Development Economics

Lecture 8: Testing the Solow approach—Mankiw, Romer and Weil (1992)

Professor Anant Nyshadham

EC 2273

Today

1. Solow with technology growth
2. Testing the Solow approach

   Mankiw, Romer, and Weil. 1992.“A Contribution to the Empirics of Economic Growth.”

Observations of MRW

1. Solow seems to do a pretty good job (Easterly and Levine will disagree)
2. Solow does even better when add in human capital
3. There seems to be only limited convergence
4. Returns on capital and interest rate differentials are not quite right

Mankiw, Romer, and Weil (1992)

- Serious paper written for other economists
  - Difficult—we will go through it carefully, but is clearest approach to Solow
  - Not all arguments are important
  - 20 years old now, new information?
- Basic idea: Take the Solow model seriously.
  - Work out some testable implications from the model
  - See how well the countries for which we have data match

Solow with technology growth

$Y = K^{\alpha}(AL)^{1-\alpha}$ $\Delta L/L = n$ $\Delta A/A = g$ $\Delta K = sY - \delta K$

Define ~~k~~ $= K/AL$ ~~y~~ $= Y/AL$

~~k~~ $= [s/(n + g + \delta)]^{1/(1-\alpha)} \rightarrow$ ~~y~~ $= ($~~k~~$)^{\alpha} = [s/(n + g + \delta)]^{\alpha/(1-\alpha)}$

$y = Y/L = A$~~y~~ in steady state $y = A[s/(n + g + \delta)]^{\alpha/(1-\alpha)}$

MRW notation: use k for ~~k~~

use continuous time $k' \approx \Delta$~~k~~ $= dk/dt$

Getting an equation to estimate

$y = A[s/(n + g + \delta)]^{\alpha/(1-\alpha)}$ in steady state

take logs (natural logarithm) ln

$\ln y = \ln A + \alpha/(1-\alpha) \ln s - \alpha/(1-\alpha) \ln (n + g + \delta)$

ln A changing over time

if know A_0 the $A_t = (1+g)^t A_0$

$\ln \mathbf{y} = \ln \mathbf{A_0} + \ln(1+\mathbf{g})^t + \boldsymbol{\alpha}/(\mathbf{1}-\boldsymbol{\alpha}) \ln \mathbf{s} - \boldsymbol{\alpha}/(\mathbf{1}-\boldsymbol{\alpha}) \ln (\mathbf{n} + \mathbf{g} + \boldsymbol{\delta})$

if we know $\mathbf{g}$, $\mathbf{A_0}$, $\boldsymbol{\delta}$

observe for each country c $\mathbf{y_c}$, $\mathbf{s_c}$, $\mathbf{n_c}$

then can estimate $\boldsymbol{\alpha}/(\mathbf{1}-\boldsymbol{\alpha})$

Assumptions and measurement

- Assume
  - **A_0** varies across countries, but **g** is same for each country c $\ln A_{0\,c} = a_0 + \varepsilon_c$
  - **g** is constant across countries
  - **δ** is constant across countries

$$(g + \delta) = .05$$

- Measure
  - s_c average net real investment share of GDP I/Y
  - n_c rate of growth of the 15-64 aged population

Expectations

$$\ln y_c = a + a_1 \ln s_c + a_2 \ln (n_c + .05) + \varepsilon_c$$

$a \rightarrow \ln a_0 + \ln (1+g)^t$ (constant across countries)

$a_1 \rightarrow \alpha/(1-\alpha)$; $a_2 \rightarrow -\alpha/(1-\alpha)$

$.05 = \delta + g$

Find a , a_1 , a_2 that provide best fit across countries

expect $a_1 = - a_2$

$a_1 = \alpha/(1-\alpha) \rightarrow \alpha = a_1/(1+a_1)$

expect $\alpha \approx 1/3$

TABLE I
ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985

	Sample:	Why?	Non-oil	Intermediate	OECD
	Observations:		98	75	22
	CONSTANT	a	5.48	5.36	7.97
			(1.59)	(1.55)	(2.48)
$\ln s_c$	ln(I/GDP)	a_1	1.42	1.31	0.50
			(0.14)	(0.17)	(0.43)
	$\ln(n + g + \delta)$	a_2	−1.97	−2.01	−0.76
			(0.56)	(0.53)	(0.84)
	$\overline{R}^2$		0.59	0.59	0.01
	s.e.e.		0.69	0.61	0.38
	Restricted regression:				
	CONSTANT		6.87	7.10	8.62
			(0.12)	(0.15)	(0.53)
	$\ln(I/GDP) - \ln(n + g + \delta)$		1.48	1.43	0.56
			(0.12)	(0.14)	(0.36)
	$\overline{R}^2$		0.59	0.59	0.06
	s.e.e.		0.69	0.61	0.37
	Test of restriction:				
	p-value		0.38	0.26	0.79
	Implied α		0.60	0.59	0.36
			(0.02)	(0.02)	(0.15)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

Basic Solow: Does a pretty good job

- Coefficients have predicted signs
  - $a_1=-a_2$ at least statistically
- The first two samples are highly statistically significant (high R^2)
- But . . .
  - OECD Sample bad R^2
  - Implied α is too high 0.6 for first two samples
    - 0.36 for OECD, but that has bad R^2
  - Constant is changing OECD much more efficient than others

Augmented Solow

$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}$

K is capital, H is human capital

Still ~~y~~ = Y/AL ~~k~~ = K/AL ~~h~~ = H/AL

so ~~y~~ = ~~k~~$^{\alpha}$ ~~h~~$^{\beta}$

Now invest in H and K: $I_H = s_H Y$ $I_K = s_K Y$

Assume depreciate at same rate

$\Delta H = s_H Y - \delta H$ $\Delta K = s_K Y - \delta K$

$$\Delta \not{k} = s_K \not{y} - (\delta+n+g)\not{k} \qquad \Delta \not{h} = s_H \not{y} - (\delta+n+g)\not{h}$$

in steady state $\Delta \not{k} = 0$ and $\Delta \not{h} = 0$

$$\not{k}^* = [s_H^{\beta}\, s_K^{1-\beta} /(\delta+n+g)]^{1/(1-\alpha-\beta)}$$

$$\not{h}^* = [s_K^{\alpha}\, s_H^{1-\alpha} /(\delta+n+g)]^{1/(1-\alpha-\beta)}$$

$$y = A\, \not{k}^{\alpha}\, \not{h}^{\beta} \quad \rightarrow \quad y = A\, (\not{k}^*)^{\alpha}\, (\not{h}^*)^{\beta}$$

$$y^* = A[s_H^{\beta}\, s_K^{1-\beta} /(\delta+n+g)]^{\alpha/(1-\alpha-\beta)}\, [s_K^{\alpha}\, s_H^{1-\alpha} /(\delta+n+g)]^{\beta/(1-\alpha-\beta)}$$

One estimating equation

$$\ln y = \ln A - (\alpha+\beta)/(1-\alpha-\beta)\ \ln(n+g+\delta) + \alpha/(1-\alpha-\beta)\ \ln s_K + \beta/(1-\alpha-\beta)\ \ln s_H$$

If we observe $\mathbf{s_K}$, $\mathbf{s_H}$, and **n** , what coefficients produce the best fit across countries?

What happens if we run this regression ***without*** *human capital measure?*

Savings rate for human capital

- Use the proportion in secondary school times the proportion of the working age population that is of school age (15-19) for s_h
- Argue that approximation is fine: as long as SCHOOL is proportional to s_h

TABLE II
ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n + g + \delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\overline{R}^2$	0.78	0.77	0.24
s.e.e.	0.51	0.45	0.33
Restricted regression:			
CONSTANT	7.86	7.97	8.71
	(0.14)	(0.15)	(0.47)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.73	0.71	0.29
	(0.12)	(0.14)	(0.33)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.67	0.74	0.76
	(0.07)	(0.09)	(0.28)
$\overline{R}^2$	0.78	0.77	0.28
s.e.e.	0.51	0.45	0.32
Test of restriction:			
p-value	0.41	0.89	0.97
Implied α	0.31	0.29	0.14
	(0.04)	(0.05)	(0.15)
Implied β	0.28	0.30	0.37
	(0.03)	(0.04)	(0.12)

Observations

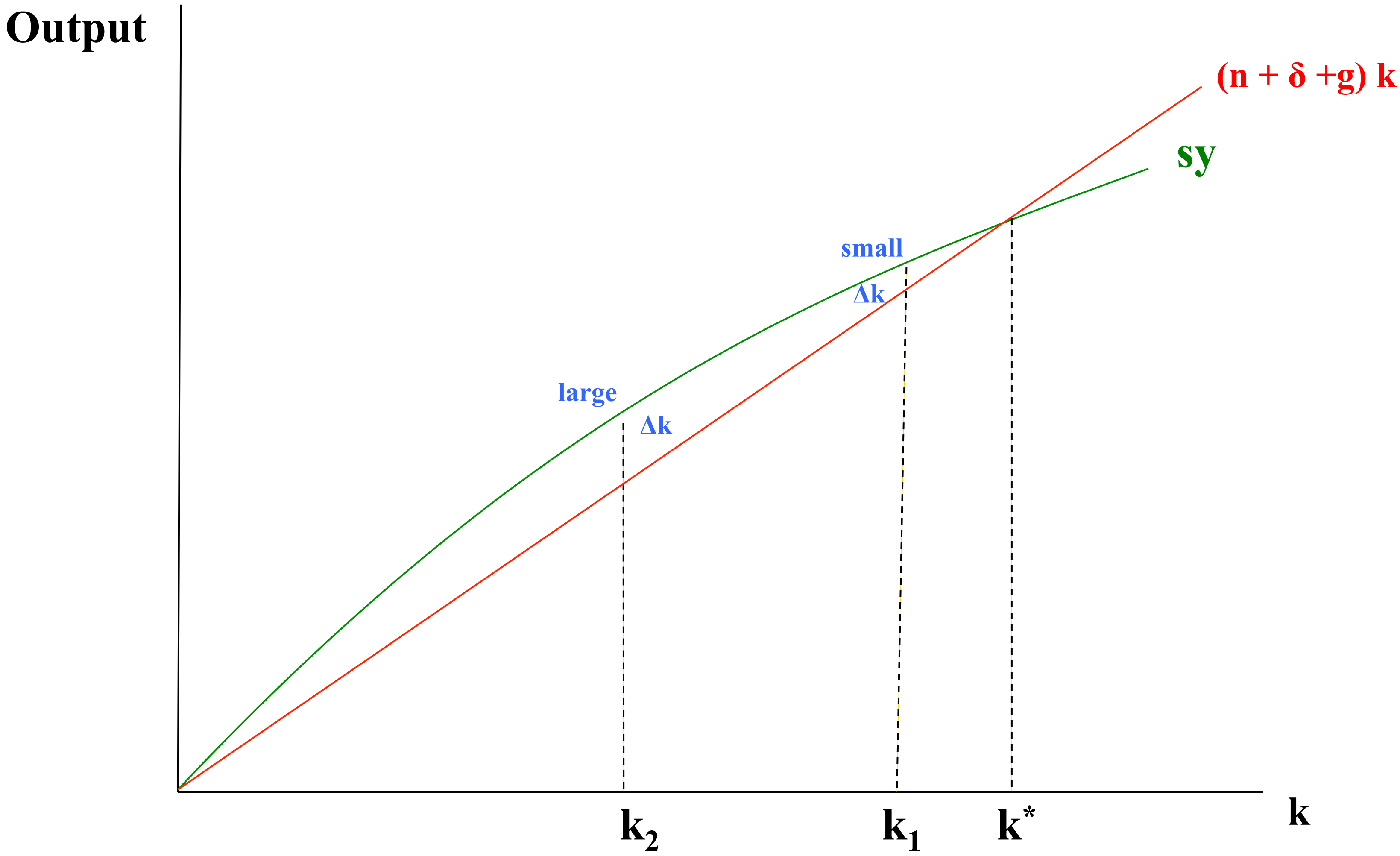
- Sum of the coefficients should be zero, and is (statistically)
- High R^2 (except for OECD)
- The implied coefficients for α and β are reasonable, both around 1/3
  - Except for (OECD)
- But . . .
  - OECD still a problem
  - Changing constant

Convergence

- Convergence is the hypothesis that poor countries should grow faster than rich countries and so catch up or converge.
- Why expect convergence:
  - Solow suggests that steady growth only depends on technological progress
  - MRW argue technological progress the same across all countries—once something discovered everyone uses it
  - Countries further from steady state should grow faster

Convergence in Solow Diagram

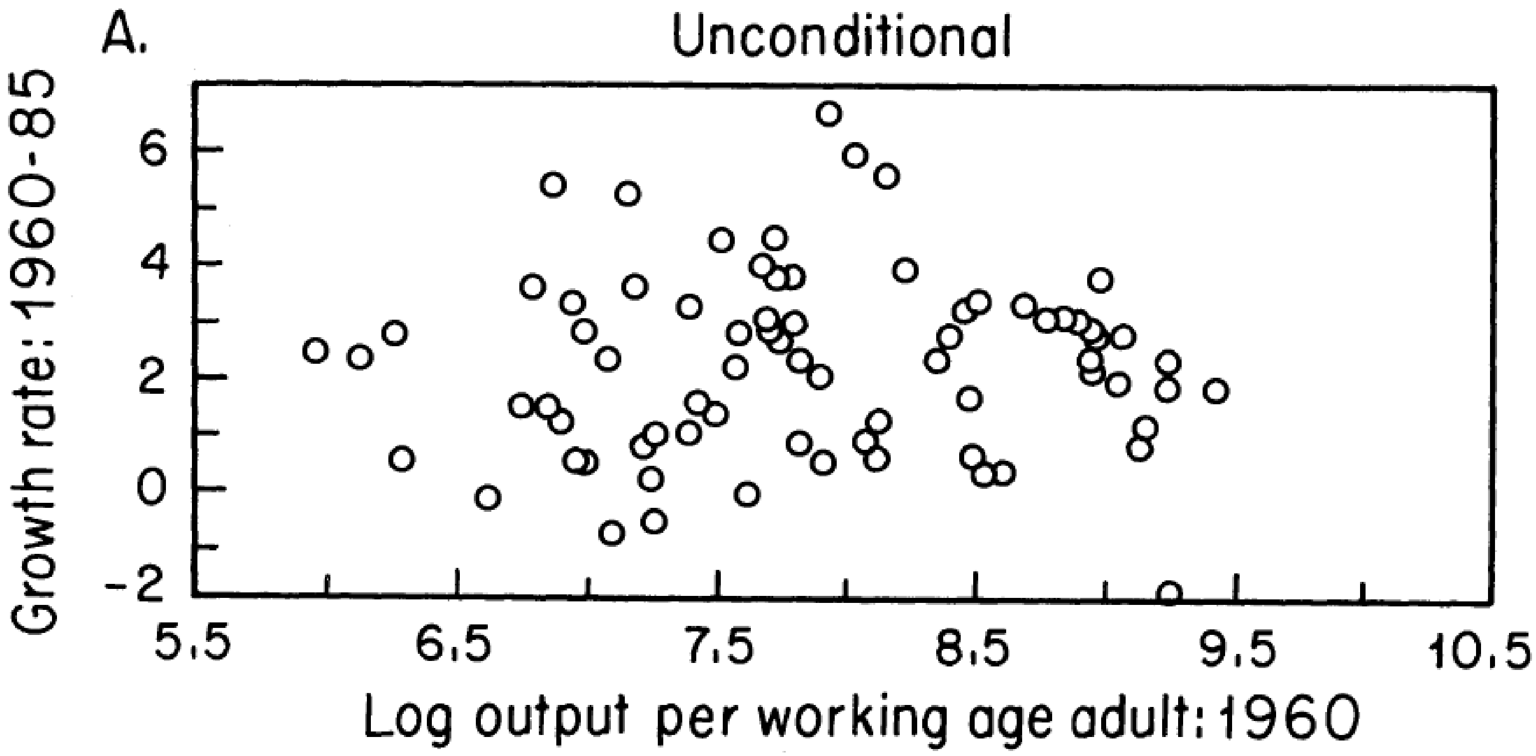
growth at k_2 > growth at k_1



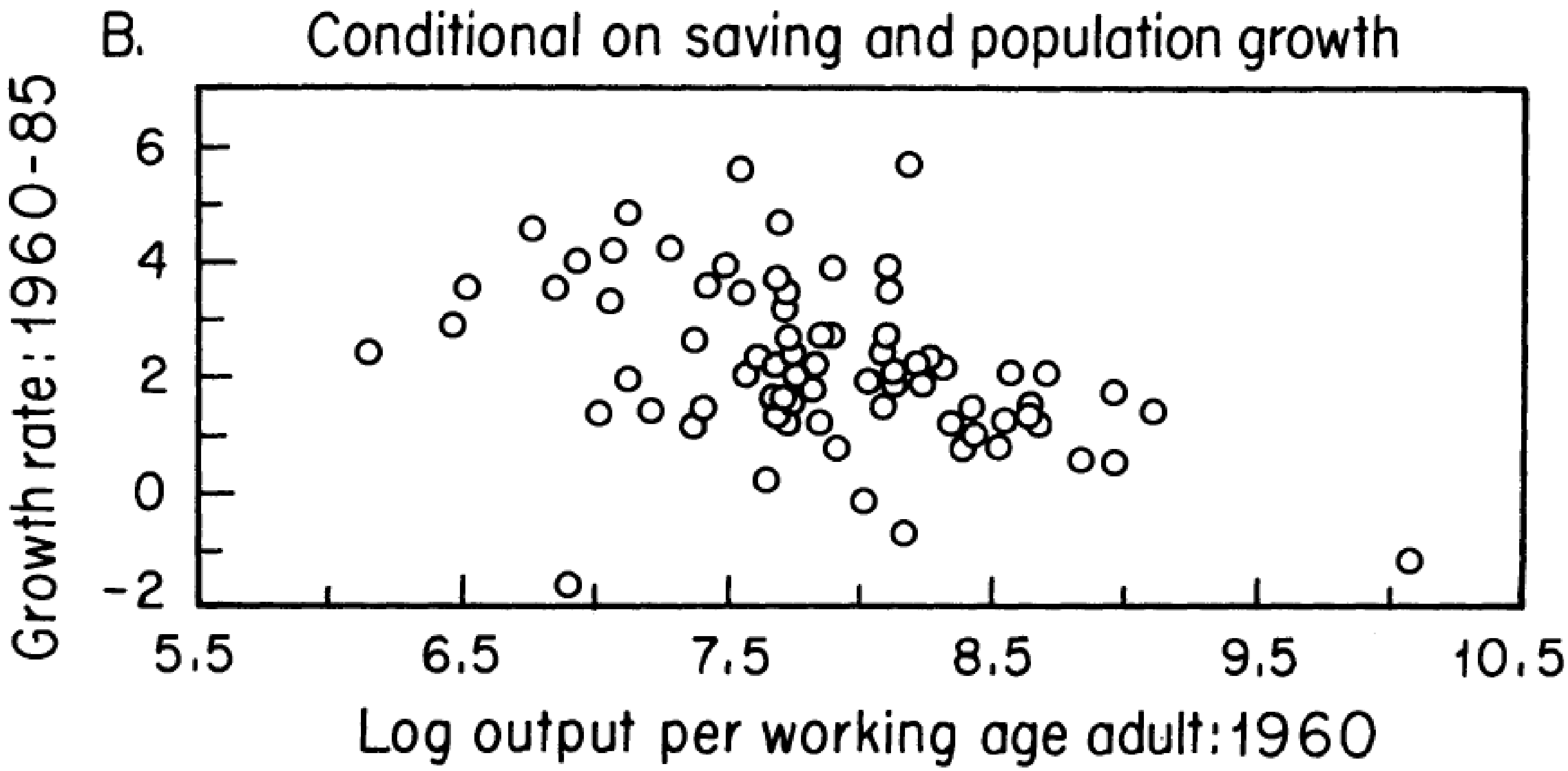
MRW: Conditional Convergence

- Countries converge to their own steady state which depends on population growth, savings rate and depreciation
  - Countries will converge *conditional* on the determinants of the steady state.
- Countries further away from their own steady state will grow faster
- Basic empirical observation (more next time)
  - Similar countries (OECD, states within US) have converged, other not.

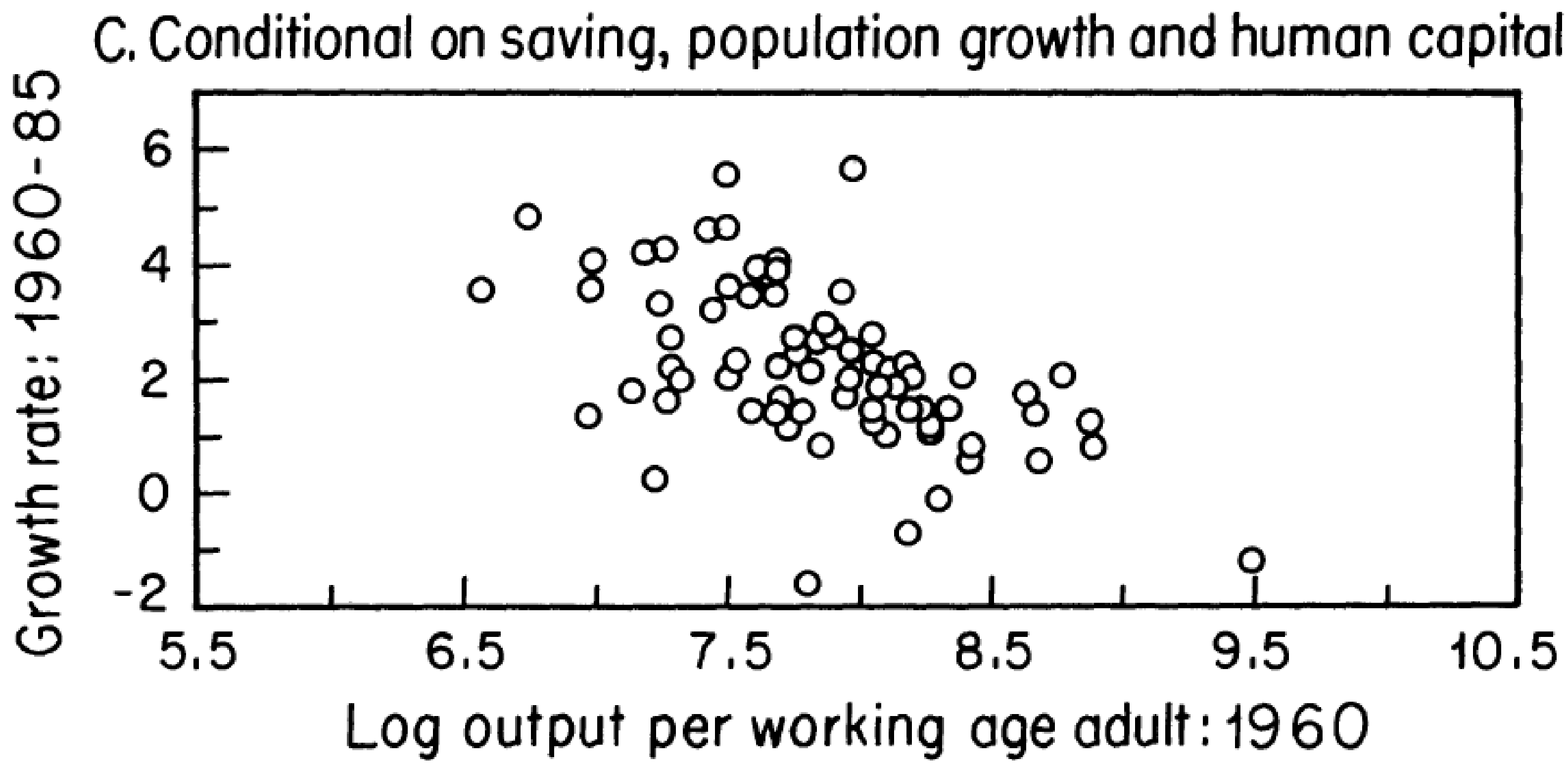
Unconditional Convergence



Partly Conditional Convergence



Conditional Convergence



Interest Rate differentials

- Saving produces new capital
- Interest rate is what borrower (firm acquiring capital) pays for savings
- If financial markets are efficient and competitive, the price pay for savings is the amount of extra value from them
- So the interest rate should be the marginal product of capital
    - Marginal return from investing more should be equal to the marginal cost of investing more.

In Solow, what is MPK?

$Y = K^{\alpha} (AL)^{1-\alpha}$ $\qquad\qquad$ $\underline{k} = K/AL$

so the marginal product of capital

$dY/dK = \alpha\, K^{\alpha-1} (AL)^{1-\alpha} = \alpha\, (K/AL)^{\alpha-1} = \alpha\, \underline{k}^{\alpha-1}$

Also: $\alpha\, Y/K = \alpha\, K^{\alpha} (AL)^{1-\alpha} / K = K^{\alpha-1} (AL)^{1-\alpha}$

so $MPK = \alpha\, \underline{k}^{\alpha-1} = \alpha$ / capital-output ratio

Since we think $\alpha < 1$, then $\underline{k}$ ***up*** → MPK ***down***

Capital Flows

- So countries with
  - High savings, should have low interest rates, and capital outflows
  - High population growth high interest rates and capital inflows
  - Large population, little capital (developing countries) further from steady state, high interest rates and capital inflows
- Sometimes description works (China has large inflows of FDI, high savings) mostly not (Africa mostly capital outflows)