Development Economics

Lecture 7: Economic Growth—Too much Solow?

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EC 2273

This Lecture

1. Introduce Solow model
    1. Production Function, Assumptions, and Flow
2. The Solow difference equation in per capita terms
3. Solow diagram
4. The steady state—understanding the determinants
5. Growth in the Solow model—away from steady state
6. Growth in the Solow model—technological progress

Solow Model

- Similar to Harrod-Domar
- Adds:
  - Labor
  - More sophisticated production
    - Decreasing marginal product in any one factor
      - Add more labor, the next unit of labor is able to produce less than the one before it
      - Add more capital, the next unit of capital is able to produce less than the one before it
- Declining marginal product has very important consequences for conclusions about growth and policy.

Neoclassical Production Function

$Y = F(K, L)$

Output is a function of capital and labor

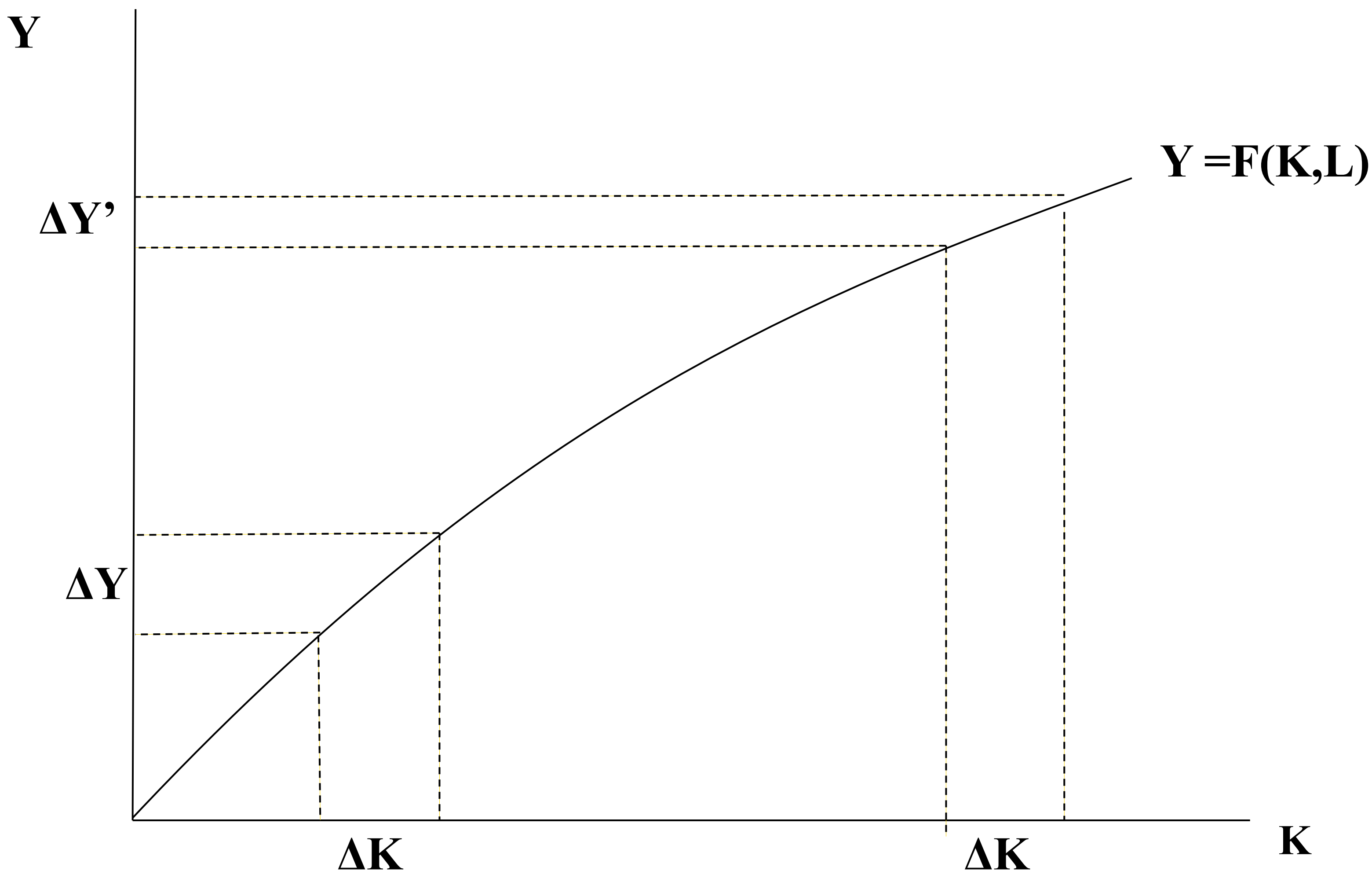
For each input:

Y is increasing with the input

$dF/dL > 0$ $dF/dK > 0$

Marginal product is decreasing

$d^2F/dL^2 < 0$ $d^2F/dK^2 < 0$



Y
ΔY'
ΔY
Y =F(K,L)
ΔK
ΔK
K

Cobb-Douglas Function

$$Y = F(K, L) = A\ K^{\alpha}L^{1-\alpha}$$

$$dF/dK = \alpha\ A\ K^{\alpha-1}L^{1-\alpha} = \alpha\ A\ (K^{\alpha-1})/(L^{\alpha-1})$$
$$= \alpha\ A\ (K/L)^{\alpha-1} = MPK$$

Per Capita

$$y = Y/L = A\ K^{\alpha}L^{1-\alpha}/L = A\ K^{\alpha}L^{-\alpha}$$
$$= A\ (K/L)^{\alpha} = A\ k^{\alpha}$$

Solow model assumptions

1. Production $Y = A\,K^{\alpha}L^{1-\alpha} = F(K, L)$

   $Y/L = y = A\,k^{\alpha} = f(k)$

2. Households save a constant fraction of income

   $S = sY$

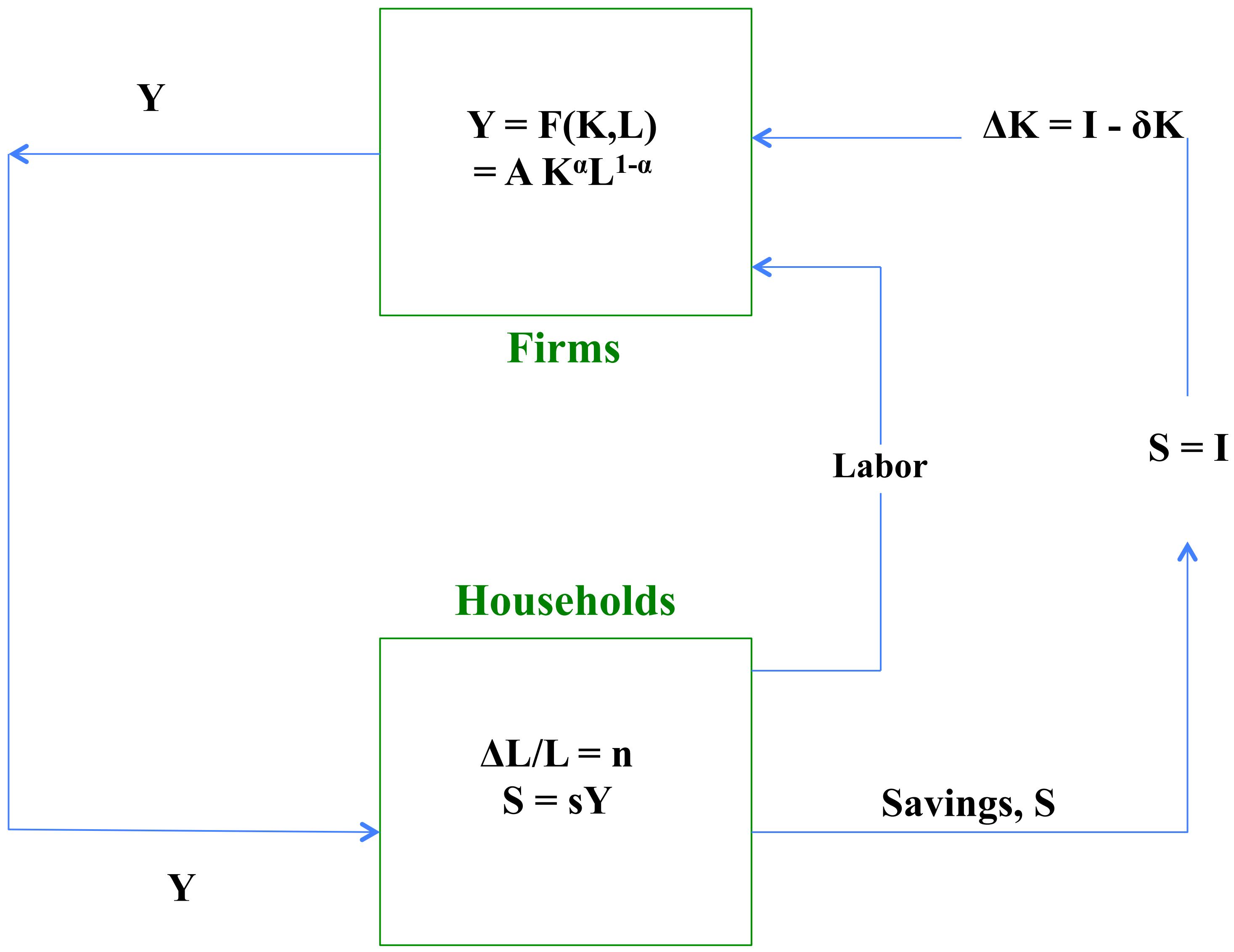
3. Savings equals investment

   $S = I$

4. $\Delta K = I - \delta\,K$
5. Labor is growing at rate n

   $\Delta L/L = n$

Solow model flow



The Solow difference equation

Find Δk in terms of

("states") k, y s, δ, α, n (parameters)

First use chain rule for Δk

$$K = k\,L \quad \rightarrow \quad \Delta K = \Delta k\,L + \Delta L\,k$$

Combine terms:

$y = A\,k^{\alpha}$, $S = I$, $\Delta K = I - \delta K$, $S = sY$, $\Delta L/L = n$

$$\Delta K = sY - \delta\,K \quad \rightarrow \quad \Delta K/L = sy - \delta\,k$$

The Solow difference equation

$\Delta K/L = sy - \delta\, k$ $\quad \Delta K = \Delta k\, L + \Delta L\, k$ $\quad \Delta L/L = n$

$(\Delta k\, L + \Delta L\, k)/L = sy - \delta\, k \rightarrow$

$\Delta k + \Delta L/L\, k = sy - \delta\, k \rightarrow$

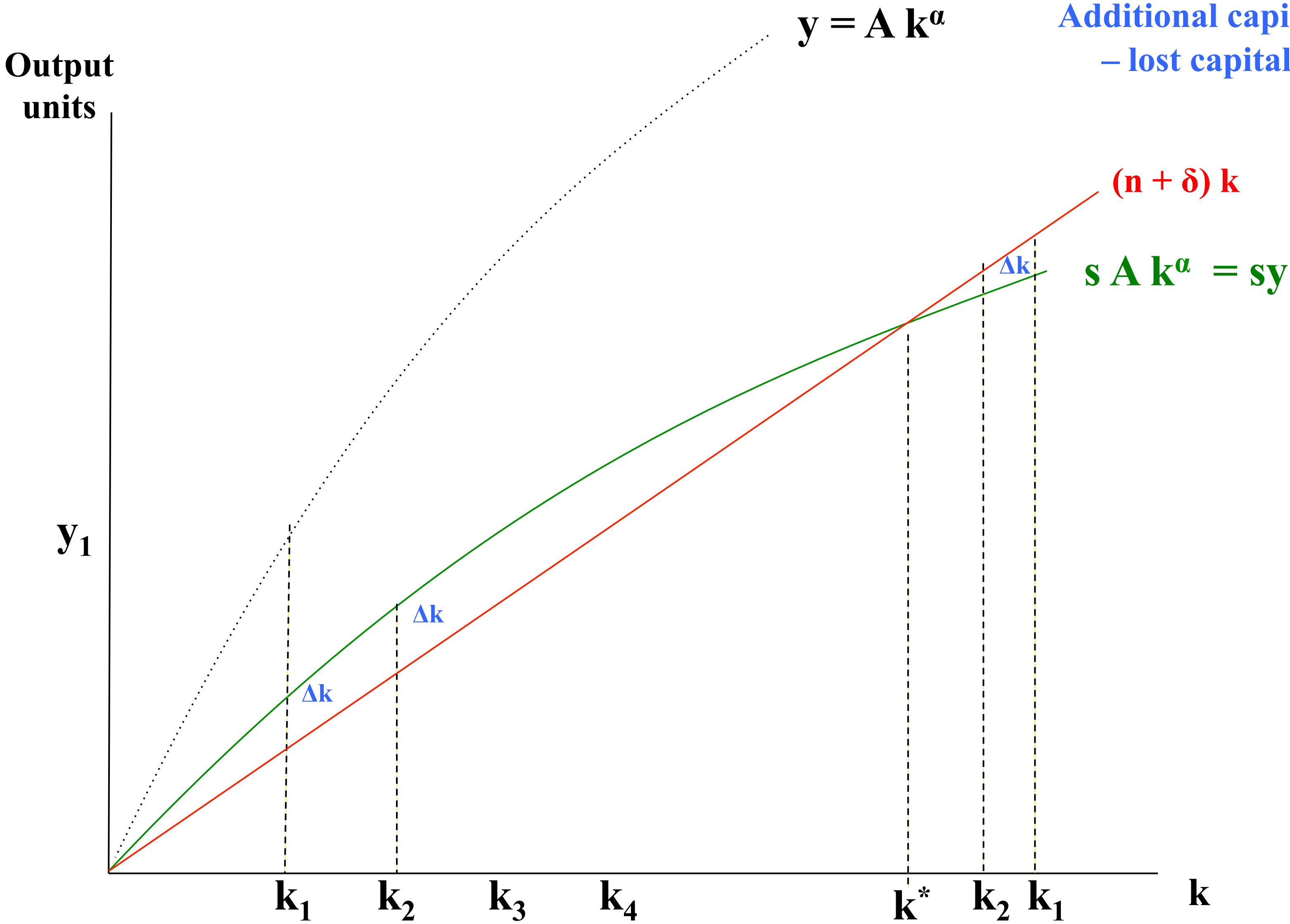
$$\Delta k = sy - \delta k - nk$$

$$\Delta k = sy - (\delta + n)k = s\, A\, k^{\alpha} - (\delta + n)k$$

The Solow Diagram

$$\Delta k = s\,A\,k^{\alpha} - (n + \delta)\,k$$

Additional capital / person
– lost capital / person



Solve for Steady State. Growth?

Solve for steady state $\Delta k = 0$ (if $\Delta k = 0 \rightarrow \Delta y = 0$)

$$\Delta k = 0 = s\,A\,k^{\alpha} - (n + \delta)\,k \rightarrow$$

$$(n + \delta)\,k = s\,A\,k^{\alpha}$$

$$(n + \delta) / s\,A = k^{\alpha-1}$$

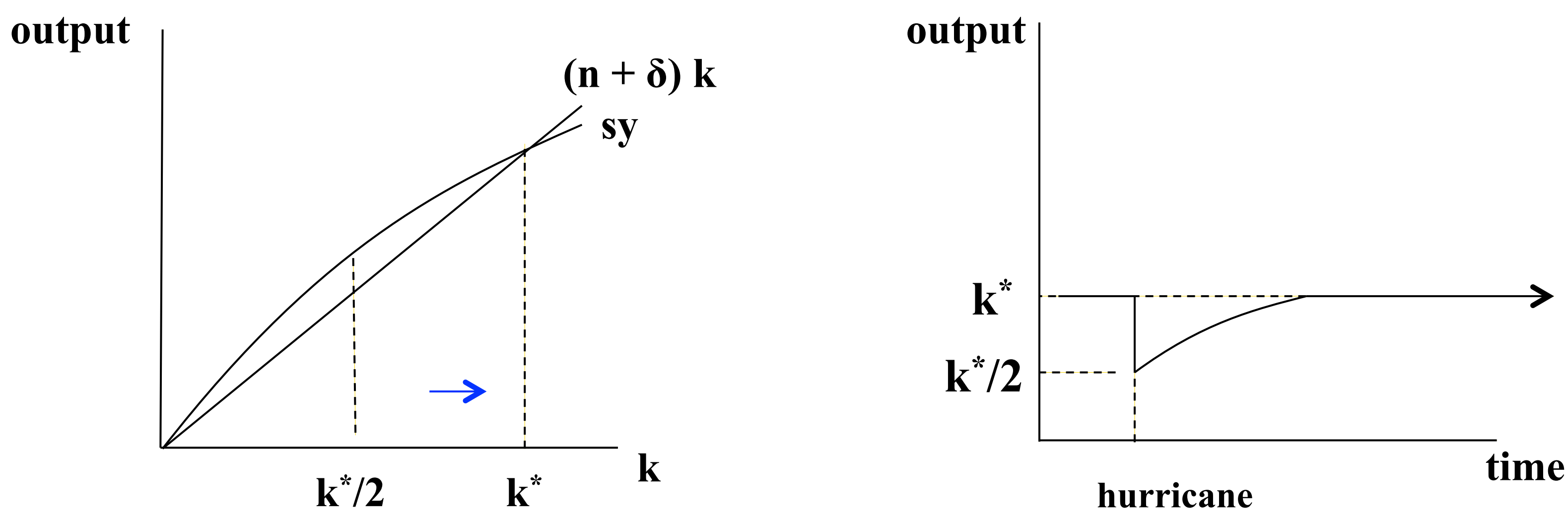
$$k^{*} = [(n + \delta) / s\,A]^{1/(\alpha-1)} = [s\,A / (n + \delta)]^{1/(1-\alpha)}$$

What is y^{*}? $y^{*} = A(k^{*})^{\alpha}$

No long term growth!

Example: Hurricane destroys capital

Growth away from steady state



Suppose hurricane destroys half of capital

Determinants of steady state

$k^* = [s\ A\ /\ (\delta + n)]^{1/(1-\alpha)}$ $\qquad$ $y^* = A\ (k^*)^\alpha$

expect α to be between 0 and 1

when will y^* and k^* be higher?

s high, A high

when will y^* be low?

n high, δ high

does $\Delta y/y$ change with s between two steady states?

No! $\Delta y/y = 0$ in steady state

Compare to Harrod-Domar

In H-D $\quad a = K/Y \quad$ constant capital output ratio

In Solow in steady state:

$$k^* / y^* = k^* / A (k^*)^\alpha = (1/A)(k^*)^{1-\alpha}$$

$$= (1/A) [(sA/(\delta + n))^{1/(1-\alpha)}]^{1-\alpha}$$

$$= s / (\delta + n)$$

If $\delta = 5\%$, $n = 3\%$, $s = 24\%$

$$k^* / y^* = 24 / (5 + 3) = 3$$

Solow Model with technology growth

Define $Y = K^{\alpha} (A\ L)^{1-\alpha} = A^{1-\alpha} K^{\alpha} L^{1-\alpha}$

consider A L “effective units of labor”

define $\underline{k} = K / A\ L$ $\underline{y} = Y / A\ L$

capital, output per effective labor

$Y / A\ L = \underline{y} = K^{\alpha} (A\ L)^{1-\alpha} / A\ L = K^{\alpha} (A\ L)^{-\alpha}$

$= (K / A\ L)^{\alpha} = \underline{k}^{\alpha}$

$\Delta A/A = g$ technology grows at rate g

Chain Rule

$K = A\ L\ \underline{k} \rightarrow \Delta K = \Delta A\ L\ \underline{k} + A\ \Delta L\ \underline{k} + A\ L\ \Delta \underline{k}$

Solow Model with technology growth

Combine with lots of algebra …

$$\Delta\underline{k} = s\,\underline{k}^{\alpha} - (\delta + n + g)\,\underline{k}$$

Steady state $\Delta\underline{k} = 0 \rightarrow \underline{k}^{*} = [s/(\delta + n + g)]^{1/(1-\alpha)}$

Find k: $k = A\,\underline{k}$ → so in steady state for $\underline{k}^{*}$

$$k = A\,[s/(\delta + n + g)]^{1/(1-\alpha)}$$

so k is growing because A is growing (rate of A)

Now we have **growth**!