transition instantaneous? Must the motion of the hand stop at any point in time?



**Figure 1.** The Zero Point

Of course, the clock hand is pointing to divisions of a circle, which involves motion in two dimensions. When these motions are represented by sine and cosine functions, plotted perpendicularly on the clock face, with their intersection at the hand's axis of rotation, they reverse numerical "direction," alternately, at either the "point" located at their intersection, where the change is from positive to negative (or vice-versa), or at the tip of the hand, where the change is from less than 1 to 1 unit, in the current unit, and from 1 to less than 1 unit, in the next unit.

Again, though, the questions are, "Where is the zero point?" "Are these transitions instantaneous, from one unit to the next?" "Must the direction of the sign or cosine plot physically stop, in order to reverse direction, at the tip of the clock's hand?" "Is the present moment partly in the past and partly in the future, at the transition point, from one unit to the next?"

The conundrum itself is probably more important to recognize than any answers to specific questions, because it indicates a possibility for taking a fresh new look at our most fundamental physical assumption: The dictionary says that the definition of motion is "an act, process, or instance of changing *place*."<sup>12</sup>

However, it should be obvious now that an instantaneous change of polarity, or an instantaneous change of direction, does not constitute a change in place, location or position, yet these are definitely changes in the properties of motion, if we accept that such properties include magnitude, dimension and direction.

The observation is that no matter how many equal divisions we decide to divide the clock face into, the clock hand always arrives at the end of the last one and at the beginning of the next one, where a transition, or a change, in the characteristics of the hand's motion takes place.

Selecting a point on the boundary separating two of these units, we can imagine subdividing them, *ad infinitum*, but regardless of how finely we determine to subdivide the unit, there will

always be a last, or final subdivision of the current unit, and a first subdivision of the next unit, that must be transitioned by the clock's hand.

Mathematically speaking, we can say that the motion of the clock hand toward the boundary we have selected represents the completion of a whole number, over time, from a fraction of a whole number, where the denominator of the fraction is the total number of unit subdivisions, and the numerator is the total number of transitioned subdivisions. The difference between the number of total subdivisions, and the number of transitioned subdivisions, at any point in time, is the number of subdivisions yet to be transitioned, in that unit.

Clearly, that number, the number of subdivisions yet to be transitioned, eventually reaches zero, when the clock hand reaches the end of the last subdivision of the current unit, yet the number of transitioned units equals the number of total units to be transitioned. In other words, when the numerator equals the denominator, the difference between them is zero.

However, the projection of the sine and cosine values upon the clock face shows that there are *two* changes occurring in the characteristics of the motion, simultaneously: One is represented by the plot of the sine (or cosine), which converges and eventually coincides with the tip of the clock's hand, when it reverses direction, and the other is at the intersection of the two plots, where there is no change in the direction of the cosine (or sine) plot, but there is a change of its polarity, when the sign of the cosine (sine) changes.

Mathematically speaking, it makes no difference how we represent these two changes. We only say that the motion of the clock hand, toward a given boundary, may be represented by either the sine function, or its inverse, the cosine function, where, the changing numerator represents one of the two aspects of the clock hand's motion, as it completes its transition of the last subdivision of the current unit, and the first subdivision of the next unit.

### **What does this mean?**

The fact that we can mathematically represent the transitional motion of the clock hand in these two ways, one where the selected datum, or reference point, is  $n/n = 1$ , and the other, where it is  $0/n$ , suggests something easily missed, but perhaps crucially important: When we divide the face of the clock's surface into  $x$  number of units, we normally think of it as dividing space into  $x$  number of equal divisions, but, in reality, we are dividing the *motion* of the clock hand, into  $x$  equal divisions, and there are two, reciprocal aspects of this motion, time and space, both of which can be represented by the sine and its inverse, the cosine, of the clock hand's changing angle, with respect to four, 90 degree quadrant divisions of the clock face. When this duality is clear, we also understand that there are two ways to represent the numerical datum of the clock's motion.

In other words, when we divide the motion of the clock face, into  $x$  number of equal units, these units are necessarily units of both space and time, the two reciprocal aspects of motion. This is

not necessarily news. It's the basis for all modern-day technology and physics, but currently it is mathematically formulated in terms that use two 0D numbers in conjunction with an *ad hoc* imaginary number 'i,' to form a 1D complex number,  $z = a + bi$ . The reference of this system, the datum of this system, is something entirely different.

True, these complex numbers and their properties amaze mathematicians, and both engineers and physicists have been carried away with their enthusiasm, and have never looked back, generation after generation. Roger Penrose puts it this way:

I think that you cannot separate physics and mathematics. One of the things that have always impressed me tremendously is how complex numbers are so fundamental to quantum theory. Complex numbers forced themselves into mathematics, even against the mathematicians' wills. Numerous mathematicians kept imagining that they didn't exist, but complex numbers kept coming back, and they became a powerful way of looking at mathematics. Obviously, one now accepts complex numbers as a very fundamental ingredient in mathematics, but they forced their way into mathematics for purely mathematical reasons, and then they were accepted. The fundamental theorem of algebra is an example — the relation between complex exponentials and sines and cosines. But that was just mathematical trickery at the time. Complex numbers have a kind of mathematical reality, which is very powerful. Here we see they actually have a fundamental role to play in physics. Whereas, up until that point, one always thought that physics dealt with real numbers. Complex numbers were kind of funny and auxiliary. But here they were, sitting there at a fundamental level in physics. So I've always been impressed by this interrelationship between mathematics and physics, which is no accident, I'm sure. The way that physical theories are so beautifully accurate — not just that the mathematics is so accurate in physics, but it's that mathematics which works well in physics which is also very fruitful within mathematics. Calculus, for example, is a tremendously powerful idea.<sup>13</sup>

However, Penrose's enthusiasm over the contribution of complex numbers to math and physics, based on *ad hoc* imaginary units, contrasts sharply with Atiyah's misgivings that, in the end, the beauty of Penrose's "mathematical reality," may only be "in the eye of the beholder."<sup>14</sup>

There is no doubt that the complex number works well, in many unforeseen ways, but the fact that we can easily understand the relationship between space and time, or motion, in another way, using rational numbers, as described above, lends credence to Atiyah's observation, previously quoted, that we need a less intricate, less complicated alternative, that "Perhaps we use all this mathematics, because we got it and there is nothing else we can do."

Nevertheless, such an iconoclastic point of view can only be fully appreciated by those who understand the trouble that theoretical physics finds itself in today, in its search for a final theory.



The truth is that “...now we are stuck,” in the words of Steven Weinberg, after the most “frustrating” years of search “in the history of elementary particle physics.”<sup>15</sup>

Fortunately, a relevant example of the power of a new way of looking at things in mathematics related to physics is found in the works of Dewey B. Larson and Xavier Borg. Larson’s work is theoretical, while Borg’s work is empirical, yet they both came to the same iconoclastic conclusion: Physical units reduce to dimensions of space over time and time over space, the dimensions of motion and inverse motion, respectively.<sup>16</sup>

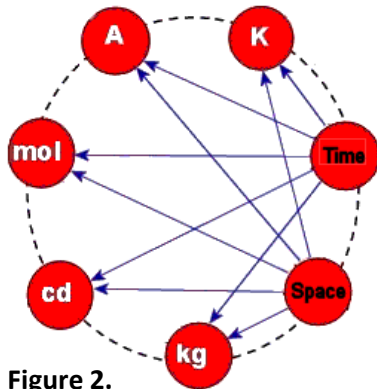


Figure 2.

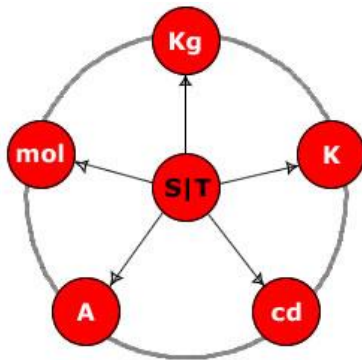


Figure 3.

However, unlike Larson, even though Borg recognizes that “only space and time are fundamental dimensions,” he doesn’t refer to the most important relation between space and time, indeed the only *known* relation of space and time, motion. Nevertheless, he proceeds to show how to redefine the International System (SI) of units, in terms of space and time only, as shown here in figure 2.

Taking it one step further, recognizing the fact that the only known relation between space and time is motion, Borg’s diagram is more naturally reconfigured, as shown in figure 3. With this alternate view of SI measuring units, a major simplification of the science of physics becomes available, because the space/time dimensions of a fundamental unit of motion can be used to derive all other physical units, as partially shown in table 1 (a full list is available on Borg’s website).

Table 1.

| Parameter                | Units                   | SI units                         | ST Dimensions    |
|--------------------------|-------------------------|----------------------------------|------------------|
| Distance <b>S</b>        | metres                  | m                                | S                |
| Area <b>A</b>            | metres square           | m <sup>2</sup>                   | S <sup>2</sup>   |
| Volume <b>V</b>          | metres cubed            | m <sup>3</sup>                   | S <sup>3</sup>   |
| Time <b>t</b>            | seconds                 | s                                | T                |
| Speed/ Velocity <b>u</b> | metres/sec              | m/s                              | ST <sup>-1</sup> |
| Acceleration <b>a</b>    | metres/sec <sup>2</sup> | m/s <sup>2</sup>                 | ST <sup>-2</sup> |
| Force/ Drag <b>F</b>     | Newtons                 | Kgm/s <sup>2</sup>               | TS <sup>-2</sup> |
| Surface Tension <b>γ</b> | Newton/meter            | Kg/s <sup>2</sup>                | TS <sup>-3</sup> |
| Energy/ Work <b>E</b>    | Joules                  | Kgm <sup>2</sup> /s <sup>2</sup> | TS <sup>-1</sup> |

## What are Numbers?

As shown above, in our modern science of mathematics, the answer to this question is complicated. We have real numbers, complex numbers, quaternion numbers and octonion numbers, recognized as the members of the only four normed division algebras known to exist.<sup>17</sup> Physicists have used these four algebras, with varying degrees of success, to bring the science of theoretical physics to its present state.<sup>18</sup>

Nevertheless, at an elementary level, numbers count things, and given two such numbers, one greater than the other, there is always another number, greater than them both.<sup>19</sup> In counting things, it's possible that the things counted are parts of a whole, where we use two numbers, which are related to each other. One number counts the total number of parts into which the whole is divided (which has no upward bound), while the other counts those parts of the whole that are under consideration. The relation between the two numbers is expressed as a ratio, where  $m/n = 1$ , when  $m = n$ .

We can write the set of all these rational numbers as a number “line,” in the following manner:

$n/m, \dots 1/3, 1/2, 1/1, 2/1, 3/1, \dots, m/n$

Notice that the ratios to the right of the whole number,  $1/1$ , are the inverses of the ratios to the left of the whole number. The magnitude,  $r$ , of the *ratios*, relative to the whole, increases in both directions, in the following manner:

$r, \dots 3, 2, 1, 2, 3, \dots, r$

While the difference,  $d$ , between the numerator and the denominator increases in like manner, but with a different result:

$d, \dots 3, 2, 1, 0, 1, 2, 3, \dots, d$

Traditionally, mathematicians assign symbols of direction, such as polarity symbols, ‘+’ and ‘-’, to distinguish between those numbers on the left and those numbers on the right:

$-d, \dots -3, -2, -1, 0, +1, +2, +3, \dots, +d$

Obviously, as is well known, the algebraic use of the ‘d’ numbers requires a negative unit,  $-1$ , so it was gradually, but eventually added to the list of rational magnitudes, by the mathematicians, even though it makes no sense and cannot be reasonably placed in the list of ‘r’ magnitudes, shown in the number “line” of rational numbers above. In other words, a number line of ‘r’ magnitudes, such as:

$-r, \dots -3, -2, \underline{-1}, +1, +2, +3, \dots, +r$

makes no sense at all.

For this reason, mathematicians originally called the negative unit an “imaginary” number, and it has complicated the field of mathematics tremendously ever since, and it has complicated the field of physics even more.<sup>20</sup>

To avoid this complication, perhaps we can use the rational numbers themselves, where their inverse magnitudes easily distinguish them as belonging to the “positive” or “negative” side of  $1/1$ , a whole number which is neither “less than one” nor “more than one,” thus, eliminating the need for polarity symbols all together, and the subsequent requirement of a negative unit that complicates algebraic operations.

However, in order to make this work, we will have to consider the fractions on the left of  $1/1$ , as the inverses of the “fractions” on the right of  $1/1$ . Fortunately, this is easily accomplished by regarding the *denominator* of the fraction as the number below the division symbol (vinculum), “/”, for the fractions on the left of the whole number,  $1/1$ , and the number above the vinculum, as the *denominator*, for the fractions to the right of the whole number.

This view of elementary numbers may seem bizarre, at first, but it also may be the start to “the new way of looking at things,” that the notable mathematician, Sir Michael Atiyah, is thinking we need, given the current prospect that mother nature’s reality might be composed of a “vastly complicated mathematical structure,” inherent in string theory.<sup>21</sup>

## The Motion of a Two-Faced Clock

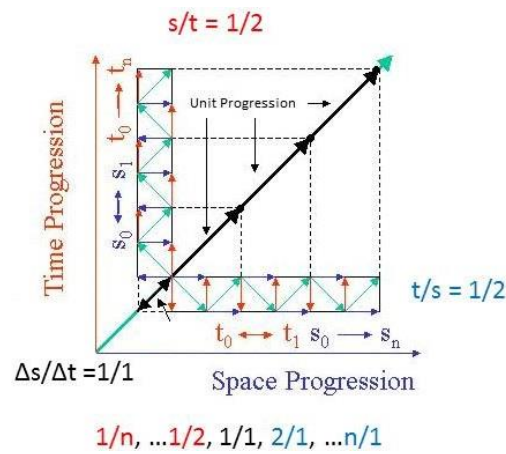
To understand this unorthodox view of elementary numbers, we need only consider a clock, with two faces, back-to-back. One face will represent the fractions, where the denominator is below the vinculum, those ordered to the left of the whole number, or the unit number, while the opposite face of the clock represents the inverse of these numbers, those ordered to the right of the unit number, where the denominator of the fraction is above the vinculum.

Of course, the first challenge will be to find a way to order these numbers on the clock face so that they repeat, *ad infinitum*, as do the 12 (or 24) numbers on our time clocks, which represent the 24 hour revolutions of the earth on its axis. In other words, we need a repeatable physical connection, a periodic motion, which this set of rational numbers can represent.

One way we can do this is to consider that the *motion* of the clock hand, in the clockwise direction on one face, and in the counter-clockwise direction on the opposite face, is actually the combination of *two* motions. One motion is the bidirectional movement of the clock’s escapement, while the other is the unidirectional movement of the clock’s escapement wheel, driving the two clock hands.<sup>22</sup>

In this analog, as the clock hand moves, the numerator, representing the escapement, repeats, or oscillates, over 1 unit, a number of times that is equal to a corresponding increase in the denominator, representing the escapement wheel. In the simplest case, the numerator oscillates

between 0 and 1 continuously, while the denominator increases simultaneously from 0 to 1 to 2. In other words, as the clock hand moves over the units on the face, the numbers in the rational number that corresponds to its motion indicate the number of units transited by the motion. The number in the numerator indicates that there is no net change as it increases and then decreases by one unit, alternately, while the number in the denominator indicates a two-unit increase, associated with the net-zero change of the numerator: Thus, the rational number, “1/2,” corresponds to this motion.



**Figure 4.**

after a one-unit change, in one of them. Since space and time are reciprocal magnitudes, in the equation of motion,  $v = s/t$ , figure 1 shows how this motion can be plotted graphically, as motion.

In the first square, in the lower left corner, a unit increase in space corresponds to a unit increase in its reciprocal, time. Therefore, the net result is motion along the diagonal. The rational number equivalent for this motion is “1/1,” meaning that there is a one-unit increase in the numerator per one-unit increase in the denominator.

However, in the second unit, which would normally be identical to the first unit, the possibility of a direction reversal occurring in one of the two reciprocal magnitudes is shown.

If the direction reversals occur in the increasing space component, the result is the rising plot shown as the vertical oscillation. If the direction reversals occur in the reciprocal time component, the result is the rightward extending plot shown as the horizontal oscillation.

In each case, the two-unit increase of the reciprocal component of the motion that does not oscillate is double the oscillating unit’s magnitude. In one case, the corresponding rational number is “1/2,” while in the other case, the rational number is “2/1,” wherein the denominator is above the vinculum, which, as the graph shows, is only distinguished in its direction, relative to the unit increase, represented by the number, “1/1.”

Of course, instead of changing the number, by turning the numerator/denominator upside down, we can exchange the space and time terms, so that, on one side of the unit number,  $s/t = 1/2$ , while on the other side,  $t/s = 1/2$ .

## Focus on the Motion

Now, given that we have these numbers that don't require polarity signs, what can we do with them? Do they too have this "deep and mysterious connection" with physics that is so much wondered at? Actually, we can see that they are an expression of motion itself, so, right off the bat, we suspect that the answer is yes. Still, it might be hard to see just how it would be worthwhile to investigate physics with such numbers.

Newton's scientific research program, into the structure of the physical world, which continues with us today, "can be summarized," writes David Hestenes, "by the dictum: Focus on the forces." He goes on to write:

This should be interpreted as the admonition to study the motions of physical objects and find forces of interaction sufficient to determine those motions. The aim is to classify the kinds of forces and so develop a classification of particles according to the kinds of interactions in which they participate.<sup>23</sup>

Clearly, the sense of "the deep and mysterious connection" between mathematics and physics has emerged from this research program of Newton's. However, as already shown above, it has been little noticed that an engineer, Xavior Borg, has recently discovered that the standard units of measure, the SI system of units of measure, can all be expressed with units of motion, or dimensions of space, and its inverse, time, only.<sup>24</sup>

This means that mathematical equations used in Newton's program of theoretical physics can now be viewed in terms of the dimensions of motion and the dimensions of inverse motion, or energy. Some important examples are:

$$E = mc^2 \text{ becomes: } t/s = (t/s)^3 \times (s/t)^2;$$

$$F = ma \text{ becomes: } t/s^2 = (t/s)^3 \times (s/t^2);$$

$$p = mv \text{ becomes: } t^2/s^2 = (t/s)^3 \times (s/t);$$

Again, it should be noted that Borg has a long list of SI units converted to space and time dimensions available on his website.<sup>25</sup>

More to the point, however, the fact that these physical units can be expressed in terms of the dimensions of motion and its inverse, energy, implies that there is something we don't understand about motion and energy. Given the motion equation,  $v = \Delta s/\Delta t$ , we see that the equation requires no object. Motion is simply defined as a change in space over a change in time.

Of course, we experience the continuous change in time, but only when we observe the distant galaxies do we see nature's continuous change in space, as well.

The motions of the graph depicted in figure 2, like the two motions of a mechanical clock, are analogs of one-dimensional motion that are symbolically represented by the inverse numbers  $1/2$  and  $2/1$ , which are derived from the unit number,  $1/1$ , a one-dimensional ratio, comprised of two, one-dimensional numbers.

Nevertheless, given Borg's multi-dimensional units of space and time, perhaps it would be worthwhile to go beyond Newton's program of research and focus on the motions, seeking to develop a classification of multi-dimensional motions, that might correspond to known multi-dimensional geometries, even though no objects, forces or interactions are relevant, at this point.

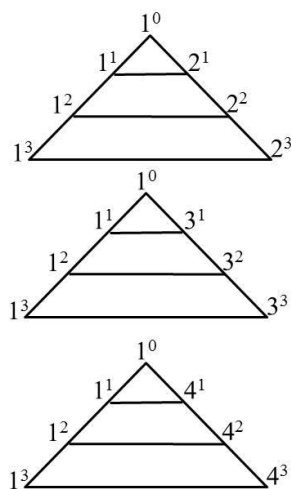
One approach, currently under investigation, defines and classifies new multi-dimensional units of motion, according to new, multi-dimensional numbers. This classification follows the structure of the Greek tetraktys, normally equated with the familiar binomial expansion, which forms the basis of Clifford algebras.<sup>26</sup>

Here, however, fundamental units of multi-dimensional numbers are defined, where the numbers 1, 2, 3 and 4 of the tetraktys are raised to exponential powers, from 0 to 3, restoring the dimensional correspondence to fundamental geometrical units, which is lost when employing only the tetraktys' binomial directions of  $2^0=1$ ,  $2^1=2$ ,  $2^2=4$  and  $2^3=8$ , as we currently do, in the normed division algebras of our four number systems, because of confusing the concept of

direction with the concept of dimension, due to our enthusiasm for the *ad hoc* invention of the negative unit.<sup>27</sup>

Here, in contrast, the idea is to equate multi-dimensional mathematical units, or multi-dimensional numbers, with corresponding multi-dimensional geometrical units of measure, which can be physically generated from multi-dimensional units of motion, as herein defined.

The Numbers of the Tetraktys



**Figure 5.** The four Numbers of the Tetraktys

The first number of the tetraktys is the number 1. When it is raised to the power of 0, it is a number that corresponds to the geometrical point, at the top of the tetraktys. When it is raised to the power of 1, it is a number corresponding to the geometric line of the tetraktys. Raised to the

power of 2, it corresponds to area and to volume when it is raised to the power of 3. These are numerical units that correspond to familiar geometrical units of points, lines, squares and cubes.

The next number is 2. It also corresponds to a point, at the power of 0, a line at the power of 1, an area at the power of 2 and a volume at the power of 3, but instead of these correspondences arising from a change in the dimensions of abstract numerical units, they arise in connection with a change in the dimensions of motion, which cause a change in position. However, since this motion necessarily involves the motion of an object's location, a unit increase in position can only be affected in one available dimension at a time.

The next number is 3. It also generates units corresponding to points, lines, areas and volumes, when raised to 0, 1, 2 and 3 powers, but these changes in the dimensions of motion correspond to the dimensions of intervals, rather than to changes of position. An example would be a 0D point, simultaneously stretched in two opposite directions to form a 1D line; a 2D line stretched in two opposite directions to form a 2D area, and a 2D area stretched in two opposite directions to form a 3D cube.

Finally, we come to the last number of the tetraktys, the number 4. Again, the change of the dimensions of this number corresponds to the geometrical changes of dimension, the dimensions of a geometrical point, line, area and volume, but here the unit change occurs as a result of changes in the dimensions of scale, rather than a change of the dimensions of position, or a change of the dimensions of interval.

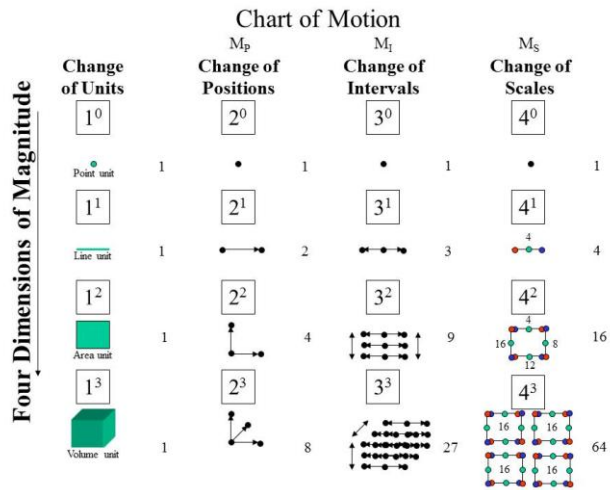
It's more difficult to explain, but the 0D scalar "point" consists of a 0D entity represented by a balance between inverse unit magnitudes, where  $4^0/4^0 = 1/1$ . Consider it symbolically, as a physical barbell, where 4 raised to the power of 1 is equivalent to an 8-pound barbell, with 4-pound weights on each end ( $4^1/4^1 = 4/4$ ). When 4 is raised to the power of 2, four of these 8-pound weights are formed, which are able to bound an area, by connecting them end-to-end, two-dimensionally

$$((4^2/4^2) = (16/16) = (4/4)+(4/4)+(4/4)+(4/4))$$

When 4 is raised to the power of 3, sixty four of these weights are formed, which can be configured into 16 of the 8-pound barbells, making it possible to bound a cube with them, by connecting them end-to-end, four times, in three-dimensions.

$$(4^3/4^3) = 64/64 = ((4/4)+(4/4)+(4/4)+(4/4)) + [(4/4)+(4/4)+(4/4)+(4/4)] + [(4/4)+(4/4)+(4/4)+(4/4)] + [(4/4)+(4/4)+(4/4)+(4/4)]$$

With this much understood, we see that each set of these multi-dimensional numbers,  $1^{0-3}$ ,  $2^{0-3}$ ,  $3^{0-3}$ ,  $4^{0-3}$ , are the numbers that form the tetraktys, and they correspond to a unique geometrical entity of the same number of dimensions, created by the corresponding class of motion, or change of space, over time, in different ways.



In other words, each of these different classifications of motion creates multi-dimensional geometrical units of space over time, as points, lines, areas and volumes, which have their equivalents in multi-dimensional numbers of corresponding dimensions, and each in its own way: By way of a change of position, by way of a change of interval and by way of a change of scale. A chart of these relationships is provided in figure 6.

Figure 6. The Chart of Motion

### Rotation – A Most Sacred Icon

But what about rotational motion, why isn't it included in the Chart of Motion? Given that rotational motion is observed in the heavens and on earth, one would think that it is surely a fundamental motion. Indeed, understanding rotational motion, from the swinging pendulum to the orbits of the planets around the sun and the stars around the galaxies, is the foundation of Newton's research program, and Western Civilization's vaunted technology.

Nevertheless, it's not included in the Chart of Motion, because it's not found in the tetraktys. It is essentially change of position motion that undergoes a constant change of direction, but change of direction, while important to understand in many respects, does not create a corresponding geometrical unit, independently of change of position motion.

The difficulty of comprehending the iconoclastic nature of this conclusion is hard to overestimate, because rotational motion is the foundation of our most powerful science and technology.<sup>28</sup> Yet, at the same time, as this author asserts, it is the conviction that rotational motion is fundamental, which goes to the root of our trouble with theoretical physics today.

The consequence of adopting the imaginary negative unit, while extremely useful, as it turns out, may nevertheless have been mankind's undoing, when it comes to understanding the mysterious connection between mathematics and physics, in the search for reality. Newton's program begins with particles, seeking to classify them, according to the forces involved in their interactions. However, now we know that particles themselves ultimately have dimensions of space and time, or motion, and force is just a changing quantity of motion, or acceleration. Consequently, we need a new program of research that classifies combinations of motion, or motions, and the relationships between them, as observed.



Presumably, such a research program would start with the implications of the charts of figures 2 and 6. The dimensions of the motion of figure 2 are unspecified, but if we assume that three-dimensional motion is represented, then the oscillating unit would be a ball, expanding and contracting, while the reciprocal component would be expanding continuously, in three dimensions. Since the magnitude of the original motion, before oscillation begins, depends on the relative magnitudes of the space and time units, a natural candidate for them would be derived from the speed of light.

Taking these magnitudes for the space and time units of the graph in figure 2, this is the “speed” represented by the diagonal line, labeled “Unit Progression,” in the graph of figure 2.

Now, given the onset of oscillation of the space component in figure 2, producing the increasing vertical time line, where each space cycle requires two units of time to complete, the frequency,  $f$ , of this entity is then one cycle for every two units of time.

But frequency, with dimensions  $1/t$ , or cycles per unit of time, is a concept of rotation, normally expressed in terms of  $2\pi$  radians per second, mathematically equivalent to the motion of waves. Even if ‘ $f$ ’ could be converted to velocity, why would we want to do so, when using the changing sine and cosine of the rotation angle, and the concept of angular momentum, underlying the wave equation, are simply indispensable to modern physics and engineering?

That is a good question, but the answer is good too: We want to look at things differently to see if the reality of nature can be expressed without the “vastly complicated mathematical structure” of modern science.<sup>29</sup> And now we see, from a study of the tetraktys that rotational motion, so crucial to our modern science, is not even a proper class of motion, and moreover, it is clear that our use of it depends on incorporating the *ad hoc* invention of an imaginary, negative unit.

Nevertheless, and notwithstanding the understandable consternation this observation might engender, the chart of figure 6 shows that there are other options. Indeed, if we regard the natural 3D motion of interval ( $2^3$  type motion), rather than trying to employ 3D motion of position through rotation (think of Lie groups and Lie algebras,) it appears that our task would be greatly simplified.

Using the interval concept of motion, the oscillations of figure 2 simply represent the expanding/contracting radius of a 3D ball. This means that the unit space volume goes from zero to unit value and back to zero, but the number corresponding to the cubic value of the three-dimensional interval motion ( $2^3=8$ ) is incompatible with the numerical equation for the volume of a ball; that is to say, nature doesn’t expand/contract in cubes.

However, we can easily quantify this oscillation for volume by recognizing that, while the number,  $2^3$  equals a  $2 \times 2 \times 2$  stack of 8, 1-unit cubes, and the unit volume of the ball equals  $4\pi/3$ , which, although it is an irrational number, it is a ball that just fits into the 8-unit stack, with diameter = 2. This means that the ratio of one of the one-eighth volumes, contained in each of

the  $2 \times 2 \times 2 = 8$  units, to the unit volume is  $1:8$ , which also means that the ratio of the radii of the two volumes is  $1:2$ .

As it turns out, then, the cube of the radius of this smaller volume is equal to the ratio of the two volumes:

$$V_1 = (4\pi/3), V_2 = V_1/8$$

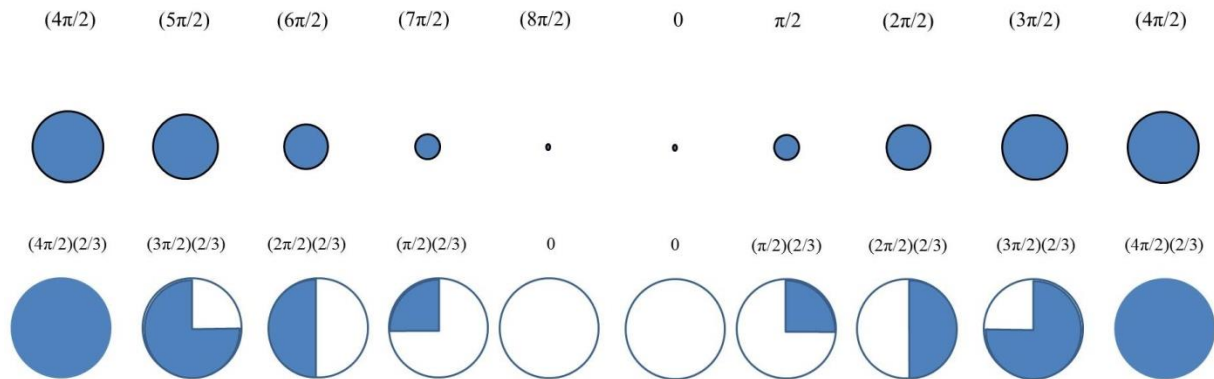
$$V_2 = (4\pi/3)r^3$$

$$r^3 = (V_2/V_1) = 1/8$$

$$r = (1/8)^{1/3} = .5$$

In other words, the *radius* of the unit volume (or 1) is the *diameter* of the  $1/8$  volume (or  $1$ ), and the *radius* of each  $1/8$  volume is therefore half of its *diameter* (or  $1/2$ ). This is fortunate, because it allows us to map the expanding/contracting volume to the equivalent of  $2\pi$  radians of rotation.

This is also important to show, if for no other reason than it takes *two*,  $2\pi$  rotations to complete one cycle of quantum spin, or  $4\pi$  radians of rotation, to get a valid solution to the wave equation. Currently, this mathematical requirement has no satisfactory physical interpretation, and it never will, until we understand that quantum spin might not be a case of 1D rotation, in 3D space, but rather a case of 3D oscillation of 3D space, equivalent to  $4\pi$  rotation, as depicted in figure 7 below:



**Figure 7.** As  $(\pi/2)/3 = .523598... = V_1/8$ , two of these volume quantities, or  $V = (\pi/2)(2/3)$ , are the equivalent of an increase, or decrease, of one,  $\pi/2$ , rotation ( $90^\circ$ )

As we see in figure 7, in the equivalent of one rotation of  $2\pi$  radians, the 3D oscillation has fully expanded, but this is only one-half of its full cycle. Contracting to the starting point, at zero, in the second half of its cycle, requires the equivalent of a second,  $2\pi$  rotation.

Thus, we can conclude that, while Roger Penrose’s enthusiasm for the beauty of “mathematical reality,”<sup>30</sup> contrasts with Sir Michael Atiyah’s misgivings that, in the end, the beauty we find

may only be “in the eye of the beholder,”<sup>31</sup> it is clear that there is a “new way of looking at [things], which will shed light and make great progress.”

There is no doubt that the complex number works well, in many unforeseen ways, but the fact that we can easily understand the relationship between space and time, or motion, in another way, using rational numbers, as described herein, which leads to gratifying results, lends credence to Sir Atiyah’s observation that we really do need a less intricate, less complicated alternative, that “perhaps we use all this mathematics, because we got it and there is nothing else we can do.”<sup>32</sup>

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<sup>3</sup> Dauben, Joseph Warren, Georg Cantor: His Mathematics and Philosophy of the Infinite, <http://tinyurl.com/pwg8a4n>

<sup>4</sup> Op Cit, Hamilton

<sup>5</sup> Baez, John, The Octonions, <http://arxiv.org/abs/math/0105155v4>

<sup>6</sup> Motl, Luboš, Why complex numbers are fundamental in physics, <http://tinyurl.com/35n46mz>

<sup>7</sup> Atiyah, Sir Michael, The Nature of Space, slides 23-25, <http://online.itp.ucsb.edu/online/lecture/atiyah/>

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<sup>9</sup> Ibid

<sup>10</sup> Ibid

<sup>11</sup> Smullyan, Raymond M., Fitting, Melvin, Set Theory and the Continuum Problem, <http://tinyurl.com/ov4cy68>

<sup>12</sup> Mirriam-Webster, Definition of motion,

<sup>13</sup> Penrose, Roger, Interview of Roger Penrose by Alan Lightman, <http://www.aip.org/history/ohilist/34322.html>

<sup>14</sup> Op Cit, Atiyah

<sup>15</sup> Dreams of a Final Theory: The Scientist’s Search for the Ultimate Laws of Nature, page 4

<sup>16</sup> Borg, Xavior, Unified Theory Foundations - The ST system of units, <http://blazelabs.com/f-u-suconv.asp>

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<sup>18</sup> Baez, John, Huerta, John, The Strangest Number in String Theory, <http://tinyurl.com/or8h8hh>

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<sup>20</sup> Auffray, Jean-Paul, On Lee Smolin’s “The Trouble with Physics,” <http://tinyurl.com/p8x3lr2>

<sup>21</sup> Op Cit, Atiyah

<sup>22</sup> Wikipedia, Escapement, <http://en.wikipedia.org/wiki/Escapement>

<sup>23</sup> Op Cit, Hestenes

<sup>24</sup> Op Cit, Borg

<sup>25</sup> Ibid

<sup>26</sup> HyperPhysics, Binomial Theorem, <http://hyperphysics.phy-astr.gsu.edu/hbase/alg3.html>

<sup>27</sup> Op Cit, Luboš

<sup>28</sup> Op Cit, Luboš

<sup>29</sup> Op Cit, Atiyah

<sup>30</sup> Op Cit, Penrose

<sup>31</sup> Op Cit, Atiyah

<sup>32</sup> Op Cit, Atiyah