EC 2273 Problem Set 2

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Spring 2017

ANSWERS

1. **Playing with Harrod-Domar** Putting numbers to H-D is a good way to understand what is going on. Suppose the national savings rate is 20% and the capital output ratio is 5. Initially assume that the depreciation is zero ($\delta = 0$).

   (a) Calculate the rate of growth of GDP (Y). What savings rate would be required for growth of 10% a year? Alternatively, for savings rate of 20% what capital output ratio would be required for growth of 10%. Explain why savings needs to rise or the capital output ratio needs to fall to get to 10

   Answer: For all of this use $g = s/a$ where a is the capital-output ratio, g is GDP growth and s is the savings rate. So for the first part g=20/5 =4% a year. To achieve 10% growth 10*5 = 50% savings rate. Or 20/2 =10 so captital output of 2. More savings means more growth. Better capital output means doing more with each additional unit of capital.

   (b) Suppose that depreciation is 2% a year. What is growth?

   Answer: $g = s/a - \delta$. So growth is 2%.

   (c) The more general H-D growth model includes population growth. Suppose the savings rate is 20%, the capital-output ratio is 5, depreciation is 2% and population grows at 3%. What is the rate of GDP growth? What is the rate of per capita growth? Do they provide different views on the long term prospects of the country? What must population growth be for per capita growth to be zero?

   Answer: $g = s/a - \delta$. GDP growth is just 2% as before. To make it per capita $g = s/a - \delta - n$ and so GDP per capita growth is -1% per year which is negative! So this country is declining. Population growth must be 2% per year to avoid long term decline.

   (d) Easterly criticizes the H-D model. But it is very useful to a policy maker because of its clear connection between savings, depreciation, efficiency, population growth, and income growth. Does this basic model help explain why population growth has played such a large role in the way people think about development? Why?

   Answer: Without labor contributing anything, population growth is a pure negative in terms of GDP per capita in this model. So any population growth reduces growth and harms living standards.

2. This question asks you to work through the basic Solow model to check your understanding. The equations that make up the standard Solow model with population growth are:

$$S = I \tag{1}$$
$$S = sY \tag{2}$$
$$\Delta K = I - \delta K \tag{3}$$
$$\Delta L/L = n \tag{4}$$
$$Y = AK^{\alpha}L^{1-\alpha} \tag{5}$$

(a) Explain in words what each of the model outcomes stand for S, I, Y, and K, and what the model parameters represent s, δ, n, A, α.

Answer:

(b) Define $k = K/L$ and $y = Y/L$. Write y in terms of k, A and α and simplify your result.

Answer: $y = Ak^{\alpha}$.

(c) The equation

$$\Delta K = k\Delta L + L\Delta k$$

relates changes in K to changes in k and L. Use it and the model assumptions to find an expression for Δk in terms of k and the model parameters.

Answer: $\Delta k = Ak^{\alpha} - (n + \delta)k$.

(d) Your answer to the previous question implies that when $\Delta k = 0$ capital per person is not increasing or decreasing. Find the k^* for which $\Delta k = 0$.

Answer: $sAk^{\alpha} = (\delta + n)k$, solving for k gives

$$k^* = \left(\frac{sA}{\delta + n}\right)^{1/(1-\alpha)}$$

(e) Is capital per person in steady state higher, lower, or the same when:

i. Savings increases
ii. Technology increases
iii. Population growth increases

Answer: Higher s means higher k. Higher A means higher k. Higher n means lower k.

(f) In steady state, how does the rate of growth of a country with high savings (s^H) compare to a country with low savings (s^L)?

Answer: Savings does not affect the rate of growth in steady state, so s^L and s^H have same growth rate.

(g) Draw the standard Solow Diagram which shows capital deepening and capital widening. Label the equilibrium k^*. Show graphically how the economy adjusts if suddenly its labor force doubles (say because the country stops oppressing women and uses their talents in the workforce) but the population growth rate remains the same otherwise.

Answer: Standard Solow diagram. L doubling halves k, which then increases back to k^*. Small variations are fine, as well as additional information, (like showing the equations, and output per person, but must have the fall and then increase over time).

3. **Cities and Solow** This question is hard. It asks you to reconsider the Solow model as it may apply to city land use. For many cities like Hong Kong, Manhattan, or Boston, land is at a premium. Finding better uses for land, and bringing in productive people to work determines growth in these space constrained cities. Suppose the production function for a city is:

$$Y = (AT)^{\alpha} H^{1-\alpha}$$

where T is land, A is the efficiency with which land is used, and H is human capital.

(a) While some cities such as Mumbai and Boston have grown by filling in land (South Bay, Back Bay and Chinatown were all "reclaimed" from the sea), others have grown upwards like Manhattan. In either case, the available space for working and living increases as new floors are added or new neighborhoods built. T is the original land, and A is how efficiently that land is used, so AT is the effective amount of land available: effectively the number of square meters of space to live and work. A sprawling city like Los Angeles has a large T, but low A, while Manhattan has a low T and high A.

Let's define output and labor in terms of "effective" density or AT. So $y = Y/(AT)$ is the output per square meter of work and living space and $h = H/(AT)$ is the human capital per square meter. Using the production function, write y in terms of h and α.
Answer: $y = h^{1-\alpha}$.

(b) City budgets put a great deal of emphasis on schools. Let's say that through death and spring break, human capital deteriorates (depreciates) at rate δ, but cities save a certain fraction of their income s to reinvest in human capital. Then $\Delta H = sY - \delta H$. Assume A is growing at rate g so that $\Delta A/A = g$. Using the chain rule we know that since $H = ATh$, then in changes $\Delta H = AT\Delta h + hT\Delta A$.

Find the Solow expression for the change in h in terms of s, h, δ, α, and g. (Hint: It should look much like the Solow difference equations for changes in k that we have considered, except here the change is in h.)
Answer: $\Delta h = sh^{1-\alpha} - (\delta + g)h$.

(c) Solve for h^*, the steady state human capital per effective land. It should be in terms of s, δ, α, and g.
Answer: $h^* = \left(\frac{s}{\delta+g}\right)^{1/\alpha}$.

(d) Draw a "Solow" diagram with h on the x-axis and y on the y-axis showing the human capital deepening and human capital widening curves, h^*, and the direction of movement of h.

(e) Do cities which invest more in schooling (have higher s) have higher steady state human capital per effective land area? Do they have higher growth in human capital per effective land area?

Answer: Steady state increases with s, so higher human capital per area. No higher growth, which is zero in steady state.

(f) By definition $H^* = ATh^*$. At what rate does H^*, the total human capital in a city, grow? Does your answer explain why many economists (your professor included) think that the most important policy failure in the United States is zoning laws that limit density?[1]

Answer: g the rate of growth of A.

[1] If you are interested in more on this topic, a great place to start is Matt Yglesias. See his article here for one quick take: http://www.vox.com/2014/4/25/5650816/nimbys-are-killing-the-national-economy. The lack of affordable housing in many cities hurts poor people the most, since they are often pushed the farthest out, often to places where there are not jobs they can get to, making it even harder to work. Recent college graduates are also hurt (since they are at the bottom of the earnings curve), and often live with their parents because housing is so expensive. The horror. http://www.vox.com/2014/7/10/5887095/2-charts-prove-millennials-really-are-living-with-their-parents-in