Development Economics Problem Set 3

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Answer Key

1. **Where does capital go?** Easterly and Levine (2001) assert that capital flows where it is already abundant. This questions asks you to do some calculations to check that.

   Go to the World Bank's data website http://databank.worldbank.org/data/databases.aspx and select the World Development Indicators. Download the series "Foreign direct investment, net inflows (BoP, current US$)", "GNI per capita, PPP (current international $)", and "Population, total" and one other variable of your choosing for 2014 for all available countries (the data for many countries for 2015 is still not available). When you download it, get the the countries on the rows and series on the columns by clicking on "Modify Orientation," "Custom" and Time on Page, Country on Row, Series on Column , and export the data into a spreadsheet.

   (a) Sort the data by GNI per capita. Note that a number of countries do not have GNI per capita data. What is the ratio of the highest to the lowest GNI per capita? (We talked about variations on this number, but it is useful to see what it is in the data you've downloaded).

   Answer: I made my calculations with a previous year of data, so the calculations from this year will be different. I have Norway/Burundi=84640/150=564. A large number is fine.

   (b) Calculate the sum of FDI in the world. What fraction of the total did the 20 poorest nations have?

   Answer: I calculate 0.0054. Again, any similar number is fine.

   (c) What fraction of the total FDI did the 20 richest nations with FDI reported have?

   Answer: I calculate 0.57.

   (d) What fraction of the total FDI does China receive? Since China is the most populous nation, and has a huge economy, even though it is not among the richest in per capita terms, does the importance of China suggest that comparing total inflows of FDI to GNI per capita is problematic?

   Answer: I calculate 0.071. China suggests that comparing FDI per capita may be more appropriate.

(e) Calculate FDI per capita. Make a scatter (X-Y) chart with GNI per capita on the x-axis and FDI per capita on the y-axis. (Exclude the two richest countries, and the countries which have no GNI.) Print the chart and include it with your problem set. Does it support the assertion that capital flows to countries that are rich already?

Answer: Yes, FDI does flow to richer countries but not very strongly.

(f) Make a scatter (X-Y) chart with GNI per capita on the x-axis and the other variable you downloaded on the y-axis. Is GNI per capita related to your variable? Why?

Answer: Getting excel to graph is hard. Reasonable attempts should get full credit.

2. **Computing residuals** The Easterly and Levine (2001) paper shows a breakdown of the determinants for growth accounting in table 1. The basic idea is to find all of the things that add into growth, and determine which changes contributed which shares. Using the Penn World Tables,[1] a compendium of national accounts for many countries over time (used by both the Easterly and Levine (2001) and the Mankiw, Romer, and Weil (1992) papers, although in different versions), this question asks you to do a growth accounting exercise for Brazil.

Suppose m is the fraction of the population P that works. Our standard Cobb-Douglas production function looks like:

$$Y = AK^{\alpha}(mP)^{1-\alpha}.$$

In class we have just used the labor force $L = mP$ and assumed that $m = 1$.

(a) Most of the time we use per capita when describing a country, so write GDP per capita y in terms of $k = K/P$, m, A, and α.

Answer: Divide by P, and everything should work out to $y = Ak^{\alpha}m^{1-\alpha}$.

(b) If many things are growing at the same time we can consider how they grow together with some simple rules. If $Y = WX^{\alpha}Z^{\beta}$, then

$$\frac{\Delta Y}{Y} = \frac{\Delta W}{W} + \alpha\frac{\Delta X}{X} + \beta\frac{\Delta Z}{Z}.$$

Use this growth rule to find the growth of income per person in terms of the growth of A, k, and m. You should end up with an equation that looks just exactly like Easterly and Levine equation 2 except with growth in m instead of n.

Answer: $\Delta y/y = \Delta A/A + \alpha\Delta k/k + (1-\alpha)\Delta m/m$.

(c) With $k = K/P$ the chain rule says that $\Delta K = P\Delta k + k\Delta P$. Write the growth in population $\Delta P/P = n$. Show how to combine this equation with the capital accumulation equation $\Delta K = sY - \delta K$ to find the standard expression for capital per person growth $\Delta k = sy - (n+\delta)k$.

[1]The data are posted at www.anantnyshadham.com/teaching in a link to the right of the link for this problem set. They are originally taken from Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 6.3, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, November 2012. datacentre.chass.utoronto.ca/pwt/

(d) The share of investment in GDP (I/Y) is savings (s). As an example: if $k_{1950} = 3000$, $y_{1950} = 1000$, $\delta = 0.05$, $n = 0.03$, and $I/Y = 0.25$, calculate k_{1951}.

Answer: $k_{1951} = 0.25 * 1000 - (0.03 + 0.05)3000 + 3000 = 3010$.

(e) From the Penn World Tables, I have downloaded data on Brazil and put it into a spreadsheet. We want to find the series that lead into our model above.

i. First find m, the share of population working or $m = L/P$. This variable does not exist in the basic PWT data, but we can figure it out by combining Real GDP per capita (rgdpch) and Real GDP per Worker (rgdpwok). Find m for all years. How has it changed from the 1950's to today? How is m related to the dependency ratio?

Answer: m has increased from 33% to 50%. The dependency ratio is the ratio of the non-working population to the working population. m is the share of working population, so the dependency ratio is m/(1-m). The exact formula is not necessary.

ii. Find the growth rate in population n for each year starting in 1951 (it is a growth rate so calculate growth in 1951 as the change from 1950 divided by population in 1950). What is the average population growth in Brazil since 1950?

Answer: I get 2.2 percent.

iii. Next find k. Capital per person is hard, since we don't know capital to start with. A common approach is to make a reasonable guess and update the guess using investment. Since capital depreciates over time, the guess becomes less and less important over time. Guess that the capital output ratio in 1950 is 3, and find k for 1950. The variable ki is the percentage of investment in GDP (I/Y). We can use the capital increase equation we found earlier to calculate k in 1951, assuming that $\delta = .05$. So

$$k_{1951} = (ki/100)y_{1950} - (n + \delta)k_{1950} + k_{1950}.$$

Find k for all years. As a check on your calculations, find the capital output ratio $a = K/Y = k/y$ for all years. It should be 3 in 1950. What is the average? Was 3 a bad guess? Does your answer imply that Brazil is more or less efficient than 3?

Answer: My calculations show average to be 2.5, which suggests 3 is probably not too bad. Since it only takes 2.5 capital to make 1 output, and Brazil is more efficient than countries with 3 (the US is 2.7). That may not be reasonable, which suggests something is being left out (like efficiency of labor).

(f) Now calculate the growth rates of the important variables in the growth accounting equation we found in b. Starting in 1960, find the $(y_{1960} - y_{1959})/y_{1959}$, and similarly for $\Delta k/k$ and $\Delta m/m$. Do the same thing for all the years after 1960. Why does it make sense to start so long after the data begins (the exact date is somewhat arbitrary)? What is the average growth in y?

Answer: I start in 1960 since we guessed the capital output ratio, but the importance of guess declines over time. Average growth in y is about 2.6%.

(g) Let's assume that $\alpha = 1/3$. Now find for each year starting in 1960 the "Solow Residual," the part left over after accounting for labor force growth and capital growth. Since we know α, $\Delta y/y$, $\Delta k/k$ and $\Delta m/m$, we can calculate $\Delta A/A$ from part (b). What is the average TFP growth rate?

Answer: Average growth in A is 1.1% (any number around this should be fine since there are multiple reasonable ways to do the calculations).

(h) Find the share of growth contributed by the residual. (Hint: Just divide growth in A by growth in y.) What is the average share of TFP in growth? Has Brazil grown by making better use of inputs, or just increasing inputs, or some of both?

Answer: The average share is about 49%. So about half of growth is coming from TFP growth.

(i) Graph the five year rolling average (add up the previous five years and divide by 5) of TFP growth, and growth in y on the same graph with year on the y axis. Print your graph and include it with your problem set. The 1980s and 1990s are sometimes thought of as lost decades for Latin America. Does your graph support that assertion, and offer a reason why?

Answer: TFP growth goes negative in the mid 1980s (5 year average) so it seems that the ability to produce more efficiently dried up. Since long run growth comes from TFP growth, the graph suggests stagnation in the economy.

(j) Brazil suffered from high inflation in 1991-1994, and the Russian currency collapse in 1998 brought a currency crisis in Brazil in 1998-1999. Since TFP picks up everything that is left, does your graph suggest that the Solow Model, which ignores financial markets and the positive or negative effects of inflation, might be leaving something out?

Answer: Big drops in TFP and growth following both. Both suggest that TFP is picking up something other than the technology of using inputs, and it may be important to understand financial markets.

3. **The O-ring in study groups** The study group can be very important in university, and especially for medical school and law school. This question considers a group of five students who come together to study five chapters in a book. The amount of the book they cover determines their grades (each group member gets the same grade). Students are of different "qualities" in that they help their group cover different fractions of the assigned chapter (no student can study more than one chapter). The five students, with the proportion of his or her assigned chapter each student can cover in parentheses, are: Annie (0.9), Bob (0.6), Carl (0.5), Damon (.6), and Ernesto (0.8).

   (a) First suppose that the book is made up of chapters on entirely different topics. So the amount of the chapter Bob covers does not affect the group's understanding of Ernesto's chapter.

      i. How much of the book do they cover, and what is the group's grade? (If they cover 85% of the book, they get a grade of 85.)
      Answer: 68 (68%). Formula for the grade is:

$$\text{Grade} = 100 * (q_A + q_B + q_C + q_D + q_E) / 5$$

      ii. Suppose the group switched out Carl for Carlita who covers 0.6 of her chapter. How much better is the group's grade?

Answer: New grade is 70, so an increase of 2 points.

iii. Suppose the group instead switched out Ernesto for Enrique who covers 0.9 of his chapter. How much better is the group's grade?
Answer: same as in (ii): 2 point increase.

iv. What is the marginal return (in terms of better group score) of getting someone who can cover a fraction 0.1 (10 percentage points) more of her chapter? Is it the same for the entire group?
Answer: Marginal return of doing so is 2 points, for everyone in the group.

(b) Now suppose that the first chapter sets up the notation and terms for the rest of the book. So how well Annie covers the first chapter determines how well Bob can cover his chapter. The rest of the chapters are unrelated, and the group is still tested on the first chapter. To be precise, if q_A is Annie's coverage, then the grade they receive is given by:

$$\text{Grade} = 100 * (q_A + q_A * (q_B + q_C + q_D + q_E))/5$$

i. Explain how the grade production function captures the description of the way the book is set up.
Answer: $\frac{\partial G}{\partial q_B} = q_A$, the same for derivative with respect to q_C, q_D, and q_E. So the marginal product of everyone (except for Annie) depends on Annie's skills.

ii. What proportion of the book do they cover?
Answer: 63.

iii. If Annie is sick and can't study ($q_A = 0$ instead of 0.9), what is the group's score?
Answer: 0.

iv. Suppose the group dropped Carl for Carlita who covers 0.6 of her chapter. How much better does the group score?
Answer: New grade is 64.8, so an increase of 1.8 points as compared to (ii).

v. Suppose the group instead dropped Ernesto for Enrique who covers 0.9 of his chapter. How much better does the group score?
Answer: Same as in (iv): an increase of 1.8 points as compared to (ii).

vi. Suppose instead the group can get Anjini, who can cover 100% of her chapter to replace Annie. How much better does the group score?
Answer: New grade is 70, so an increase of 7 points as compared to (ii).

vii. With Anjini in the group, suppose the group dropped Carl for Carlita. How much better does the group score? Is the marginal value of Carlita the same with Anjini as it was with Annie? Why or why not?
Answer: New grade is 72, so an increase of 2 points as compared to (vi). Carlita's marginal value with Annie (answer in (iv)) is 1.8, and her marginal value with Anjini is 2. So with Annie, her marginal value is lower. This happens because one's productivity depends on the first person's skills. Anjini is more skilled than Annie, so Carlita's marginal value is higher with Anjini than with Annie.

(c) Compare the two grade production functions, calling the first one additive and the second one multiplicative to separate them. Let's assume that there are two groups which are identical except that one is lead by Anjini and the other by Annie (so the rest of each group is composed of Bob, Carl, Damon, and Ernesto). In both production functions

Annie and Anjini would like to work with better students. Let's suppose both Carl and Carlita are paid tutors, who help the study group, but do not get graded. The rest of the group is graded the same way, but Carl and Carlita go wherever the money is best.

i. What is the marginal value in grades of Carlita (dropping Carl for Carlita) for the group with Annie, and for the group with Anjini under each production function?
Answer: Marginal value of Carlita under Additive production function (PF): $\frac{\partial G}{\partial q_C} \times (q_Carlita - q_Carl) = \frac{100}{5} \times (12 - 10) = 2$, whether it is Annie or Anjini in the group. Marginal value of Carlita under Multiplicative PF: $\frac{\partial G}{\partial q_C} \times (q_Carlita - q_Carl) = \frac{100}{5} q_A \times (12 - 10) = 2q_A$, so Carlita's marginal value is $2 \times 0.9 = 1.8$ with Annie, and $2 \times 1 = 1$ with Anjini.

ii. If both Annie and Anjini offer Carlita $100 times the improvement in grades (so 100 times the difference in grades between when she is in the group and when Carl is in the group), how much are they offering under each production function?
Answer: Under Additive PF, they both offer $\$100 \times 2 = \$200$. Under Multiplicative PF, Anjini offers $200 and Annie offers $\$100 \times 1.8 = \$180$.

iii. If Carlita goes to the group where she is offered more, will she end up with Anjini or Annie (or is she indifferent) for each production function?
Answer: If Annie leads with the group with Multiplicative PF, and Anjini leads the group with Additive PF, then Carlita will choose Anijni, because her payoff with Anjini ($200) is higher than with Annie ($180). If Annie leads with the group with Additive PF, and Anjini leads the group with Multiplicative PF, then Carlita is indifferent between the two groups, because her payoff is $200 in either case.

(d) Under either grade production function, is it better to go to a school with lots of good students with whom to study? Under which production function is the incentive for good students to congregate stronger? Does this model explain why good students tend to go to the best schools they can get into (and why there is such a premium on getting into the best schools), even though a better student may get more attention and resources if he or she goes where talent is scarce? (It may help to consider what the average score in each production function is if all of the students are 0.5, and if all of the students are 0.6).

Answer: Under both PFs, it is better to go to a school with lots of good students, because in either case, the group grade increases if we replace one student with his/her more competent counterpart (replace Carl with Carlita).

Under Multiplicative PF, the incentive for good students to congregate is stronger, because the marginal product of a student depends on the aptitude of other students (Carlita prefers to work with Anjini under Multiplicative PF). Therefore this PF explains why good students tend to go to the best schools they can get into. It also helps explain why there is such a premium to get into such schools. To see why: under Additive PF. if all students have $q = 0.5$, then average grade is 50, but if $q = 0.6$, then average grade is 60. That is if each student can do 20% better, the school will do 20%. But under Multiplicative PF, if each student can do 20% better, the school will do 36% better. Schools that have lots of good students will do better and can bid for better students with premiums, such as higher GPA to get admitted.

(e) Some state schools are willing to offer a free ride plus a stipend to very good students. Does the multiplicative grade production function explain why good students (who can presumably get into better schools) have to be offered such a sweet deal, and why it is in the best interests of the school to offer such a deal?

Answer: Yes. As explained in (d), a good student has strong incentive to go to the best school she can get into. A less competitive school if wants a good student will need to offer her an attractive scholarship package to compensate for her alternative options, given that she stands high chance of getting into better schools. It is also in the school's best interest to offer good students with generous scholarship, because a good student will also contribute to other students' success, which in turn will make the school better off.