Society of Decision Professionals
November 14, 2012

Five Rules – The Theoretical Foundation of Decision Analysis

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Decision theory is analogous to Euclidean geometry in that it is founded on a small set of rules, or “axioms”.

- The theory of good decision-making rests on just five rules of rational behavior.
  - Whereas the axioms (i.e., self-evident truths) of geometry are statements about the external world, the rules of decision making are statements about how the decision maker chooses to behave.

- These rules satisfy the requirements we would want for any set of fundamental rules (such as the rules of arithmetic).
  - They are correct when applied to simple situations
  - They remain valid when extended to more complicated situations

- The theory states that we must make decision choices in a certain way for our actions to be consistent with the rules.

- We call these the rules of *actional thought* – the thoughts we have about taking action.
Several somewhat different formulations of rules for decision making have been proposed.

- John von Neumann and Oskar Morgenstern stated four axioms of rational decision making in their 1944 ground-breaking work, *Theory of Games and Economic Behavior*.
  - Von Neumann and Morgenstern showed that the axioms imply the existence of a utility function and that decisions must be made to maximize expected (i.e., mean) utility to be consistent with the axioms.

- Leonard Savage in 1954 proposed an axiomatic theory of decision making that explicitly includes the use of subjective probabilities.

- In this talk, we use the formulation of decision theory created by Professor Ronald Howard of Stanford University. This formulation is based on five rules of decision making behavior.

- The rules apply to both personal and organizational decision making.
Rule 1: The Probability Rule

- The rule states: I can describe any decision alternative as a set of possibilities, to each of which I can assign a distinct probability.
  - A decision alternative is a “deal” that can be expressed as a probability tree.

- Because I cannot change the past, the only consequences of interest to me in any decision are those occurring in the future.
  - I choose to avoid “sunk cost” thinking

- Saying “I have no probability for that event” violates this rule.
  - It is OK to say “I have to think hard to get the probability of that event.”
For example, consider how we might apply the Probability Rule to describe an alternative in a vacation decision.

The distinctions of weather and level of campground crowdedness must be defined to pass the Clarity Test.

The probabilities assessed for the possibilities express our beliefs regarding the relative likelihood of occurrence.

The preference ordering of the various prospects is the concern of the next rule.
Rule 2: The Order Rule

- The rule states: I can arrange any set of prospects in a list from best to worst according to my preferences.
  - A prospect is defined as my future life assuming that I have selected an alternative in a decision and that a specific combination of the relevant possibilities have occurred.
- Prospects may be tied in order of preference.
- If uncertainty in a prospect makes it difficult to rank in order, distinctions should be added so that the prospects can be ordered.
  - For example, I may have difficulty ranking a new dog and a new stereo because I am unsure how much care the dog will require.
  - In this case, I should add hours per week of care to the definition of the new dog prospect and introduce enough possibilities to be able to rank them.
For example, how would you put the following prospects in order of preference?

• Prospect A – Be invited to join the President and First Lady for a private dinner at the White House (all expenses paid).

• Prospect B – See your favorite sports team win a major championship.

• Prospect C – Receive a non-transferrable 50% discount on everything you buy at Starbucks, valid for one year.

• Prospect D – Receive a non-transferrable lifetime subscription to the National Geographic magazine.
The Order Rule keeps me from becoming a “money pump”.

- A prospect may not occupy two different positions in the preference order list. So if A is higher on the list than B, and B is higher than C, then A must be higher than (i.e., preferred to) C.

- Violating the Order Rule would allow someone to extract money from me endlessly.
  - Suppose that I own a VW, but I prefer a Volvo because it is safer. I would willingly pay money to someone to take the VW in exchange for the Volvo.
  - But then I prefer a Jaguar to the Volvo because it is more stylish. So I willingly pay money to someone to take the Volvo in exchange for the Jaguar.
  - But then I prefer the VW to the Jaguar because it is more reliable. So I willingly pay money to someone to take the Jaguar in exchange for the VW.
  - I have paid money three times and wind up with the same car! I could be taken around the cycle again and again.
Rule 3: The Equivalence Rule

• The rule states: For any three prospects A, B, and C such that I prefer A to B and B to C, I can specify a probability $p$ such that I would be indifferent between receiving B for sure or a deal with probability $p$ of receiving A and probability $(1-p)$ of receiving C.

• The probability $p$ that I specify to make the deal between A and C equal in preference to B for sure is called a “preference probability” because it expresses only my preferences and not my beliefs about future events. My preference probability might be quite different from the probability that expresses my belief about the likelihood that prospect A will occur.
For example, what probability $p$ makes you indifferent between owning and not owning the following deal?

Uncertain deal

- $A$: Cash prize of $500 (after-tax).
- $B$: Status quo = $p$
- $C$: Bad cold symptoms for one week

$1 - p$
Rule 4: The Substitution Rule

• The rule states: If I face a situation as described in the Equivalence Rule in which my probability of receiving prospect A happens to be equal to the preference probability that I have assessed, I will be indifferent between receiving the uncertain deal or the middle prospect B for sure.

• This rule is sometimes called the “Do you really mean it?” rule. It means that the preference probabilities that I assess can be treated as probabilities when analyzing my decision choices.
Rule 5: The Choice Rule

- The rule states: Given a choice between two deals offering different probabilities of receiving the same two prospects, I must choose the deal having the higher probability for the prospect that I prefer more.
- This rule is so transparent and self-evident that any violation of it would be considered nonsensical.
In this example, the Choice Rule states that I must choose Deal 1, assuming that I prefer $500 to having bad cold symptoms.

Deal 1
- 80% Cash prize of $500 (after-tax).
- 20% Bad cold symptoms for one week

Deal 2
- 60% Cash prize of $500 (after-tax).
- 40% Bad cold symptoms for one week
Any decision can be made by direct application of the Five Rules.

• It can be shown by construction that the Five Rules imply the existence of a metric (called “u-value”) such that the best alternative is the one with the highest probability-weighted average of that metric.

• However, direct application of the rules is not a practical method for helping decision making. Instead, we apply a more streamlined method (as we shall soon see).
Compliance with the Five Rules places some strong restrictions on decision making.

- The non-informational deletion of an alternative from the set to be considered for a decision cannot change the preference ordering of the remaining alternatives.
- A non-informational addition of a new alternative cannot change the rank ordering of the other alternatives.
- Since decisions can affect only the future, sunk costs cannot be a consideration in any decision.
- Possible future regret for what “might have been” cannot be a consideration in any decision.
Regret about what “might have been” is a thought that we need to discipline ourselves to disregard.

Suppose that you are plaintiff in a civil lawsuit that is just about to be decided in court. The defendant has offered to pay you a considerable sum to settle out of court. In deciding whether or not to accept the settlement, you should NOT consider the regret that you would feel if you refuse the settlement and then lose the case in court.
Mapping all prospects in a decision to a value measure is useful.

- The value measure must refer to a resource that is both *fungible* and *alienable*.
  - A resource is *fungible* if equal-valued quantities are freely exchangeable with each other.
  - A resource is *alienable* if ownership can be transferred from one person to another.

- Money is the most useful value measure, because each of us has a lifetime of experience in judging the value of prospects in monetary terms.

- Expressing all prospects in a decision in monetary terms increases the power of decision analysis.
  - We can use a streamlined method for finding the optimal alternative.
  - We can compare the values of alternatives.
  - We can determine if getting additional information is worth the cost.
The Order Rule means that we can map (nearly) any prospect to an equivalent monetary value.

Example non-monetary prospect

See your favorite sports team win a major championship

The Order Rule states that any set of prospects can be put in preference order. So, the non-monetary prospect must fit somewhere in the ordered list of monetary prospects. Its position in the list specifies its equivalent monetary value.
The mapping of monetary value to the u-value metric is called a “u-curve”.

The u-curve maps any monetary value to its corresponding u-value and vice versa.

The u-value scale is unit-less and is similar to temperature in that it can be multiplied and shifted by any constants while retaining its essential properties.
With a u-curve, we can easily find the best alternative in any decision and calculate its *certain equivalent*.

1. Use the u-curve to find the u-value of each monetary outcome.

2. Calculate the mean u-value for each alternative.

3. Use the u-curve to find the monetary certain equivalent for each alternative.
The shape of the u-curve depends on the decision maker’s attitudes toward risk-taking.

- The only restriction on the shape of the u-curve imposed by the Five Rules is that it must be non-decreasing in the value measure.
- Different shapes of u-curves express different preferences regarding risk-taking, what we call *risk attitude*.
- For most decision makers, the u-curve is concave downward, expressing some degree of *risk aversion* – a preference for a sure amount of value rather than an uncertain amount. For a risk averse person, the certain equivalent of any uncertain deal is always less than the monetary mean (i.e. expected value).
Many people find the ‘delta property’ to be an attractive additional rule to observe.

Suppose that a person has established a certain equivalent for a specific deal:

\[ \text{CE} = 50\% \times 50\% \times 10,000 + 50\% \times 50\% \times 0 = 4,000 \]

The delta property says that if a constant monetary amount $\Delta$ is added to each possible outcome of the deal, the certain equivalent of the deal increases by $\Delta$.

\[ \text{CE} = 50\% \times 50\% \times (10,000 + \Delta) + 50\% \times 50\% \times (0 + \Delta) = 4,000 + \Delta \]

The delta property is hard to argue against as long as delta is not too big.
Someone who accepts the delta property as a sixth rule has a u-curve of a specific form.

- The u-curve for someone who observes the delta property is either linear or exponential in form.
- Someone with a linear u-curve is risk-neutral. His certain equivalent for any deal is equal to the monetary mean. We say that he “plays the averages”.
- The exponential form of u-curve has only one parameter of importance, called “risk tolerance”.

The risk tolerance parameter specifies the person’s level of risk aversion.

Risk tolerance is in monetary units.

Small risk tolerances express high aversion to risk-taking. Large risk tolerances express low aversion to risk-taking. Infinite risk tolerance expresses risk neutrality.

Risk tolerance should not be interpreted as “the most I am willing to risk”.

Exponential u-curves at various risk tolerances

<table>
<thead>
<tr>
<th>Risk tol</th>
<th>u-value</th>
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<tbody>
<tr>
<td>25</td>
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<tr>
<td>50</td>
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<td>100</td>
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Monetary outcome
A decision maker’s risk tolerance can be assessed via one question.

For what value \( X \) does this deal have zero value?

\[
\begin{align*}
0.5 & \quad 0.5 \\
X & \quad -0.5X
\end{align*}
\]

The amount \( X \) in the deal above that makes the decision maker just indifferent between owning and not owning the deal is a very close approximation (within 4%) of that decision maker’s risk tolerance.
When assessing risk tolerance, beware of falling into the trap of the “zero illusion”.

- The “zero illusion” is the bias in which we are convinced that our current financial status quo is somehow very special and any negative departure from it is to be avoided.

- A more realistic view is that one’s total wealth goes up and down daily by amounts that are far larger than ‘pocket money’.
  - Value of retirement funds, 401K, etc.
  - Value of home and other fixed assets
  - Unexpected expenses

- So we should not think that any particular deal is the only threat to the status quo. It’s already in a state of flux.
To avoid the zero illusion, remember that any deal is incremental to one’s total wealth.

Any deal that is described as:

\[ \begin{align*} & 50\% & X \\ & 50\% & -\frac{1}{2}X \end{align*} \]

is actually the following deal:

\[ \begin{align*} & 50\% & \text{Current total wealth} + X \\ & 50\% & \text{Current total wealth} - \frac{1}{2}X \end{align*} \]
For example, would you willingly take the following deal?

- 50% Increase my net worth by 4%
- 50% Decrease my net worth by 2%
The certain equivalent diverges from the monetary mean as the size of the deal increases.

For this deal:

The difference between monetary mean and certain equivalent is called the “risk premium”.

Note that for deals that are “small”, the certain equivalent is almost the same as the monetary mean.
In practice, we often calculate the monetary mean as a first approximation of the certain equivalent.

- For any decision maker, there is a range of monetary outcomes for which he is essentially risk neutral.
- A useful rule of thumb is that one might as well “play the averages” (i.e., act as if risk neutral) for deals whose range of monetary outcomes (best to worst) is less than 5% of his risk tolerance.
- For deals whose range of outcomes is between 5% and 100% of risk tolerance, one should calculate certain equivalents using an exponential u-curve.
- For deals whose range of outcomes is bigger than the risk tolerance, one should think carefully whether the delta property applies.
When the delta property does not apply, the “one-switch” rule seems attractive.

• The delta property implies a constant level of risk aversion for any level of wealth. It is convenient to assume that the delta property holds for deals that are small enough.

• But as a person’s wealth increases substantially, we would expect that person’s risk aversion to decrease. So, as wealth increases, the preference ranking of deals can change.

• The “one-switch” rule states that the preference ranking of any pair of deals can switch positions at most once as wealth increases.

• There is only one form of u-curve that is consistent with the one-switch rule, called the “linear plus exponential”.
The Five Rules provide the foundation for calculating the certain equivalent of any uncertain deal.
When the delta property applies, the assessment of risk preference is simplified.