Development ECON 2273 Problem Set 5

Professor Nyshadham

Spring 2017

Answer Key

1. **Poverty Gap Index** The Foster-Greer-Thorbecke poverty index is

$$P_\alpha = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_p - y_i}{y_p}\right)^\alpha 1(y_i < y_p)$$

where N is the size of the total population, y_p is the poverty line, y_i is the consumption of person i, α is an index which determines how much we care about inequality among the poor, and $1(y_i < y_p)$ is a function which is 1 if $y_i < y_p$ and 0 otherwise.

   (a) Todaro and Smith define the same index, but only sum from $i = 1$ to H and leave off the function $1(y_i < y_p)$. What is H and why are the two definitions equivalent?

   Answer: N is the number of people in poverty. So T&S are just summing over the people in poverty. $1(y_i < y_p)$ is zero if the person is not in poverty so the two definitions are equivalent.

   (b) The Poverty Gap Index or Normalized Poverty Gap is the Average Poverty Gap divided by the poverty line. Show that the poverty gap index is the FGT index with $\alpha = 1$.

   Answer: With $\alpha = 1$ we can pull y_p out of the summation so the PGI which is the Total poverty gap $\frac{1}{N}\sum_{i=1}^{N}(y_p - y_i)1(y_i < y_p)$ divided by Ny_p is just P_1. Students should get that you are just pulling out y_p from the sum.

2. **Returns to education** On Canvas I have posted an Excel file based on data from Psacharopoulos (1995) showing age and earnings in Bolivares per year for different education levels in Venezuela in 1989.[1] The same data is shown in Figure 8.1 in the Todaro and Smith.

   (a) Secondary school seems to last from ages 13 through 17 in the data (five years). Assuming there are no out-of-pocket expenditures during secondary school, calculate the difference between earnings with a secondary school education and earnings with a primary school education in each year starting from age 13. What is the lifetime difference in earnings from age 13 to age 60 (just add up all of the differences so we can compare it with better approaches)?

[1] The original is here http://omulga.riversstate.org.ng/Doc/psacharopoulos1995.pdf.

Answer: I calculate 1,237,295 bolivares.

(b) If we discount each year in the future at a rate of 5%, what is the net present value of the difference in earnings (take age 13 to be the first so that there is no discount on the first year). Why is the NPV so much lower than the raw sum?

Answer: I calculate 202,495 bolivares. The NPV is lower because we discount earnings 50 years in the future a great deal.

(c) Find the discount rate r which makes the NPV zero using trial and error. (Your answer will be approximate, but try to get at least the first decimal correct.)

Answer: 10.174%. Anywhere between 10.1 and 10.3 is fine.

(d) Psacharopoulos (1995) reports that the cost to the government of providing the public secondary education are 12,170 Bolivares for each student per year. The social return on education includes all of the costs of education, so find the discount rate r which makes the social NPV zero.

Answer: I calculate 7.92%. Anything 7.9 is fine.

3. In *Portfolios of the Poor*, Daryl Collins, Jonathan Morduch, Stuart Rutherford, and Orlanda Ruthven collected financial diaries from many households in the developing world. Go to http://www.portfoliosofthepoor.com/portfolios.asp and find the household data for Siddhanath and Shyamnath in Rural India (you may need to use the drop down menu). Also find one other family that interests you. Read the descriptions of the households and (briefly) answer the following questions:

   (a) Can anyone in the household read? Any women? Are the children in school (that may not be applicable for your household).

   Answer: For this and the other questions just check that they have made a reasonable effort.

   (b) What are the major sources of income for the household?

   (c) What were the major expenses? Were there any large expenses?

   (d) What financial services does the household use? Do they save? Do they borrow and and who from?

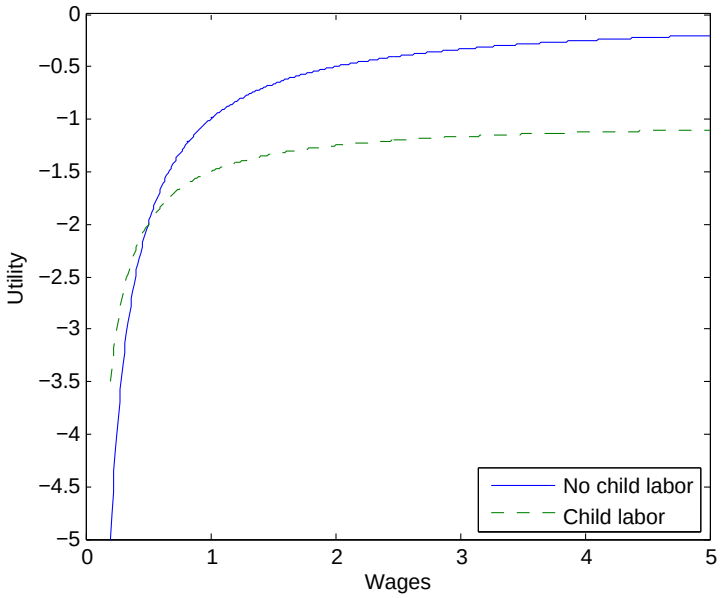
4. **Child labor as a bad equilibrium** There are 10 families in a slum in India, each of which has one adult and one child. The adults would prefer not to have their children work, but the family needs to eat, and children's wages contribute to the food available. Adults, however, will always work. Children are exact substitutes for adults—a child earns the same wage as an adult. Each family immediately consumes all its income, whether from adults or children. If the going wage is w the income of a family is

$$I = w + wC$$

where C is 1 if the family sends it child to work, and 0 if not. The utility of the each family is given by

$$u(I, C) = -1/I - C = -\frac{1}{w + wC} - C.$$

Figure 1: Utility from child labor



(a) What is the expression for the utility of a family if it does not send its child to work so C=0? What is the expression for the utility of the family if it does send its child to work so C=1? Sketch the two curves as w changes (put w on the x-axis, utility on the y axis; your sketch does not need to be from a computer, but should get how the curves relate to each other; negative numbers for utility are fine, utility is ordinal not cardinal).

Answer: Not sending children to work gives $u(w, C = 0) = -1/w$, while sending them to work gives $u(w, C = 0) = -1/(2w) - 1$. These curves are shown in figure 1. The important point is that the curves cross. At a high wage not sending children to work is better. But the value of the extra income becomes more and more important at low wages, when it is better to send children to work.

(b) At some wage the utility from sending the child to work, and getting extra income, is equal to the utility with no child labor. What is this wage?

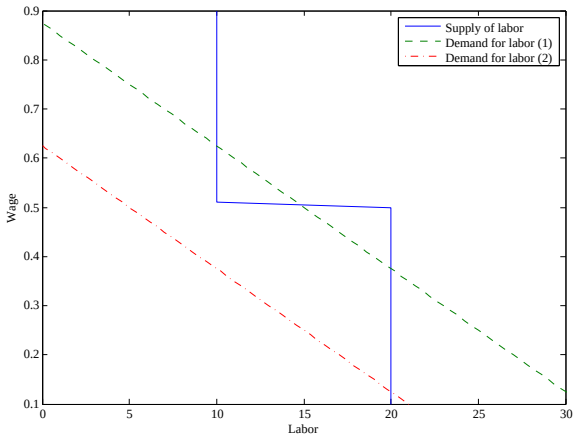
Answer: Set $-1/w = -1/(2w) - 1$. A little algebra shows that $w = 1/2$, which is where the two curves cross in the figure.

(c) Draw the aggregate labor supply curve. (For each wage on the y axis, what is the labor that is supplied by all families?)

Answer: All adults work, so at high wages the labor supply is 10. At low wages the families send their children to work, so the labor supply is 20. The transition is at 1/2. Figure 2 shows the supply of labor curve. The actual supply at 1/2 is anywhere between 10 and 20 since families are indifferent between sending their children to work or not. Note that unlike normal supply curves, the supply increases as the price decreases.

(d) Suppose aggregate demand for labor is downward sloping and is given by $L = -40(w - 1/2) + 15$. Find the wages if all children work, and the wages if only adults work. Draw

Figure 2: Labor demand and supply



the labor demand curve with the aggregate labor supply curve and indicate the (stable) equilibria.

Answer: Figure 2 shows demand for labor, which intersects the supply of labor at three points. The middle one is unstable, since if the wage were slightly higher, or supply slightly lower, everyone would prefer to not send their children to work. The figure also suggests how to solve for the wages: since we know the supply is either 10 or 20, by putting 10 and 20 into the labor demand equation it is possible to find the wages. Rearranging labor demand: $w = (-1/40) * (L - 15) + 1/2$, and plugging in $L = 10$ gives $w = 5/8$ when no children work. Setting $L = 20$ gives $w = 3/8$ when all children work.

(e) What is the utility of a family in the equilibrium where no children work? What is the family income? Does any individual family want to send its child to work?

Answer: u(5/8,C=0)=-1/(5/8) = -8/5 =-1.6. Income is 5/8. Since wage $5/8 > 1/2$, no family wants to send a child to work.

(f) What is the utility of a family in the equilibrium where all children work? What is the family income? Would any individual family prefer to not have its child work (given that all the other families are sending their children to work)?

Answer: u(3/8,C=1)=-1/(2*3/8)-1 = -7/3 =-2.3. Income is 2*3/8=4/3 which is larger than when no children work, although utility is lower.

(g) If the families could coordinate, for example by passing a ban on child labor, would it improve the welfare of the families?

Answer: Yes. Even though income is lower when children do not work, utility is higher, so the families prefer the equilibrium where no children work.

(h) Now suppose the demand for labor is lower: $L = -40(w - 1/2) + 5$. What is the sole equilibrium wage and employment? Does it involve child labor?

Answer: The new demand for labor is in figure 2 which intersects the supply of labor at only one point for $L = 20$. So the sole equilibrium is for child labor. Plugging in $L = 20$ gives that the wage is very low w=1/8.

(i) With the lower labor demand curve, suppose there is a ban on child labor, so that no children work. If one family can send its child to work illegally, will it do so? Does your answer explain why child labor still sometimes exists despite bans?

Answer: A ban on child labor forces the supply curve to be fixed at 10. But every individual family, whose preferences are summarized in the supply curve, would prefer to send its child to work at a wage of 3/8 (where the new demand curve crosses 10). So they may be willing to send their children to work illegally. Note that utility of all families may be lower if they all disobey the ban, but for any individual family the incentive to disobey the ban is still there. While the higher demand curve had two (stable) equilibria, the lower demand curve has only one, so the ban must be enforced despite parents' interests, which is very hard to do.